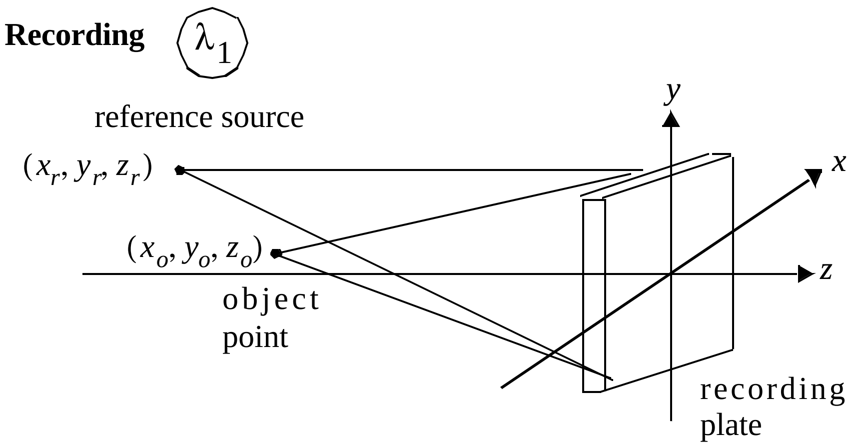
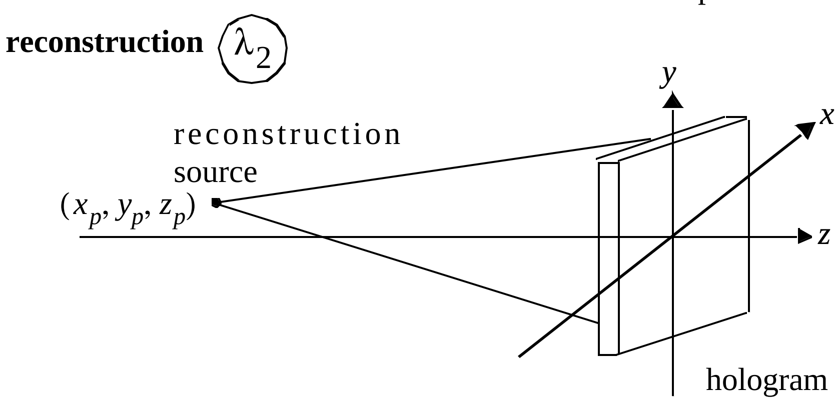
Chapter 17
Holography – Part 2

Generalized treatment

More commonly the reference and reconstruction waves are generated by point sources. (In practice, this is a pinhole spatial filter.) Also, the reconstruction wavelength can differ from the reference wavelength.



It is sufficient to consider one object point. We obtain the result for a general object by superposition



The z coordinate is negative for a point source to the left of the recording plane (diverging wave).

We will work in the paraxial approximation (quadratic phase for spherical waves).

The total field at the recording plate:

$$U(x, y) = A\exp\left\{-j\frac{\pi}{\lambda z_r}[(x-x_r)^2 + (y-y_r)^2]\right\} + a\exp\left\{-j\frac{\pi}{\lambda_1 z_o}[(x-x_r)^2 + (y-y_r)^2]\right\} \quad (17.1)$$

The amplitude transmittance of the developed plate is proportional to its *Intensity.* Again we get four components $t_1 + t_2 + t_3 + t_4$. Assuming an adequate angular separation, t_1, t_2 are unimportant $(|A|^2, |a|^2)$.

$$t_3 = \beta' A a^* \exp\left\{-j\pi\left[\frac{(x-x_r)^2 + (y-y_r)^2}{\lambda_1 z_r} - \frac{(x-x_o)^2 + (y-y_r)^2}{\lambda_1 z_o}\right]\right\} \quad (17.2)$$

t_4 = complex conjugate

<u>Reconstruction:</u> illuminate the plate by a spherical wave:

$$U_p = B\exp\left\{-j\pi\left[\frac{(x-x_p)^2+(y-y_p)^2}{\lambda_2 z_p}\right]\right\} \quad (17.3)$$

Then $U_3 = t_3 U_p$; $U_4 = t_4 U_p$

U_3 and U_4 can be seen to represent quadratic phase approximations to spherical waves as well. We now wish to find the locations of the real (or virtual) points. Take an image wave:

(17.4)

U_3, U_4 can be written this way (with a possible extra constant phase factor absorbed into C). We must expand out the U_3, U_4 expressions and compare them to U_i. We get z_i by equating x^2, y^2 terms.

(17.5)

upper sign: U_3; lower sign: U_4

The *(x, y)* coordinates of the image points are found by equating the linear terms in *x, y*.

$$x_i = \mp\frac{\lambda_2 z_i}{\lambda_1 z_o}x_o \pm \frac{\lambda_2 z_i}{\lambda_1 z_o}x_r + \frac{z_i}{z_p}x_p \quad (17.6)$$

$$y_i = \text{similar}$$

Substituting these expressions for x_i, y_i, z_i into the U_i expression, we get U_3, U_4 expressions with an extra phase term independent of (x, y) which is absorbed in C.

Since the coordinates in image are *rescaled*, there is a magnification factor. An interesting point is that the transverse *(x,y)* magnification is different from the axial magnification *(z)*.

(17.7)

The negative sign of M_t indicates that the image is inverted.

The axial magnification is:

$$M_a = \left|\frac{\partial z_i}{\partial z_o}\right| = \left|\frac{\partial}{\partial z_o}\left(\frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \mp \frac{\lambda_2}{\lambda z_o}\right)^{-1}\right| = \frac{\lambda_1}{\lambda_2}|M_t|^2 \qquad (17.8)$$

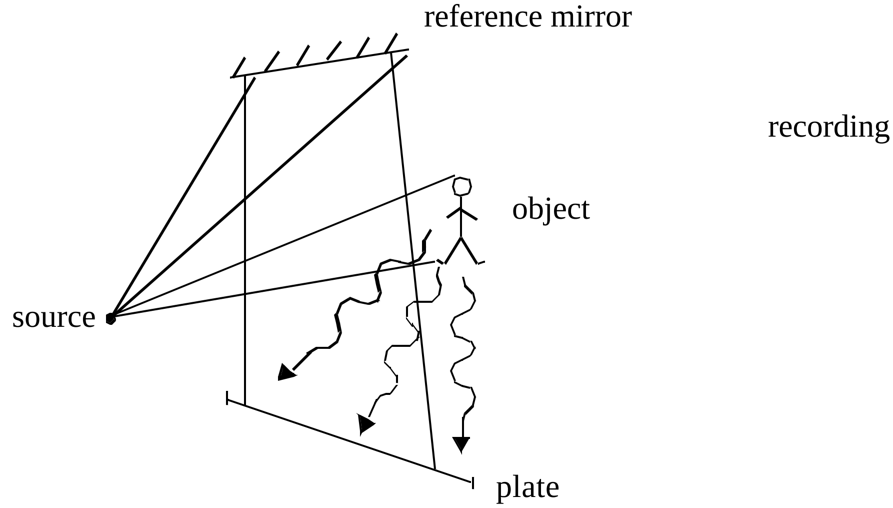
So $M_a \neq M_t$, in general. This results in a distortion of the image.

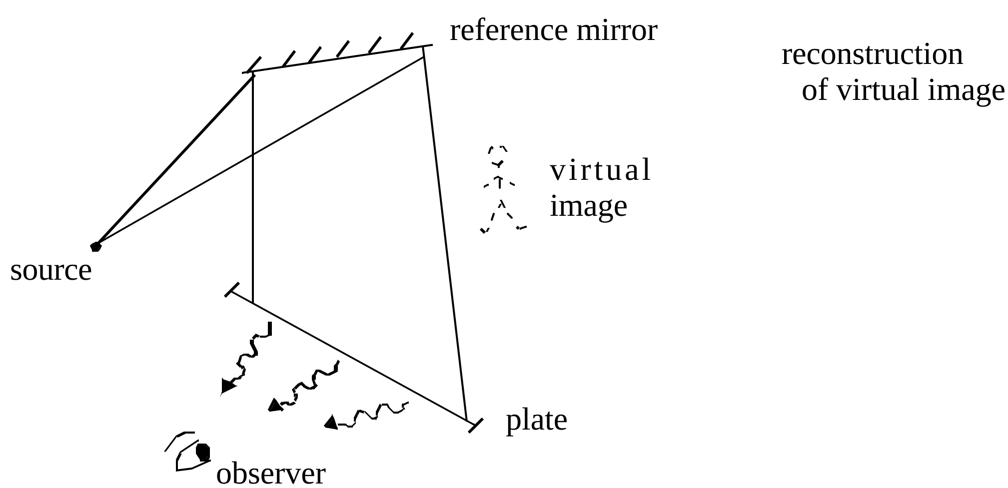
A simple case is to use the same wavelength and make the reconstruction wave identical to the reference wave:

$$\lambda_2 = \lambda_1 \qquad (x_p, y_p, z_p) = (x_r, y_r, z_r) \qquad (17.9)$$

Real image:		Virtual image:	
	$\frac{1}{z_i} = \frac{2}{z_r} - \frac{1}{z_o}$ $M_t = \frac{z_i}{z_o}$ distorted		$z_i = z_o$ $M_t = 1$ faithful

common geometry





The virtual image retains the full three dimensional properties of the object since the complete object wave is reconstructed. The observer can move around and look behind the object.

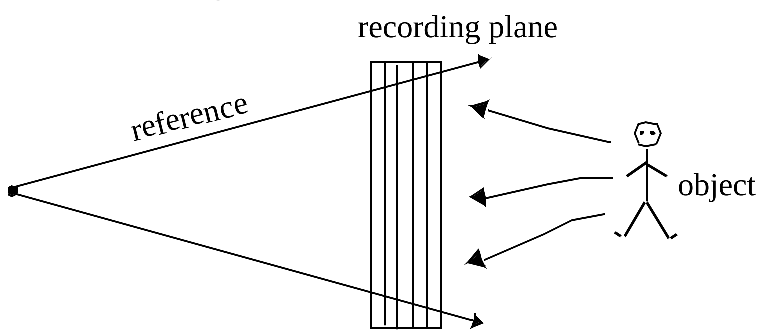
Another approach to combat distortion is to rescale the hologram (this is suitable if it is recorded electronically). Hologram scaling *m* gives

$$M_t = m\left|1 - \frac{z_o}{z_r} \mp m^2\frac{\lambda_1}{\lambda_2}\frac{z_o}{z_p}\right|^{-1} \qquad (17.10)$$

(17.11)

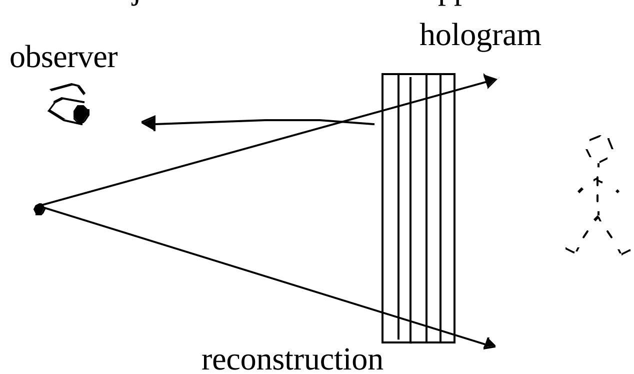
A suitable choice of *M* can make these equal.

Reflection holograms



Interference fringes form standing waves in emulsion with a period of about $\lambda/2$. They are nearly parallel to the surface.

The reference and object waves come from opposite sides of the recording medium.



The virtual image is formed by the light Bragg reflected by the hologram

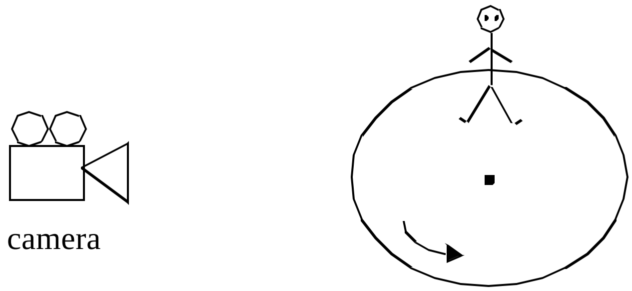
This can be viewed in white light since the Bragg condition is wavelength selective. More on this later.

Multiplex holograms

A primitive holographic “movie”:

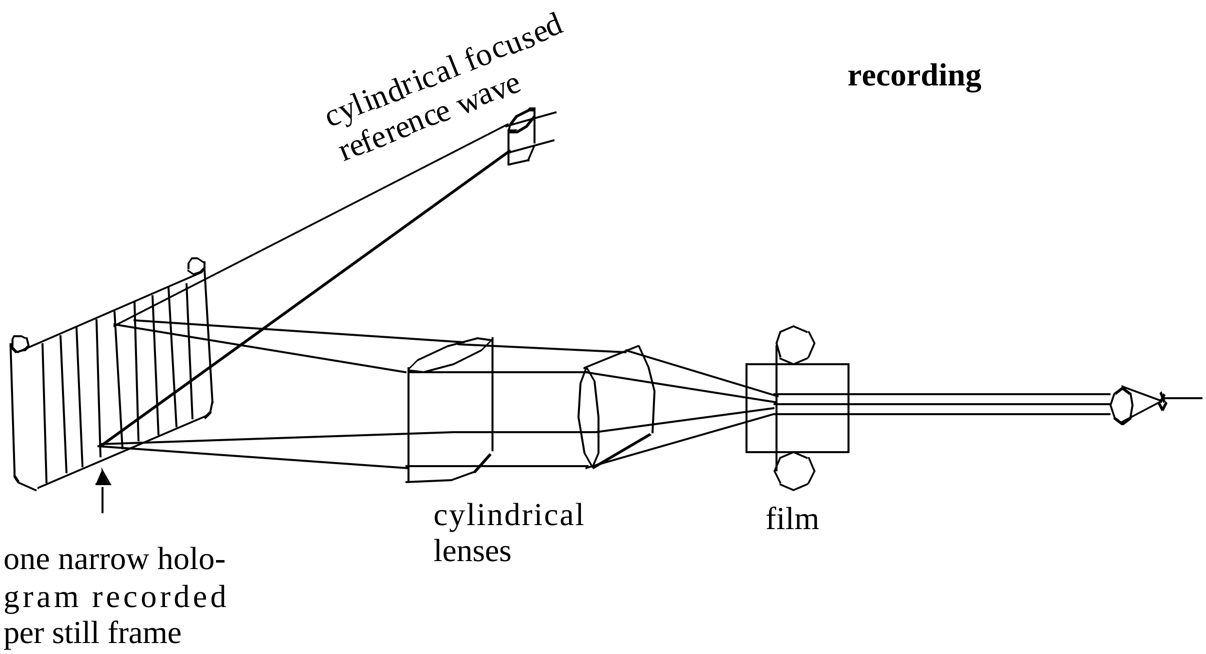
2 steps

1. Still a frame sequence, with conventional lighting

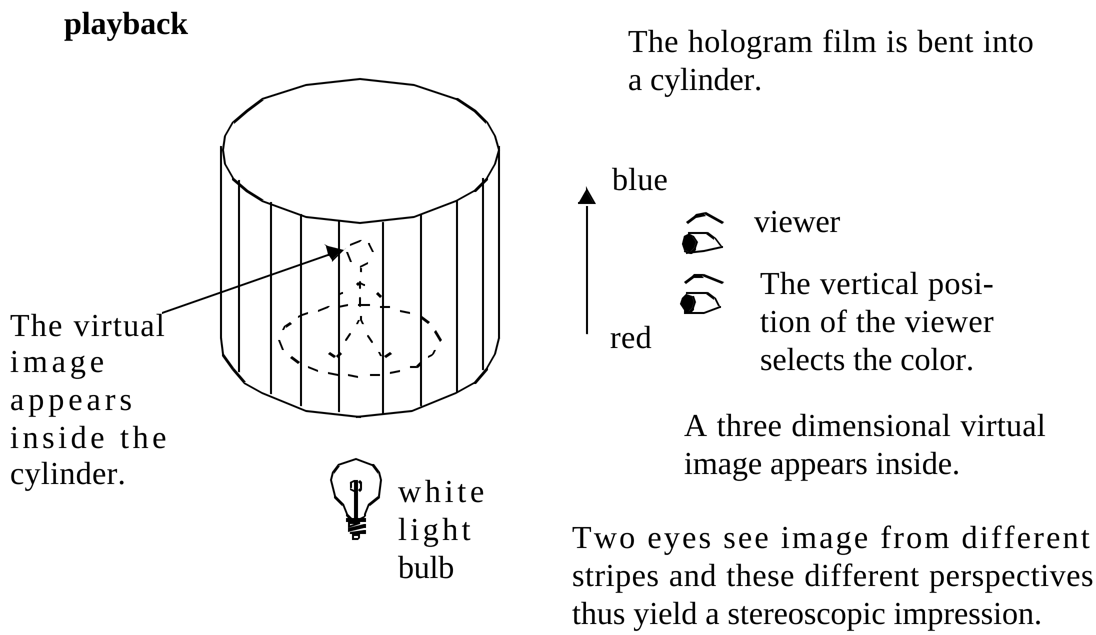


The object rotates on a turntable - the subject moves at the same time.

record about 3 frames/degree rotation



Each hologram stripe has an image taken from a different perspective.

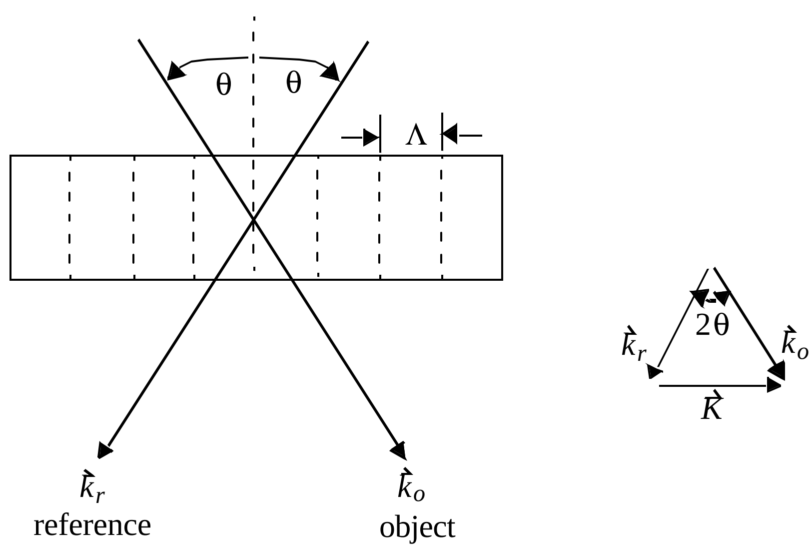


As the observer moves around the cylinder it appears that the subject moves.

Thick or volume holograms

We have assumed that the hologram is ideally thin. In most cases, the emulsion or other recording medium has a non-negligible thickness. This has significant effects.

Consider the simple case of plane object and reference waves. (This is easily generalized.) Suppose both are incident at angle θ to surface normal.



The intensity distribution in the recording medium is:

$$I(\vec{r}) = |A|^2 + |a|^2 + 2|A||a|\cos[(\vec{k}_r + \vec{k}_o)\cdot\vec{r} + \phi] \qquad (17.12)$$

Let $\vec{K} \equiv \vec{k}_r - \vec{k}_o$ which is the wavevector of the interference grating

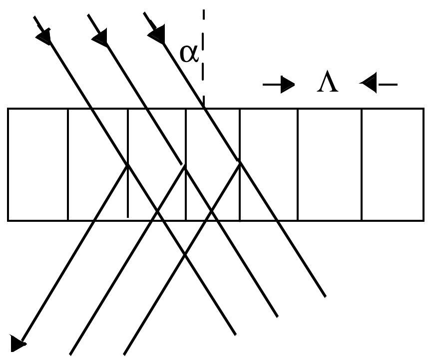
Then

(17.13)

is the grating period.

Now, in reconstruction, if the hologram is *thick*, then Bragg diffraction plays a role.

$\vec{k}_p$ reconstruction wave

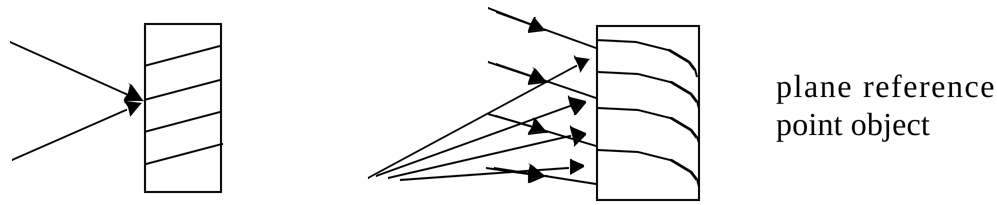


If the Bragg condition is satisfied, there is constructive interference in the formation of the reconstructed wave.

The Bragg condition is: $\sin\alpha = \pm\frac{\lambda}{2\Lambda}$, which means that $\alpha = \begin{cases} \pm\theta \\ \pm(\pi - \theta) \end{cases}$.

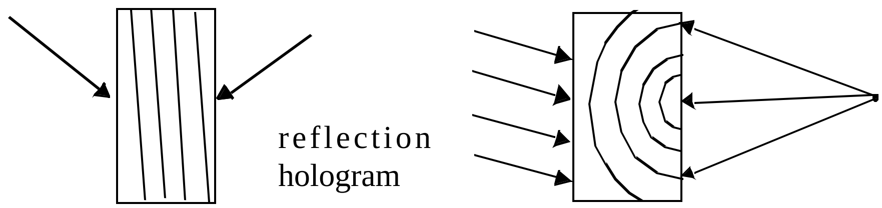
This is a special condition on the reconstruction wavevector such that there is a particularly *strong* reconstructed intensity.

More generally, a complex object leads to a curved grating, but locally lines are parallel.



slant fringes

curved fringes



What is the difference between thick and thin holograms (the effect of finite thickness)?

A simple argument: let $g(\vec{r})$ represent the local absorption of a hologram. We can express this as a Fourier decomposition:

(17.14)

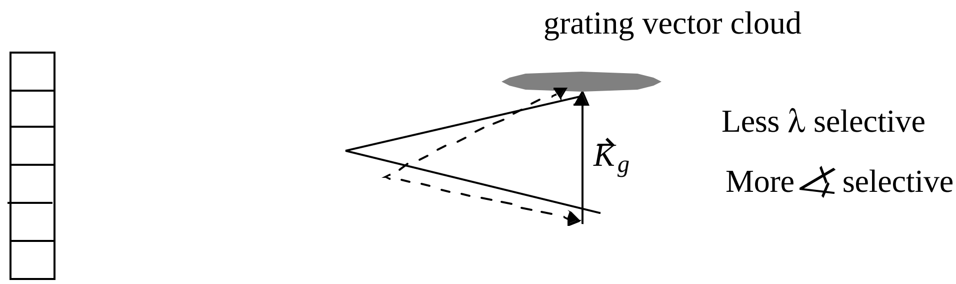
Suppose the hologram is a simple cosinusoidal grating in a certain direction.

$$g(\vec{r}) = [1 + m\cos(\vec{k}_g \cdot r + \phi_o)]rect\left(\frac{x}{X}\right)rect\left(\frac{y}{Y}\right)rect\left(\frac{z}{Z}\right) \tag{17.15}$$

The grating is a cube with dimensions $X \times Y \times Z$.

Then $$G(k) = \left[\delta(k) + \frac{m}{2}\delta(\vec{k} - \vec{k}_g) + \frac{m}{2}\delta(\vec{k} + \vec{k}_g)\right] \otimes XYZ\,\mathrm{sinc}\frac{Xk_x}{2\pi}\mathrm{sinc}\frac{Yk_y}{2\pi}\mathrm{sinc}\frac{Zk_z}{2\pi}$$

Thus the finite grating size blurs out a grating vector “tip”



grating "thin"
in direction ⊥ to fringes