PROBLEM SET 6

(Due Wed. Nov. 21 at 5PM in 508 Cory)

1.

a) Prove that aberrations can only decrease the modulation transfer function (MTF) of an imaging system. In other words:

$$|\mathcal{H}(f_x, f_y)|^2_{\text{with aberrations}} \leq |\mathcal{H}(f_x, f_y)|^2_{\text{diffraction limited}}$$

b) Show that the Strehl ratio, **S**, is given by the normalized volume under the OTF of the imaging system:

$$\mathbf{S} = \frac{\int\int_{-\infty}^{\infty} \mathcal{H}(f_x, f_y)_{\text{with aberrations}} df_x df_y}{\int\int_{-\infty}^{\infty} \mathcal{H}(f_x, f_y)_{\text{diffraction limited}} df_x df_y}$$

2. The gas mixture in a helium-neon discharge emits light at 633 nm with a Doppler broadened spectral width of about 1.5 x 10^9 Hz. Calculate the coherence time and the coherence length for this light. Repeat for the 488 nm line of the argon ion laser which has a Doppler broadened line width of about 7.5 x 10^9 Hz.

3. A 1 mm pinhole is placed immediately in front of an incoherent source. The light passed by the pinhole is to be used in a diffraction experiment, for which it is desired to illuminate a distant 1 mm aperture coherently. Given λ = 550 nm, calculate the minimum distance between the pinhole source and the diffracting aperture.

4. The sun subtends an angle of about 32 minutes of arc on earth. Assuming a mean wavelength of 550 nm, calculate the diameter of the coherence area of sunlight observed on earth (assume quasimonochromatic conditions).

5. Consider Young's interference experiment performed with *broadband* light.

a) Show that the field incident on the observing screen can be expressed as

$$u(Q, t) = K_1 \frac{d}{dt} u\left(P_1, t - \frac{r_1}{c}\right) + K_2 \frac{d}{dt} u\left(P_2, t - \frac{r_2}{c}\right)$$

where
where A_i is the area of the *i*th pinhole.
b) Using the result of part a), show that the intensity of the light striking the screen can be

$$K_i \cong \int\int_{P_i} \frac{\cos(n, r_i)}{2\pi c r_i} ds \cong \frac{A_i \cos(n, r_i)}{2\pi c r_i} \qquad i = 1, 2$$

expressed as

$$I(Q) = I^{(1)}(Q) + I^{(2)}(Q) - 2K_1 K_2 \mathrm{Re}\left\{ \frac{\partial^2}{\partial \tau^2} \Gamma_{12}\left(\frac{r_2 - r_1}{c} \right) \right\}$$

where

$$I^{(i)}(Q) = K_i^2 \left\langle \left| \frac{d}{dt} u\left(P_i, t - \frac{r_i}{c} \right) \right|^2 \right\rangle \qquad i = 1, 2$$

c) Show that the preceding expression reduces to the one given in the class notes for narrow-band light.

6. Prove that the image intensity, $I_i(u,v)$, can be directly obtained from the 4D Fourier spectrum of the mutual intensity, $\mathcal{J}_i(\nu_1,\nu_2,\nu_3,\nu_4)$ as:

$$I_i(u, v) = \iiiint_{-\infty}^{\infty} \mathcal{J}_i(\nu_1, \nu_2, \nu_3, \nu_4) \exp\{-j2\pi[u(\nu_1 + \nu_3) + v(\nu_2 + \nu_4)]\} d\nu_1 d\nu_2 d\nu_3 d\nu_4$$

7. Prove that the image spectrum, $I_i(\nu_U, \nu_V)$ can be directly obtained from the 4D Fourier spectrum of the mutual intensity, $\mathcal{J}_i(\nu_1,\nu_2,\nu_3,\nu_4)$ as

$$I_i(\nu_U, \nu_V) = \iint_{-\infty}^{\infty} \mathcal{J}_i(\nu_1, \nu_2, \nu_U - \nu_1, \nu_V - \nu_2) d\nu_1 d\nu_2$$