Chapter 16
Holography

Read Goodman, *Fourier Optics,* Chapter 9

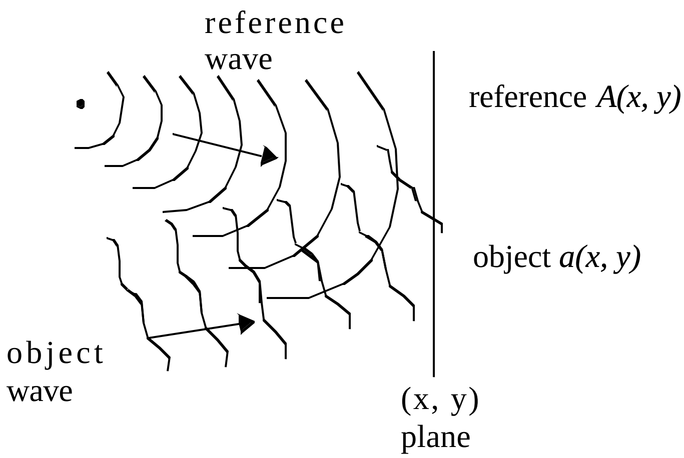
Virtually all recording devices for light respond to light intensity.

Problem: How to record, and then later reconstruct both the amplitude and phase of an optical wave. [This question sensibly arises only for highly coherent illumination.] The same issue can also be raised for acoustic and seismic waves.

The challenge is to figure out how to convert phase information to intensity.

Interferometry

Create a second wavefront with known amplitude and phase that is coherent with the wave to be recorded (the object wave). Add this to the object wave.



The intensity of the sum contains the interference pattern

$$\text{let} \quad a(x, y) = |a(x, y)| \exp[-j\phi(x, y)] \tag{16.1}$$

(16.2)

$$I_s(x, y) = |A(x, y)|^2 + |a(x, y)|^2 + 2|A(x, y)||a(x, y)| \cos[\psi(x, y) - \phi(x, y)] \tag{16.3}$$

A recording of this interference pattern is a hologram.

The recording medium for holography is typically some type of film emulsion. The transmission of the developed film can be linear in absorbed energy over a limited dynamic range. See Goodman, *Fourier Optics,* chapter 7.

Under these conditions, the transmittance of film can be written

(16.4)

Where we assume that $|A|^2$ is constant and uniform, which gives the bias t_b . β' is the sensitivity parameter of the film.

<u>Reconstruction:</u> illuminate transparency by reconstruction wave $B(x, y)$.

Transmitted light is: $B(x, y)t_A(x, y)$

$$= Bt_B + \beta'|a|^2 B + \beta' A^* Ba + \beta' ABa^* \qquad (16.5)$$

$$\equiv U_1 + U_2 + U_3 + U_4 \qquad (16.6)$$

If *B* is a duplicate of *A*, then *B* = *A*. Then:

. (16.7)

If $|A|^2$ is uniform, then U_3 is a duplication of *a*. We could also arrange that $B = A^*$. Then

(16.8)

the conjugate of the original wavefront

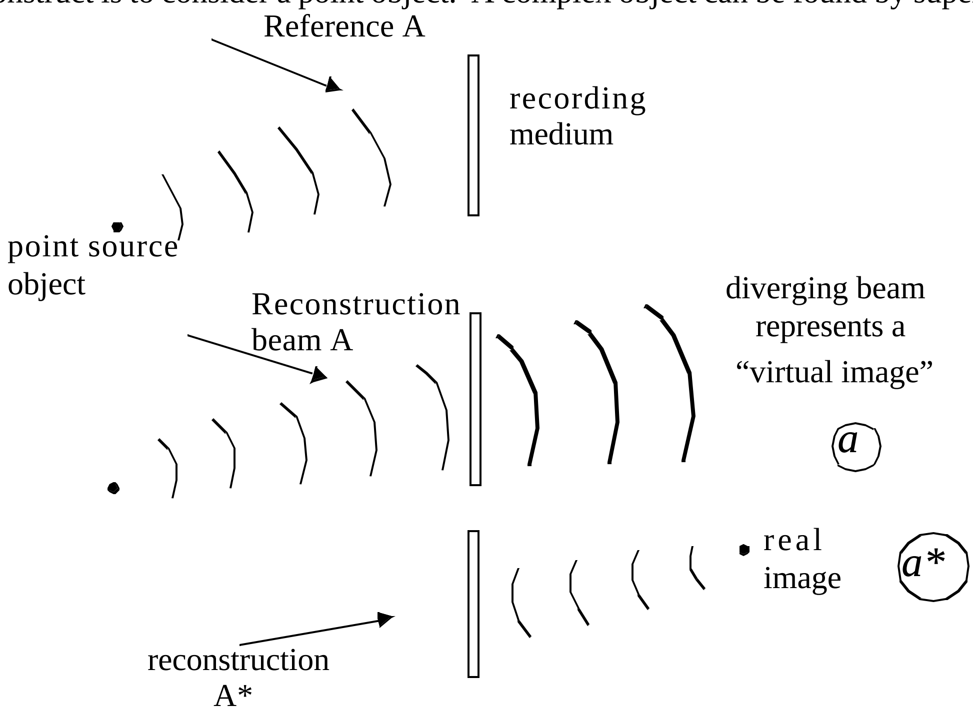
This process is a two dimensional analog of amplitude modulation.

Here we have three extraneous signals which lead to unwanted interference. If we want *a* or *a**, we have to filter out the other components.

Real and virtual images

A general principle in holography is linearity. We have a coherent system which is linear in field.

A useful construct is to consider a point object. A complex object can be found by superposition.



The point source object is

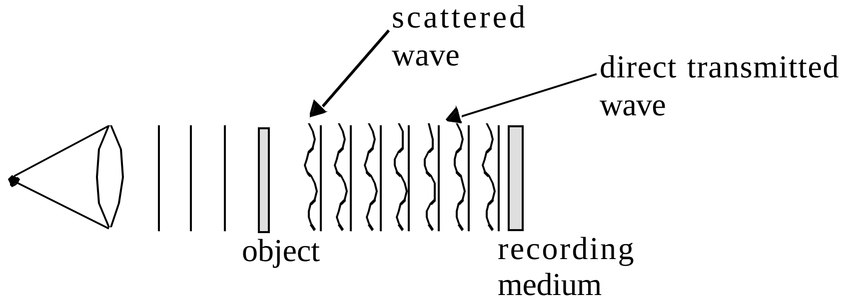
$$ \qquad (16.9)$$

where r_o is the position of the point source. When the A* reconstruction wave is used.

$$U_4(\vec{r}) = \beta'|A|^2 a^*(\vec{r}) = \beta'|A|^2 \frac{a_o^* e^{-jk(\vec{r} - \vec{r}_o)}}{|\vec{r} - \vec{r}_o|} \qquad (16.10)$$

We get a converging spherical wave toward point $(-\vec{r}_o)$. But we still have not specified how to exclude the three unwanted components.

Original Gabor Hologram (1948)



recording

object must be highly transmissive

$$\Delta t \ll t_o \quad (16.11)$$

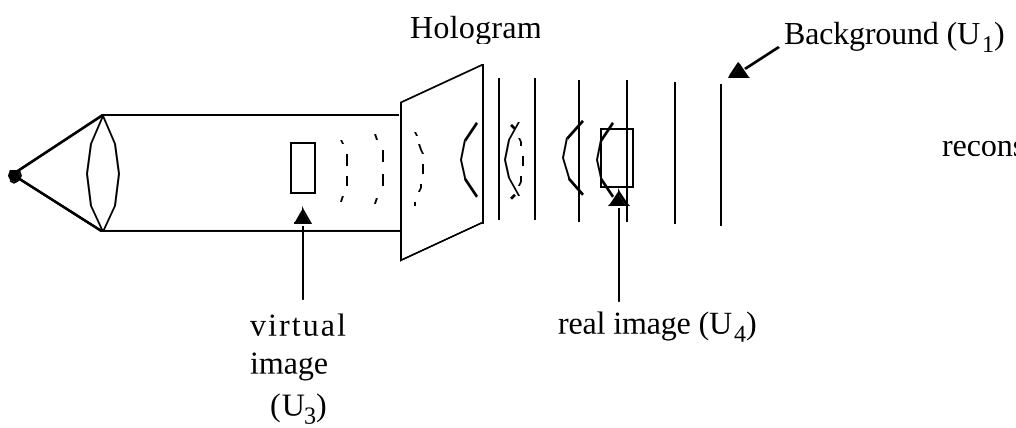
The reference wave is a plane wave which comes from the t_o component.

The object wave is scattered by the variations $\Delta t(x_o, y_o)$

The scattered wave is weak compared to the reference plane wave

(16.12)

Thus we can neglect the U_2 term.

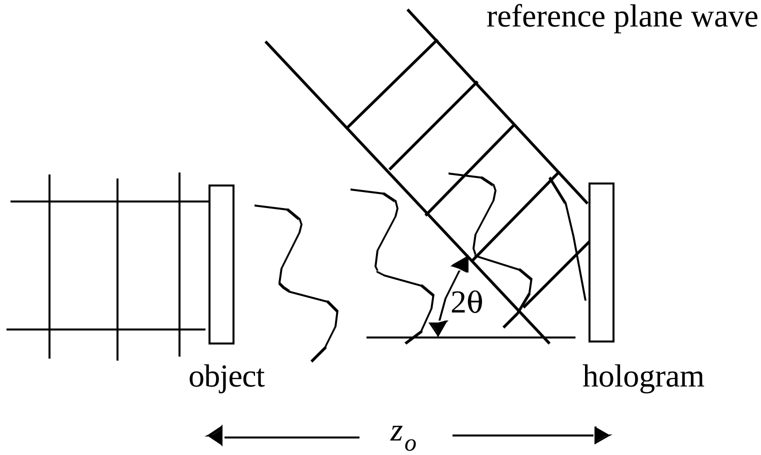


reconstruction

"twin" images

In a Gabor hologram there are three overlapping components, the <u>real image</u>, the <u>virtual image</u>, and the <u>background</u>.

Leith-Upatnieks (offset reference) hologram (1962)



The reference beam is tilted. This is a real “hero” experiment without a laser. Holography was made practical after the invention of the laser.

Now the field at the recording plate consists of a scattered wave from the object $a(x, y)$ plus the reference plane wave.

(16.13)

As usual, $k \equiv \frac{2\pi}{\lambda}$. The intensity at the plate is

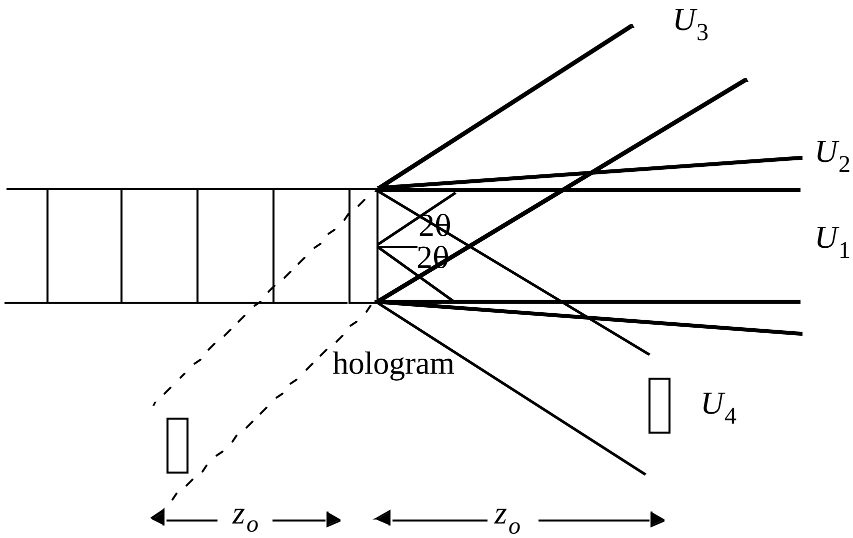
$$I(x, y) = \left|A^2\right| + |a(x, y)|^2 + A^* a(x, y) \exp(jky \sin 2\theta) + A a^*(x, y) \exp(-jky \sin 2\theta) \tag{16.14}$$

The developed film transmittance has four terms:

$$t_A = t_B + \beta'[|a(x, y)|^2 + A^* a(x, y) \exp(jky \sin 2\theta) + A a^*(x, y) \exp(-jky \sin 2\theta)] \tag{16.15}$$

$$\equiv t_1 + t_2 + t_3 + t_4 \tag{16.16}$$

Reconstruction:



The reconstruction beam is a plane wave at normal incidence, with amplitude B.

Four components in the transmitted wave:

(16.17)

(16.18)

$$U_3 = \beta' B A^* a(x, y) \exp(jky \sin 2\theta) \qquad (16.19)$$

$$U_4 = \beta' B A a^*(x, y) \exp(-jky \sin 2\theta) \qquad (16.20)$$

U_1: attenuated version of the reconstruction beam

U_2: scattered wave by object $|a(x, y)|^2$. It stays close to the axis.

U_3: original wave a, modulated by the exponential phase factor. This modulation causes deflection by the angle 2θ. Proportionality to a causes the real image to be formed at the distance z_o.

U_4: a^* is modulated. By a similar argument, we get the virtual image deflected by -2θ at the distance z_o.

The twin images (real and virtual) are now angularly separated from each other as well as from the background components of U_1 and U_2.

Angular separation of the four components

Each of the four components has an angular divergence (bandwidth) in the y - direction. Adequate separation requires a sufficiently large angle 2θ. What is the minimum angle? $2\theta_{min}$ is the minimum.

To deal with this question, we go to the Fourier domain, where we have a one-to-one correspondence between angle and spatial frequency.

U_3 and U_4 have a simple frequency offset.

We write

$$\exp(jk\sin 2\theta y) \equiv \exp(j2\pi f_{\alpha} y) \ . \qquad (16.21)$$

The frequency offset is

$$. \qquad (16.22)$$

Now we take Fourier transforms of the individual components in the scattered field:

$$\mathcal{F}[U_1(x, y)] \equiv G_1(f_x, f_y) \ = \ Bt_b\delta(f_x, f_y) \qquad (16.23)$$

which is simply a plane wave

$$\mathcal{F}[U_2(x, y)] \equiv G_2(f_x, f_y) \ = \ B\beta' G_a(f_x, f_y) \otimes G_a(f_x, f_y) \qquad (16.24)$$

where

$$(16.25)$$

which is the object spectrum.

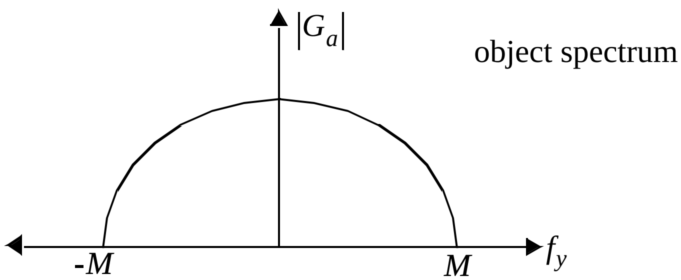
$$\mathcal{F}[U_3(x, y)] \equiv G_3(f_x, f_y) \ = \ B\beta' A G_a(f_x, f_y - f_{\alpha}) \qquad (16.26)$$

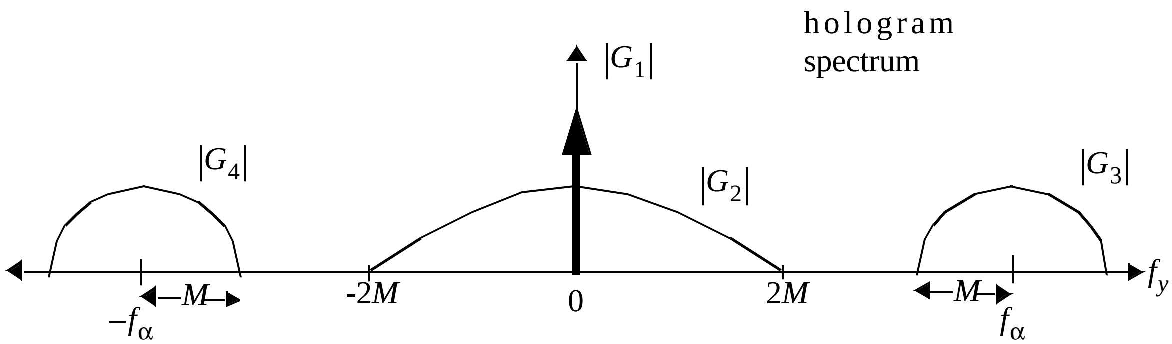
which gives a shifted object spectrum. Finally,

$$\mathcal{F}[U_4(x, y)] \equiv G_4(f_x, f_y) \ = \ B\beta' A G_a(-f_x, -f_y - f_{\alpha}) \qquad (16.27)$$

is also a shifted object spectrum.

Assume the object is band limited, with bandwidth M in the y direction.





For separation of all components, we need $f_\alpha > 3M$, or

(16.28)

However, if the recording reference wave *A*, is much stronger than the scattered object wave *a*,

$$|A| \gg |a| \quad (16.29)$$

then the $|G_2|$ component is very weak compared to G_3, G_4. Then a smaller offset is tolerable:

(16.30)