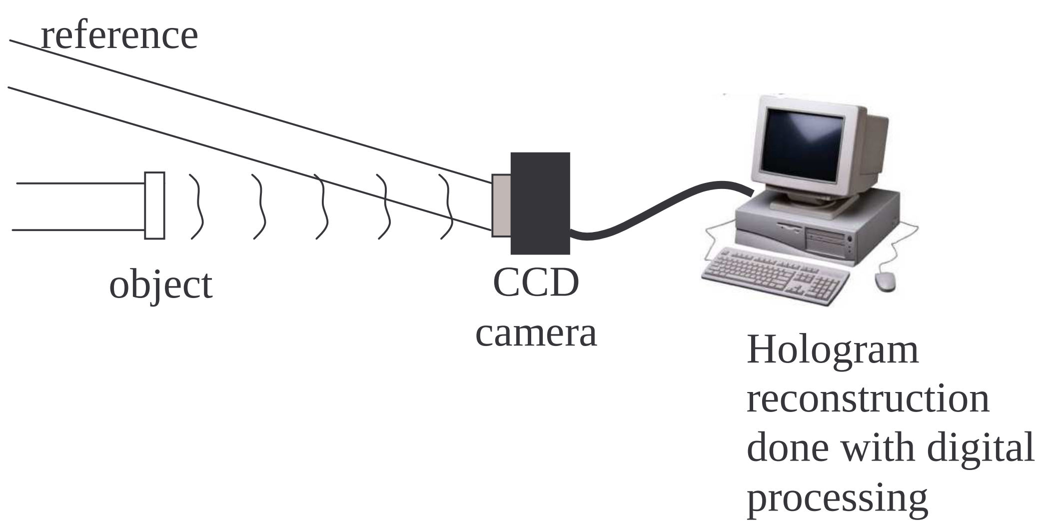
ELECTRONIC HOLOGRAPHY

CCD-camera replaces film as the recording medium.

Electronic holography is better suited than film-based holography to quantitative applications including:

- phase microscopy
- metrology
- optical signal processing



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CCD-Imposed Resolution Limit

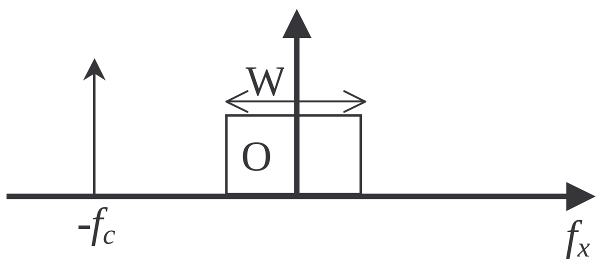
Consider spatial-frequency content at CCD plane (Fourier analysis)

Reference wave = $\delta(f_x - f_c)$

Object wave has double-sided bandwidth W, model its

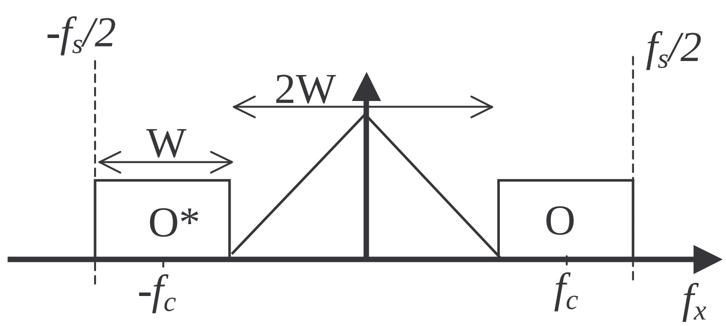
spectrum O as $\mathrm{rect}(f_x/W)$

Amplitude spectrum at CCD = $\delta(f_x + f_c) + \mathrm{rect}(f_x/W)$



Intensity spectrum at CCD = autocorrelation of amplitude spectrum

$[\delta(f_x+f_c) + \mathrm{rect}(f_x/W)] \otimes [\delta(f_x+f_c) + \mathrm{rect}(f_x/W)]^*$



f_s = CCD sampling rate [1/(pixel pitch)]

Requirements for f_c :

$f_c \geq 1.5W$ - to prevent zero-order overlap

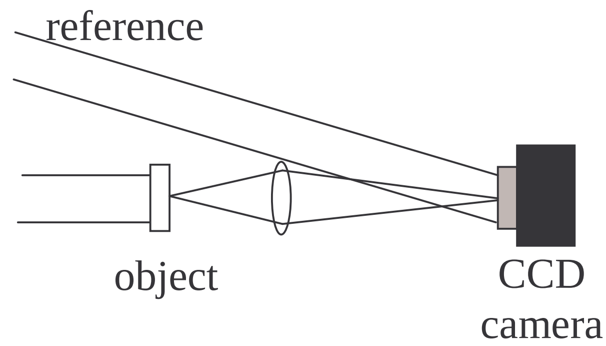
$(f_c + 0.5W) \leq f_s/2$ - to prevent aliasing errors

For strict adherence to term separability, object spectrum must be limited to ¼ of CCD bandwidth. i.e. only ¼ of CCD's pixels contribute to reconstructed image.

Reconstruction Process

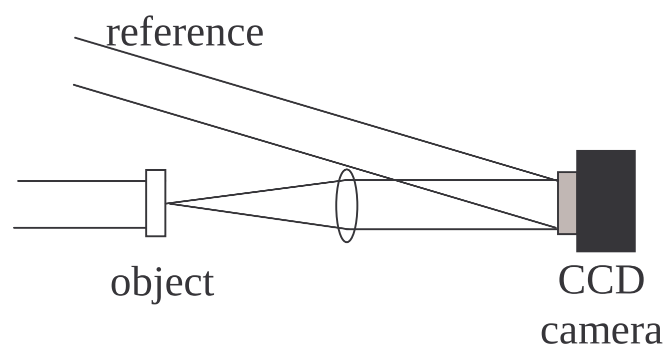
Image-plane hologram:

FFT -> Filter -> IFFT



Fraunhofer/Fourier hologram

FFT -> Filter



Fresnel hologram

FFT -> Filter -> Fresnel Transform

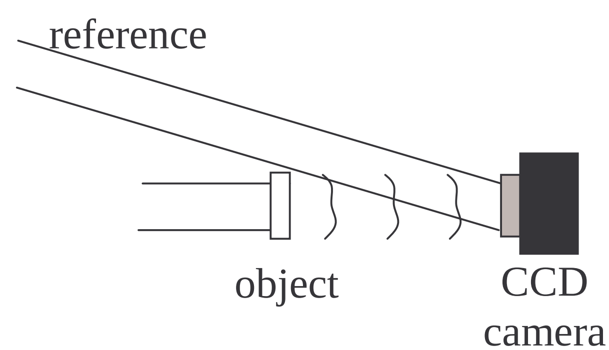
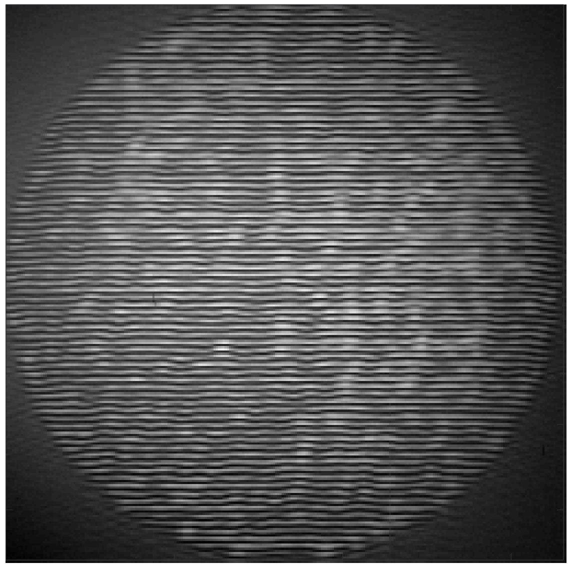
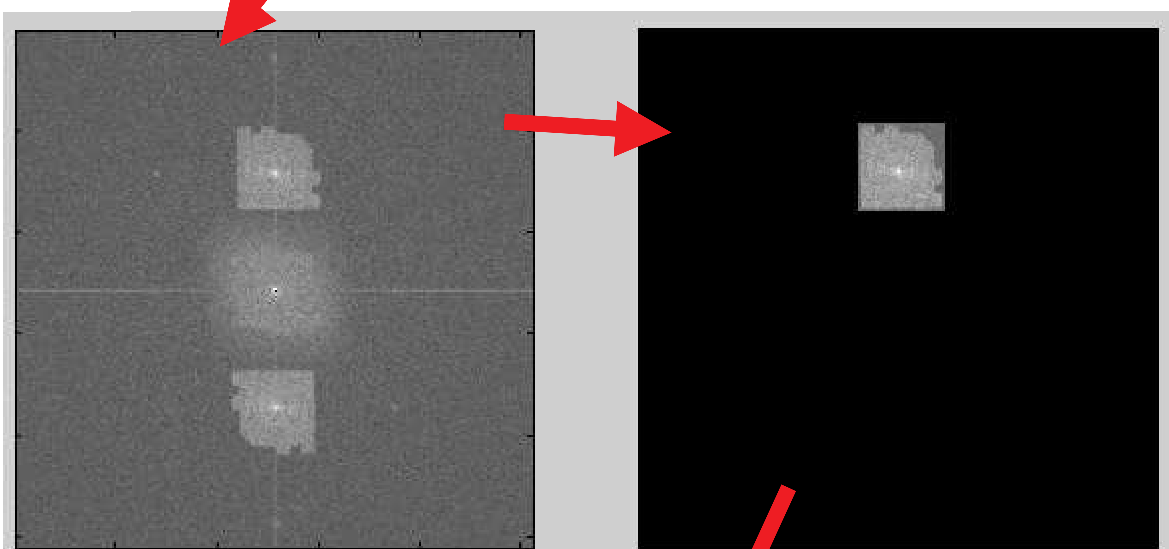


Image-plane hologram of optical system pupil

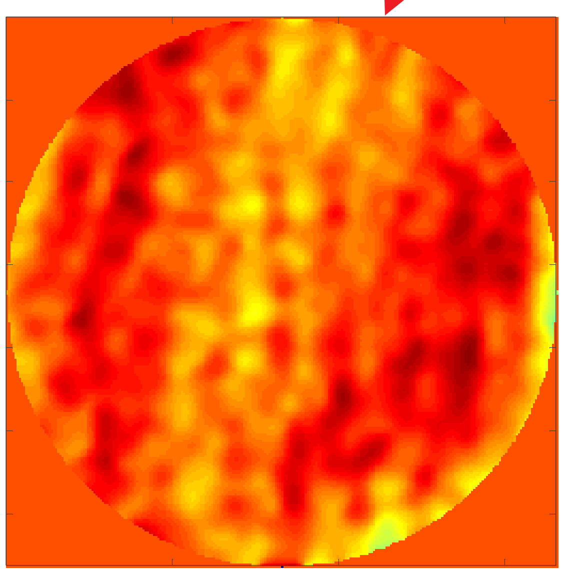


FFT

Filter



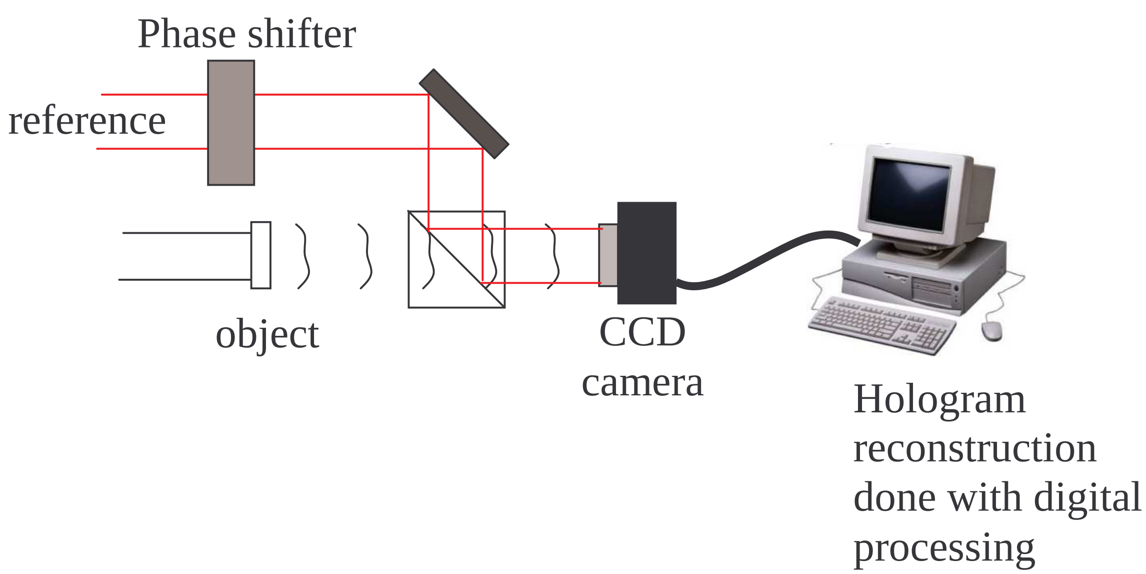
Phase after IFFT



Extending the CD-Imposed Resolution Limit

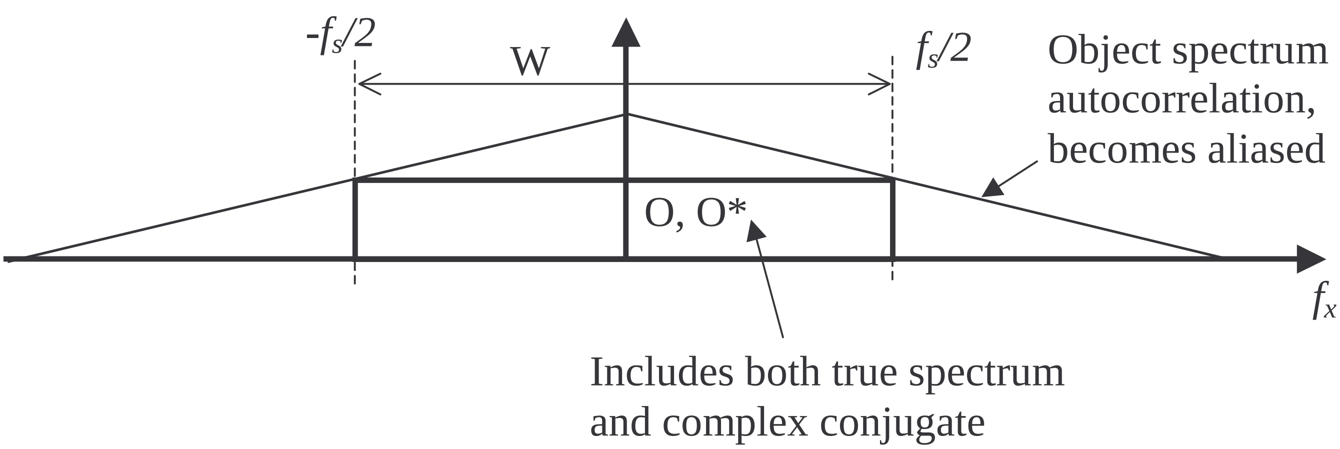
Use a temporal carrier instead of spatial (phase shift holography)

Time sequence of images required instead of just 1



In the spatial frequency domain signals become inseparable

$[\delta(f_x) + \mathrm{rect}(f_x/W)] \otimes [\delta(f_x) + \mathrm{rect}(f_x/W)]^*$

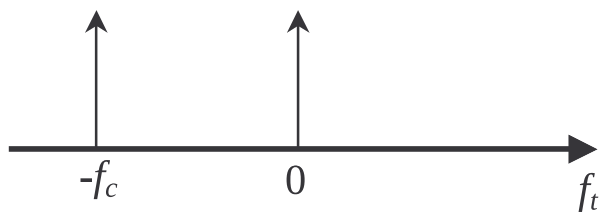


If we instead look in the temporal domain

Reference wave = $\delta(f_t - f_c)$

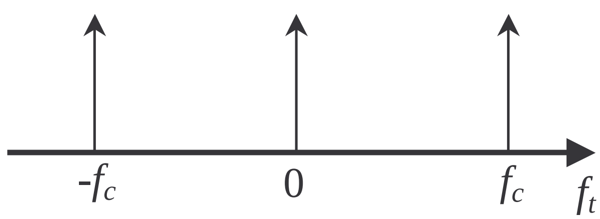
Object wave is stationary in time = $\delta(f_t)$

Temporal spectrum at a single pixel on CCD = $\delta(f_t + f_c) + \delta(f_t)$



Intensity spectrum at CCD = autocorrelation of amplitude spectrum

$$[\delta(f_t+f_c) + \delta(f_t)] \otimes [\delta(f_t+f_c) + \delta(f_t)]^*$$



Time varying interference creates a sinusoidal intensity in time

Temporal filtering used to extract the amplitude and phase of the sinusoid, which is directly proportional to the amplitude and phase of the object beam at given pixel.

Note that the object need not be stationary in time; hence real-time temporal holography is also possible.

COMPUTER-GENERATED HOLOGRAPHY

The holographic pattern of a desired optical field is calculated and then reproduced onto a physical device using, for example:

- Conventional printing
- Microelectronics patterning techniques (lithography)
- Spatial Light Modulators (SLM) (Goodman section 7.2)

Calculation process:

1) Calculate or generate desired field in hologram plane
2) Depending on encoding method, calculate actual holographic pattern, in this step we model both the interference (if any) and the recording steps. Possible final holographic pattern configurations include:
    - Pure amplitude (binary or grayscale)
    - Pure phase (binary or continuous)
    - Amplitude and phase

Required Bandwidth for Step 1:

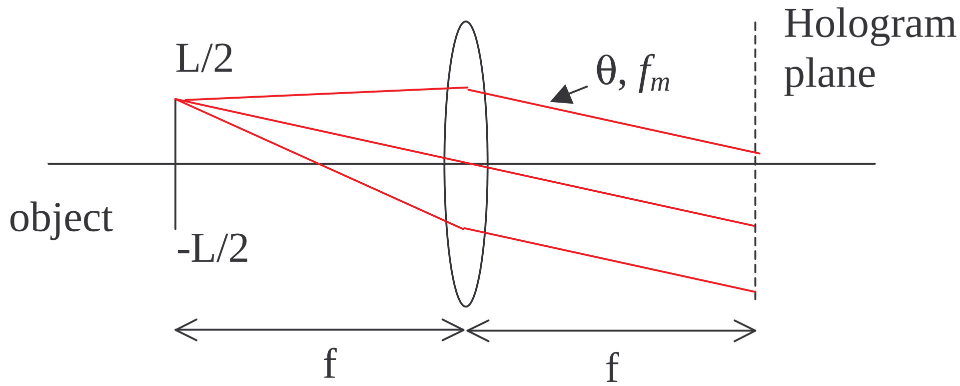
1) Image-plane hologram

Simply the bandwidth in the image itself.

No calculation of hologram-plane field required for “thin” object

2) Fourier hologram

Calculation of hologram-plane field involves Fourier transforming object



$f_x = \sin(\theta)/\lambda \sim \theta/\lambda$
$\tan(\theta) = L/(2f) \sim \theta$
$f_m = L/(2\lambda f)$
Full bandwidth (2B) = $2 f_m = L/(\lambda f)$

3) Fresnel hologram

Calculation of hologram-plane field involves Fresnel transforming object

$$U(x,y)=\frac{\exp(jkz)}{j\lambda z}\exp\left[\frac{jk}{2z}\left(x^2+y^2\right)\right]\times$$

Multiplicative quadratic phase term

$$\underbrace{\iint\left\{U(\xi,\eta)\exp\left[\frac{jk}{2z}\left(\xi^2+\eta^2\right)\right]\right\}\exp\left[\frac{-j2\pi}{\lambda z}\left(x\xi+y\eta\right)\right]d\xi d\eta}$$

FT of object times quadratic phase, bandwidth same as for FT of object alone (Fourier hologram case)

By the convolution theorem, multiplication in real space is equivalent to convolution in Fourier space, therefore, final bandwidth will be approximately the sum of the two individual bandwidth terms.

Bandwidth of first term set by derivative of phase (see section 2.2)

$f_x = 1/(2\pi)\ \delta/\delta_x[\pi x^2/(\lambda z)]$, $\quad k = 2\pi/\lambda$

$f_x = 1/(2\pi)\ 2\pi x/(\lambda z) = x/(\lambda z)$

$f_m = f_x$ evaluated at $x = L_h/2$

$f_m = L_h/(2\lambda z)$

$2B = (L_h + L)/(\lambda z)$, L = object size, L_h = hologram size

Encoding Methods:

1) Full amplitude and phase

    Simple calculation but difficult to implement.

    No additional calculation required beyond calculation of field.

    This method assumes that a device can be fabricated to directly reproduce the full complex field.

    This could conceptually be achieved through a series of lithography steps or cascaded phase and amplitude SLMs.

    This is rarely done in practice.

2) Pure amplitude grayscale encoding

    Analogous to intensity-detector based recording of hologram, requires grayscale fabrication or output capabilities.

    Process:

    a) Add desired reference (i.e. off-axis plane wave) to field calculated in previous section.
    b) Calculate intensity of reference + object field
    c) Fabricate grayscale device with calculated intensity pattern, or display pattern on grayscale amplitude SLM.

    Drawback: Complicated fabrication process.

3) Pure amplitude binary encoding

Binarized version of grayscale amplitude hologram, much simpler fabrication or display requirements.

Process:

a) Add desired reference (i.e. off-axis plane wave) to field calculated in previous section.
b) Calculate intensity of reference + object field.
c) Binarize calculated intensity.
d) Fabricate binary device with calculated intensity pattern, or display pattern on binary amplitude SLM.

Drawback: Amplitude information of recorded wavefront is lost. Note however, that amplitude information can be recovered through duty cycle modulation of binary pattern at the cost of calculation complexity.

4) Pure phase continuous encoding

Analogous to film-based “bleached” hologram, requires continuous phase fabrication or output capabilities. Preferable to (2) due to increased efficiency.

Process:

a) Add desired reference (i.e. off-axis plane wave) to field calculated in previous section.
b) Calculate intensity of reference + object field.
c) Convert normalized intensity to phase
d) Fabricate continuous phase device with calculated pattern, or display pattern on continous phase SLM.

Drawback: Complicated fabrication process.

5) Pure phase binary encoding

Binarized version of continous phase hologram, much simpler fabrication or display requirements. Preferable to (3) due to increased efficiency.

Process:

a) Add desired reference (i.e. off-axis plane wave) to field calculated in previous section.
b) Calculate intensity of reference + object field.
c) Binarize calculated intensity.
e) Convert binarized intensity to phase
d) Fabricate binary phase device with calculated pattern, or display pattern on binary phase SLM.

Drawback: Amplitude information of recorded wavefront is lost. Note however, that amplitude information can be recovered through duty cycle modulation of binary pattern at the cost of calculation complexity.

Synthesizing Phase-Only Holograms:

From the point of view of efficiency and ease of fabrication, the binary phase encoding method is preferred. However, in many cases we need to get around the problem of the lost amplitude information.

One particularly common application for CGHs is in the generation of a particular far-field intensity pattern for specialized illumination systems. In the special case where far-field phase is not of concern, it is possible to synthesize a phase-only hologram that generates precisely any arbitrary far-field intensity pattern using an iterative process (the Gerchberg/Saxon and Fienup methods)*.

Parameter definitions

Known:

1) Amplitude of the Fourier transform of the modulating function (the desired diffraction pattern)

2) Amplitude of the modulating function (unity because it is a phase-only device).

Unknown:

1) Phase of the modulating function (parameter to be determined)

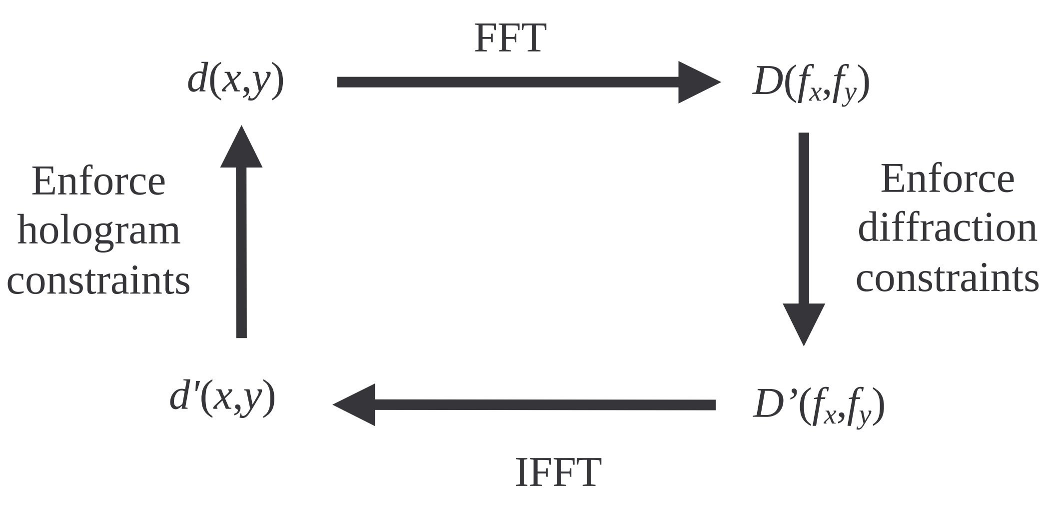
2) Phase of the Fourier transform of the modulating function (don't care)

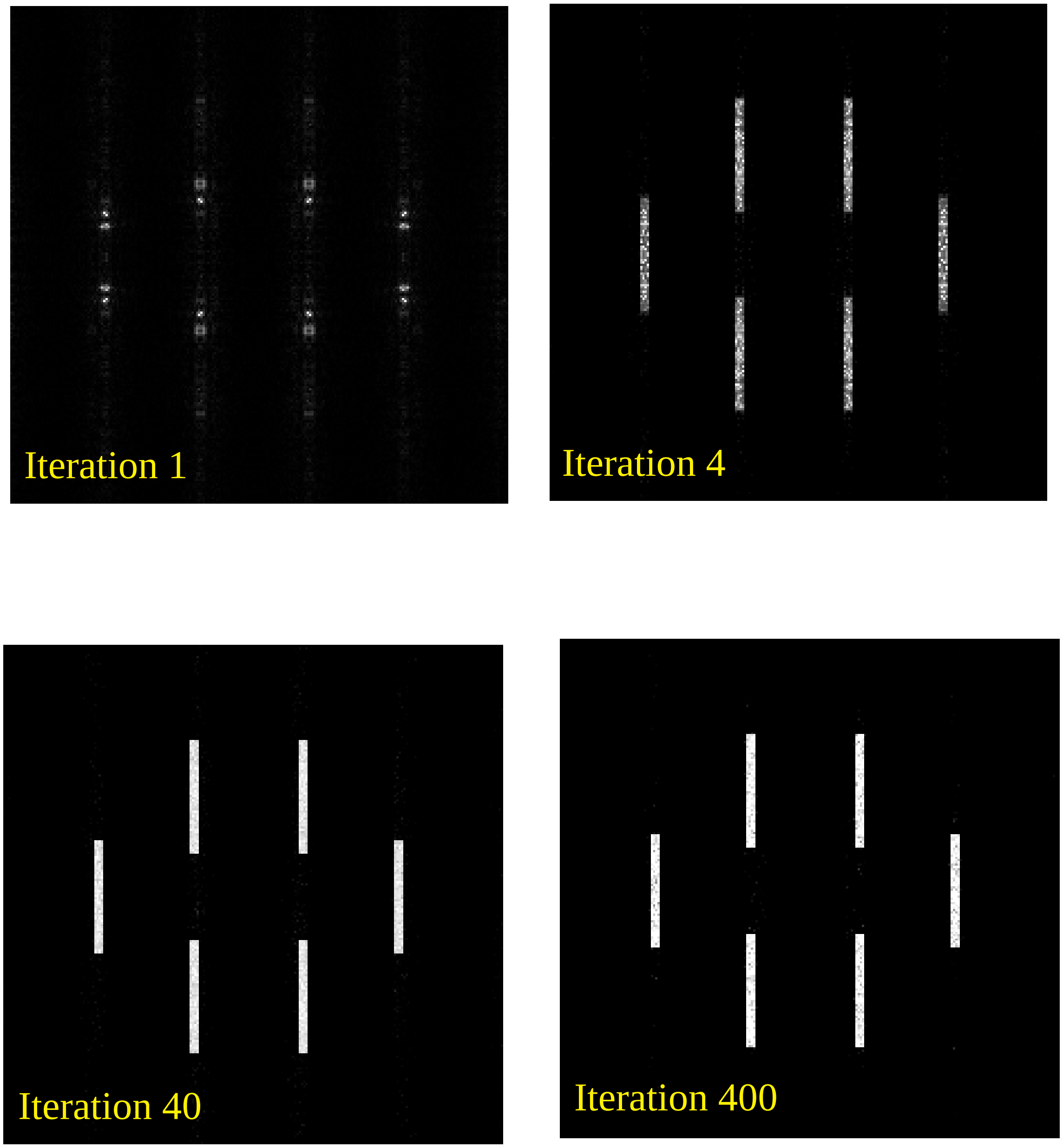
*R. Gerchberg and W. Saxon, "A practical algorithm for the determination of phase from image and diffraction plane pictures," Optik, **35**, 237-246 (1972).

J. Fienup, "Reconstruction of an object from the modulus of its Fourier transform," Opt. Lett., **3**, 27-29 (1978).

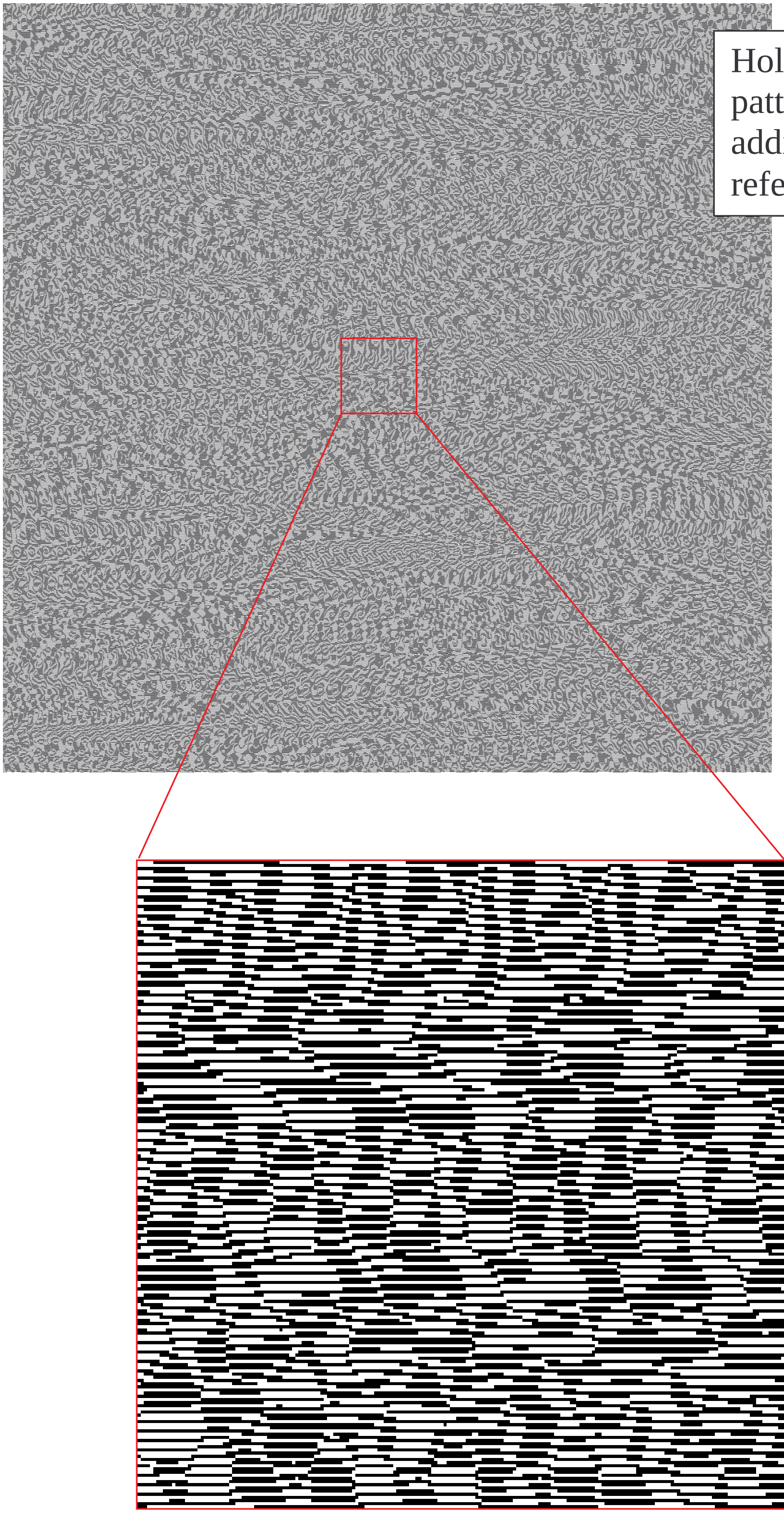
Iterative-method process (the error-reduction method):

1) Generate the desired magnitude diffraction pattern, $A(f_x,f_y)$
2) Generate the seed diffraction pattern for calculation process, $D(f_x,f_y)$ (most conveniently unity)
3) Enforce constraints on $D(f_x,f_y)$ [multiply by the desired magnitude diffraction pattern, $A(f_x,f_y)$] to get $D'(f_x,f_y)$
4) Inverse Fourier transform $D'(f_x,f_y)$ obtaining guess at the modulating signal, $d'(x,y)$
5) Enforce constraints on $d'(x,y)$ (force the amplitude to be unity) to get $d(x,y)$
6) Fourier transform $d(x,y)$ to generate resulting diffraction pattern, $D(f_x,f_y)$
7) Repeat steps 3 through 6 until the magnitude of the result of step 6 matches the desired magnitude diffraction pattern.
8) Now ready to calculate encoded CGH.

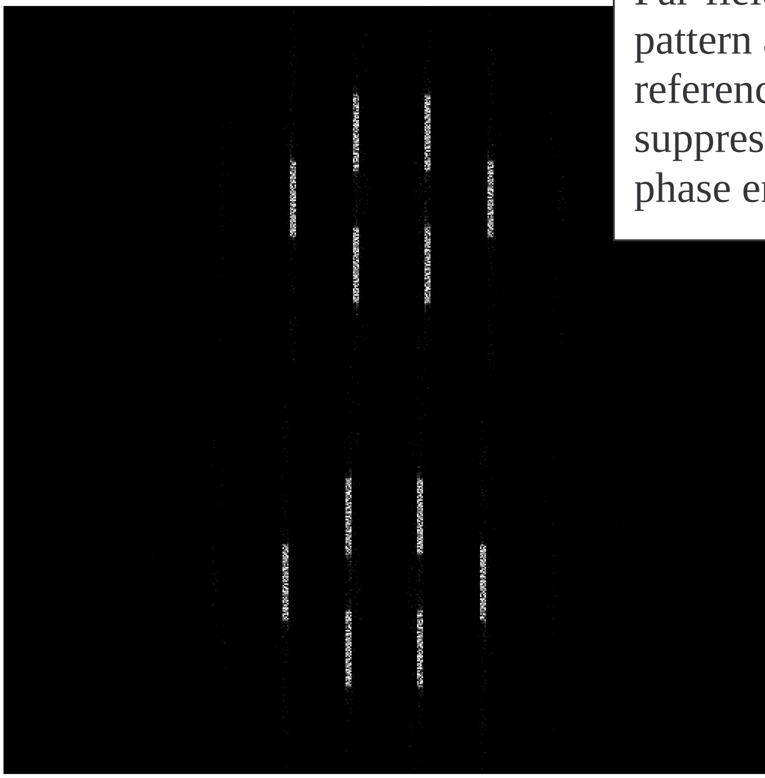




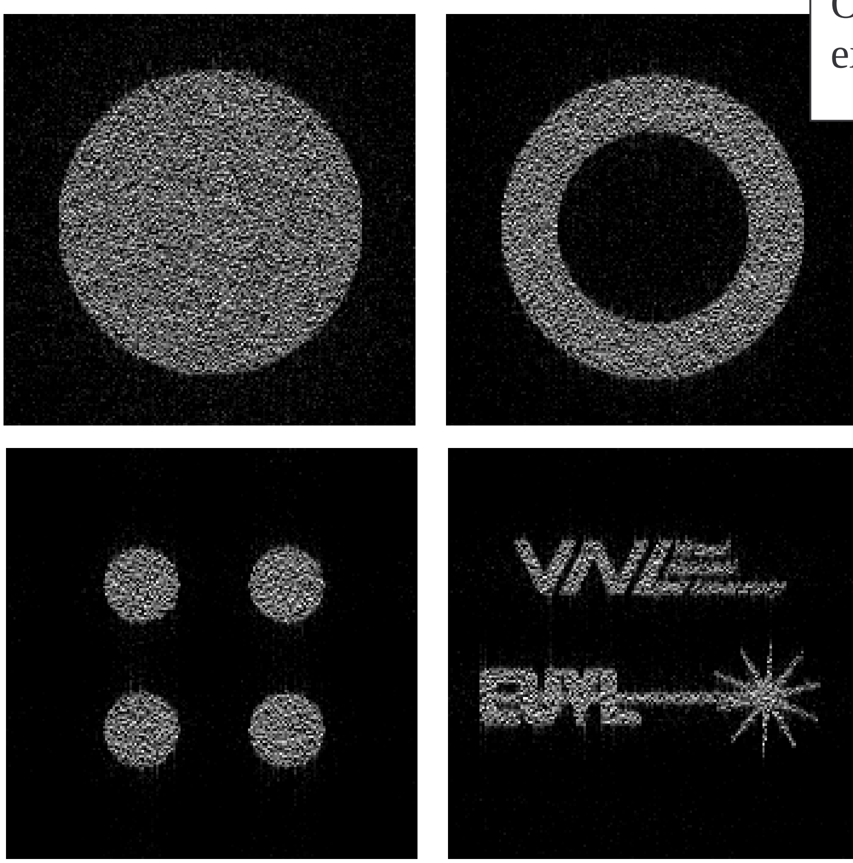
Iteration 1
Iteration 4
Iteration 40
Iteration 400



Hologram pattern after addition of reference.



Far-field diffraction pattern after adding reference. Zero-order is suppressed due to pure phase encoding.



Other examples.