6-1. We can answer the questions posed in this problem if we find the intensity point-spread function. From Eqs. (6-4) and (6-5), we know that the intensity point-spread function of an incoherent system is the squared magnitude of the (properly scaled) Fourier transform of the exit pupil illumination. The amplitude transmittance of the exit pupil in this case can be written

$$t_A(x,y) = \mathrm{circ}\left(\frac{2r}{d}\right) \otimes [\delta(x - s/2, y) + \delta(x + s/2, y)]$$

where $r = \sqrt{x^2 + y^2}$. The Fourier transform of this expression is

$$\mathcal{F}\{t_A(x,y)\} = \pi \left(\frac{d}{2}\right)^2 2\frac{J_1(\pi d\rho)}{\pi d\rho} \times 2\cos(\pi s f_X),$$

where $\rho = \sqrt{f_X^2 + f_Y^2}$. Taking the squared magnitude of this expression, using the identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$, and introducing the scaling parameters appropriate for the optical Fourier transform, we obtain the following expression for the intensity point-spread function (under the assumption that the intensity of the wave at the exit pupil is unity):

$$I(u,v) = |h(u,v)|^2 = \frac{\pi^2 d^4}{16\lambda^2 z_i^2}\left[2\frac{J_1\left(\frac{\pi d\sqrt{u^2+v^2}}{\lambda z_i}\right)}{\frac{\pi d\sqrt{u^2+v^2}}{\lambda z_i}}\right]^2 \left[1 + \cos\left(\frac{2\pi s u}{\lambda z_i}\right)\right]^2.$$

We can now answer the specific questions of the problem:

(a) The spatial frequency of the fringe is clearly given by

$$f_0 = \frac{s}{\lambda z_i}.$$

Note that the fringe frequency increases as the separation between the two apertures increases.

(b) The envelope of the fringe pattern is seen to be an Airy pattern of the form

$$E(u,v) = \left[2\frac{J_1\left(\frac{\pi d\sqrt{u^2+v^2}}{\lambda z_i}\right)}{\frac{\pi d\sqrt{u^2+v^2}}{\lambda z_i}}\right]^2,$$

where the scaling factor preceding the Airy pattern has been neglected.

6-2. The physical quantities to follow are amplitudes in the case of a coherent system and intensities in the case of an incoherent system. $p(x,y)$ represents the (amplitude or intensity) point-spread function.

(a) A line excitation lying along the x axis would be represented by

$$o(x,y) = \delta(y).$$

The response to such an excitation would be

$$\begin{aligned} i_1(x,y) &= p(x,y) \otimes o(x,y) = p(x,y) \otimes \delta(y) \\ &= \iint_{-\infty}^{\infty} p(\xi,\eta)\, \delta(y-\eta)\, d\xi d\eta = \int_{-\infty}^{\infty} p(\xi,y)\, d\xi = l(y) \end{aligned}$$

(b) Consider a one-dimensional Fourier transform of the line-spread function:

$$\begin{aligned} \mathcal{F}\{l(y)\} &= \iint_{-\infty}^{\infty} p(\xi,y) \exp[-j2\pi f y]\, d\xi dy \\ &= \iint_{-\infty}^{\infty} p(\xi,y) \exp\left[-j2\pi((\xi f_X + y f_Y)\right]\, d\xi dy \; \Big|_{\substack{f_X=0 \\ f_Y=f}} = P(0,f). \end{aligned}$$

(c) The unit step function will be represented by

$$s(x,y) = \begin{cases} 0 & y < 0 \\ 1 & y > 1 \end{cases}.$$

Therefore the response of the system will be

$$i_2(x,y) = p(x,y) \otimes s(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{\infty} p\xi,\eta)\, d\xi d\eta = \int_{-\infty}^{x} l(\eta) d\eta$$

Thus

$$\text{step response} = \int_{-\infty}^{x} l(\eta) d\eta.$$

6-3. (a) The the f_X-axis and f_Y-axis sections of the OTF of a clear square pupil are already known to be identical triangle functions, dropping linearly to zero at frequency $2f_o = \frac{2w}{\lambda z_i}$ from value unity at the origin. Such a curve is included in part (a) of the figure. More interesting is the case with a central obscuration. We can calculate either the f_X section or the f_Y section, since they are identical. Note that the total area of the obscured pupil is $4w^2 - w^2 = 3w^2$, which must be used as a normalizing factor for the autocorrelation function. In calculating the autocorrelation function of the pupil, we shift one version of the pupil in the x direction with respect to the other version. As the shift takes place, the area of overlap drops from $3w^2$ with no shift, linearly to $3w^2/2$ at a shift of $f_0/2$. With further shift, the curve changes slope, dropping linearly to value w^2 at shift f_o. Continuing shift results in no change of overlap until the shift is $3f_o/2$, following which the curve falls linearly to zero at $2f_o$. Part (a) of the figure shows the properly normalized OTF that results.

(b) Suppose that the width of the stop is $2w - 2\epsilon$. The total clear area of the pupil become $4w^2 - (2w - 2\epsilon)^2 = 8w\epsilon - 4\epsilon^2 \approx 8w\epsilon$. As the two pupils are shifted, the overlap area quickly drops to $2(2w - \epsilon)\epsilon \approx 4w\epsilon$ after a shift of ϵ. The overlap then continues to drop linearly, but with a shallower slope, reaching value $4\epsilon^2$ for a shift of $2w - 2\epsilon$. Continued shifting results in a rapid linear *rise* in the overlap to a value of $2w\epsilon$ when the displacement is $2w - \epsilon$, following which it falls linearly to zero at displacement $2w$. After proper normalization, the resulting OTF is as shown in part (b) of the figure.

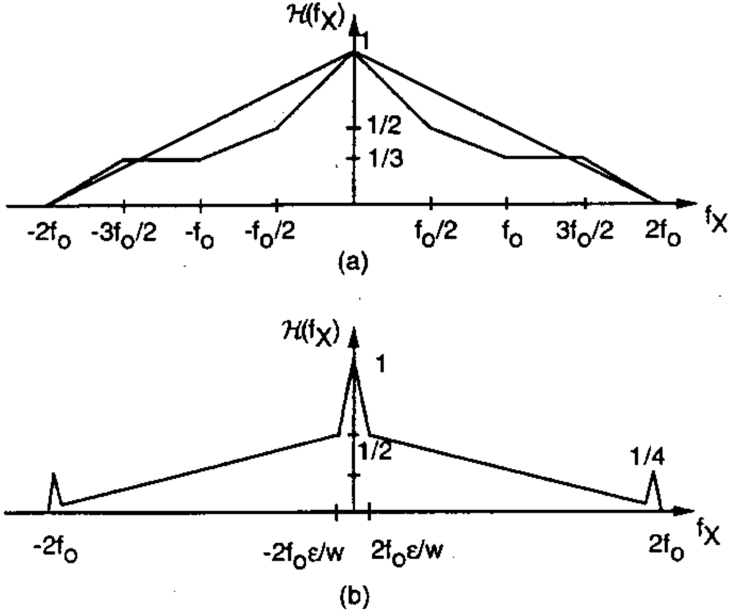

Figure 6-3: .

6-7. To find the OTF of this system under various assumptions, we first find the intensity point-spread functions under those conditions. If the object is a point source, then under the assumption that z_o is very large, we can assume that the pinhole is illuminated by a normally-incident plane wave.

(a) Under the assumption that geometrical optics can be used when the pinhole is large, the point-spread function is in this case simply a projection of the pupil function onto the image plane. Since the incident wave has been approximated as plane, the diameter of the circular spread function is the same as the diameter of the circular pupil. Thus the point-spread function is given by

$$s(u, v) = A \operatorname{circ}\left(\frac{r}{w}\right)$$

where A is an arbitrary constant, and $r = \sqrt{u^2 + v^2}$. The corresponding OTF is the normalized Fourier transform of $s(u, v)$, so

$$\mathcal{H}(\rho) = 2\frac{J_1(2\pi w\rho)}{2\pi w\rho},$$

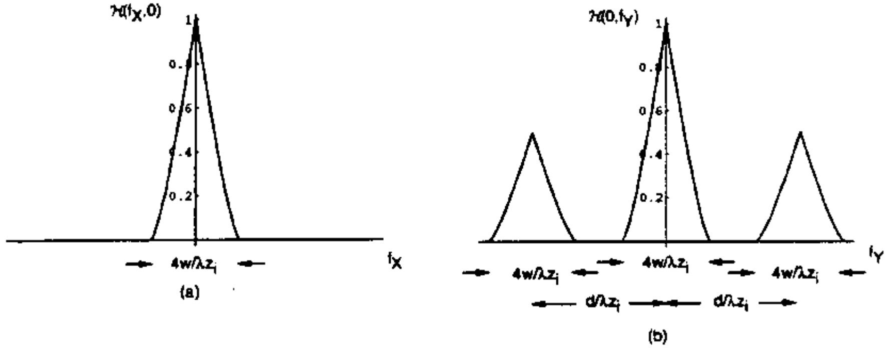

Figure 6-6: .

where $\rho = \sqrt{f_X^2 + f_Y^2}$. The first zero of this OTF occurs at

$$\rho_{o1} = \frac{0.61}{w}.$$

Note that the cutoff frequency *decreases* as the pinhole size increases.

(b) Now the pinhole is assumed to be so small that Fraunhofer diffraction occurs between the aperture and the image plane. The point-spread function of the system will now be the scaled optical Fourier transform of the circular aperture distribution, namely

$$s(u,v) = I_o \left[2\frac{J_1(2\pi wr)/\lambda z_i}{2\pi wr/\lambda z_i} \right]^2.$$

A scaled and normalized Fourier transform of this function yields the OTF

$$\mathcal{H}(\rho) = \frac{2}{\pi} \left[\cos^{-1}\left(\frac{\rho}{2w/\lambda z_i}\right) - \left(\frac{\rho}{2w/\lambda z_i}\right) \sqrt{1 - \left(\frac{\rho}{2w/\lambda z_i}\right)^2} \right]$$

which vanishes at

$$\rho_{o2} = \frac{2w}{\lambda z_i}.$$

Note that this cutoff frequency *increases* as the diameter of the pinhole increases.

(c) If we start with a large pinhole, geometrical optics will hold, and the cutoff frequency will increase as we make the pinhole smaller. However, eventually the pinhole size will be so small that geometrical optics does not hold, and eventually the Fraunhofer approximation will be valid. In this case the cutoff frequency will decrease as we make the pinhole smaller. A good approximation to the optimum choice of pinhole diameter can be found by equating the two expressions for cutoff frequency,

$$0.61/w = 2w/\lambda z_i,$$

yielding a solution for the radius w given by

$$w_{\text{optimum}} = \sqrt{0.305}\sqrt{\lambda z_i}.$$

This solution has chosen the smallest pinhole size possible before diffraction spreads the point-spread function appreciably.

6-17. The intensities in the two cases are as follows:

$$\begin{aligned} I &= |A+a|^2 = A^2 + 2Aa + a^2 && \text{coherent} \\ I &= A^2 + a^2 && \text{incoherent.} \end{aligned}$$

It follows that in the two cases

$$\begin{aligned} \frac{\Delta I}{|A|^2} &= \frac{2Aa + a^2}{A^2} && \text{coherent} \\ \frac{\Delta I}{|A|^2} &= \frac{a^2}{A^2} && \text{incoherent.} \end{aligned}$$

Since $A \gg a$, it is clear that the perturbation of the desired intensity is much greater in the case of coherent noise than in the case of incoherent noise.