Chapter 12
Effects of Partial coherence on imaging systems

Read Goodman, *Statistical Optics,* Chapter 7

Coherence effects are described by the mutual intensity function $J(x_1, y_1; x_2, y_2)$. (under quasi-monochromatic conditions) We now have the tools to describe the evolution of mutual intensity.

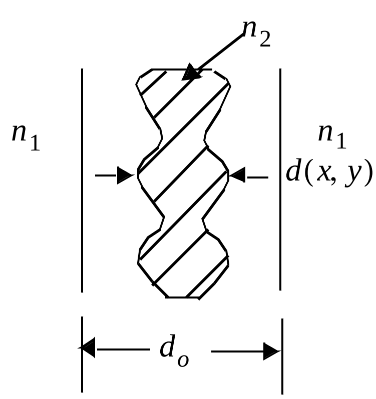
<u>The propagation law for mutual intensity:</u>

$$J_i(x_1, y_1; x_2, y_2) = \frac{e^{j\psi}}{(\bar{\lambda}z)^2}\iiiint J_o(\xi_1, \eta_1; \xi_2, \eta_2)\exp\left\{-j\frac{\pi}{\bar{\lambda}z}[(\xi_2^2+\eta_2^2)-(\xi_1^2+\eta_1^2)]\right\}$$
$$\times \exp\left\{j\frac{2\pi}{\bar{\lambda}z}[x_2\xi_2+y_2\eta_2-x_1\xi_1-y_1\eta_1]d\xi_1 d\eta_1 d\xi_2 d\eta_2\right\} \tag{12.1}$$

Transmission through the object:

$$\boxed{} \tag{12.2}$$

Can (12.2) be used to describe the effect of a lens on mutual intensity? Or on another strong phase object?



The delay through this structure, neglecting the constant, is

$$\delta = \frac{[n_2(x,y)-n_1]d(x,y)}{c}$$

Assuming a possible amplitude transmission factor $B(x, y)$,

$$u_t(x, y; t) = B(x, y)u_i[x, y; (t - \delta(x, y))]$$

Now we evaluate the mutual coherence of the transmitted field:

$$\boxed{}$$

Here, we assume no deviation of rays, only a phase shift. Then,

$$\Gamma_t(P_1, P_2;\tau) \;=\; B(P_1)B(P_2)\langle u_1(P_1, t+\tau-\delta(P_1))u^*{}_i(P_2, t-\delta(P_2))\rangle \tag{12.3}$$

Thus

$$\Gamma_t(P_1, P_2;\tau) = B(P_1)B(P_2)\Gamma_i(P_1, P_2;\tau-\delta(P_1)+\delta(P_2)) \,. \tag{12.4}$$

With narrowband light, we can write:

$$\boxed{} \tag{12.5}$$

So,

$$\begin{aligned}\Gamma_t(P_1, P_2;\tau) \;=\; & B(P_1)e^{j2\pi\bar{\nu}\delta(P_1)}B(P_2)e^{-j2\pi\bar{\nu}\delta(P_2)} \\ & \times \langle U_i(P_1, t+\tau-\delta(P_1)+\delta(P_2))U_i^*(P_2, t)\rangle e^{-j2\pi\bar{\nu}\tau}\end{aligned} \tag{12.6}$$

If $\left|\delta(P_1)-\delta(P_2)\right| \ll \tau_c \approx \dfrac{1}{\Delta\nu}$, *then* the time average is independent of $\delta(P_1), \delta(P_2)$, so

$$\Gamma_t(P_1, P_2;\tau) \;=\; B(P_1)e^{j2\pi\bar{\nu}\delta(P_1)}B(P_2)e^{-j2\pi\bar{\nu}\delta(P_2)}\Gamma_i(P_1, P_2;\tau) \tag{12.7}$$

$$\Gamma_t(P_1, P_2;\tau) \;=\; t_A(P_1)t_A^*(P_2)\Gamma_i(P_1, P_2;\tau) \tag{12.8}$$

with $t_A(P) \equiv B(P)e^{j2\pi\bar{\nu}\delta(P)}$. This is the same result we had for mutual intensity for a thin aperture.

This is strictly valid only when the delay differences induced by the object are much less than the coherence time. The relation holds even when the time delays through the object fail the quasi-monochromatic condition, as long as the condition holds for *net* time delay differences in the full system.

Recall the thin lens transmission function:

$$t_l(x, y) \;=\; \exp\left\{-j\frac{\pi}{\bar{\lambda}f}(x^2+y^2)\right\}P(x, y) \tag{12.9}$$

<u>Object-Image coherence relation</u>

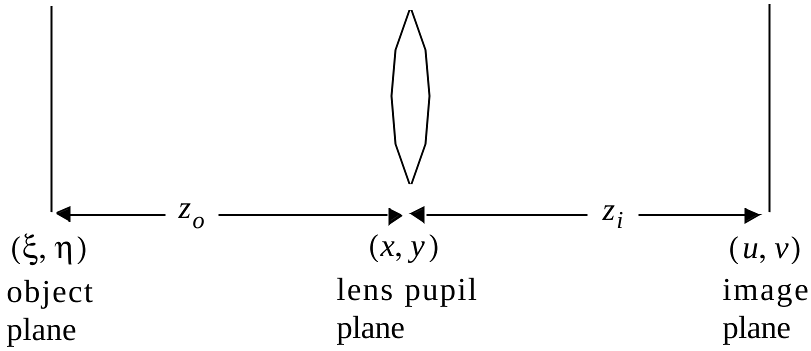
We assume the quasi-monochromatic condition. Our interest is in the relationship between the mutual intensities in the object and image planes.

First, assume that the mutual intensity of the object is known.

(1) *Propagate* to the lens

(2) *Transmit* through the lens.

(3) *Propagate* to the object plane.



We use the propagation law for mutual intensity, in the paraxial approximation.

$J'_o(\xi_1, \eta_1; \xi_2, \eta_2)$: mutual intensity leaving object plane

$J_l'(x_1, y_1; x_2, y_2)$: mutual intensity leaving lens

$$J_l'(x_1, y_1; x_2, y_2) = \frac{1}{(\bar{\lambda} z_o)^2} P(x_1, y_1) P^*(x_2, y_2) \exp\left\{-j\frac{\pi}{\bar{\lambda}}\left(\frac{1}{z_o} - \frac{1}{f}\right)[(x_2^2 + y_2^2) - (x_1^2 + y_1^2)]\right\}$$
$$\times \iint\limits_{-\infty}^{\infty}\iint J_o'(\xi_1, \eta_1; \xi_2, \eta_2) \exp\left\{-j\frac{\pi}{\bar{\lambda} z_o}[(\xi_2^2 + \eta_2^2) - (\xi_1^2 + \eta_1^2)]\right\} \tag{12.10}$$
$$\exp\left\{j\frac{2\pi}{\bar{\lambda} z_o}[x_2\xi_2 + y_2\eta_2 - x_1\xi_1 - y_1\eta_1]\right\} d\xi_1 d\eta_1 d\xi_2 d\eta_2$$

Now propagate to the image plane

$$J_i(u_1, v_1, u_2, v_2) = \frac{1}{(\bar{\lambda} z_o)^2}\frac{1}{(\bar{\lambda} z_i)^2} \exp\left\{-j\frac{\pi}{\bar{\lambda} z_i}[(u_2^2 + v_2^2) - (u_1^2 + v_1^2)]\right\} \tag{12.11}$$
$$\times \iint\limits_{-\infty}^{\infty}\iint d\xi_1 d\eta_1 d\xi_2 d\eta_2 J_o'(\xi_1, \eta_1; \xi_2, \eta_2) \exp\left\{-j\frac{\pi}{\bar{\lambda} z_o}[(\xi_2^2 + \eta_2^2) - (\xi_1^2 + \eta_1^2)]\right\}$$
$$\times \iint\limits_{-\infty}^{\infty}\iint dx_1 dy_1 dx_2 dy_2 P(x_1, y_1) P^*(x_2, y_2) \exp\left\{-j\frac{\pi}{\bar{\lambda}}\left(\frac{1}{z_o} + \frac{1}{z_i} - \frac{1}{f}\right)[(x_2^2 + y_2^2) - (x_1^2 + y_1^2)]\right\}$$
$$\times \exp\left\{j\frac{2\pi}{\bar{\lambda}}\left[x_2\left(\frac{u_2}{z_i} + \frac{\xi_2}{z_o}\right) + y_2\left(\frac{v_2}{z_i} + \frac{\eta_2}{z_o}\right) - x_1\left(\frac{u_1}{z_i} + \frac{\xi_1}{z_o}\right) - y_1\left(\frac{v_1}{z_i} + \frac{\eta_1}{z_o}\right)\right]\right\}$$

Go to the usual image plane $\frac{1}{z_o} + \frac{1}{z_i} - \frac{1}{f} = 0$. Define the amplitude point spread function. This is basically the same as before:

$$h(u, v;\xi, \eta) = \frac{1}{(\bar{\lambda} z_i)(\bar{\lambda} z_o)} \exp\left\{j\frac{\pi}{\bar{\lambda} z_i}(u^2 + v^2)\right\} \exp\left\{j\frac{\pi}{\bar{\lambda} z_o}(\xi^2 + \eta^2)\right\} \tag{12.12}$$

$$\times \int_{-\infty}^{\infty}\int P(x, y) \exp\left\{-j\frac{2\pi}{\bar{\lambda} z_i}\left[\left(u + \frac{z_i}{z_o}\xi\right)x + \left(v + \frac{z_i}{z_o}\eta\right)y\right]\right\} dx dy$$

The result now simplifies considerably to look like this:

$$J_i(u_1, v_1;u_2, v_2) = \int_{-\infty}^{\infty}\int\iint d\xi_1 d\eta_1 d\xi_2 (d\eta_2) J_o'(\xi_1, \eta_1;\xi_2, \eta_2) \tag{12.13}$$

$$\times \quad h(u_1, v_1;\xi_1, \eta_1) h^*(u_2, v_2;\xi_2, \eta_2)$$

Image intensity

$$I_i(u, v) = \int_{-\infty}^{\infty}\int\iint J_o'(\xi_1, \eta_1;\xi_2, \eta_2) h(u, v;\xi_1, \eta_1) h^*(u, v;\xi_2, \eta_2) d\xi_1 d\eta_1 d\xi_2 d\eta_2 \tag{12.14}$$

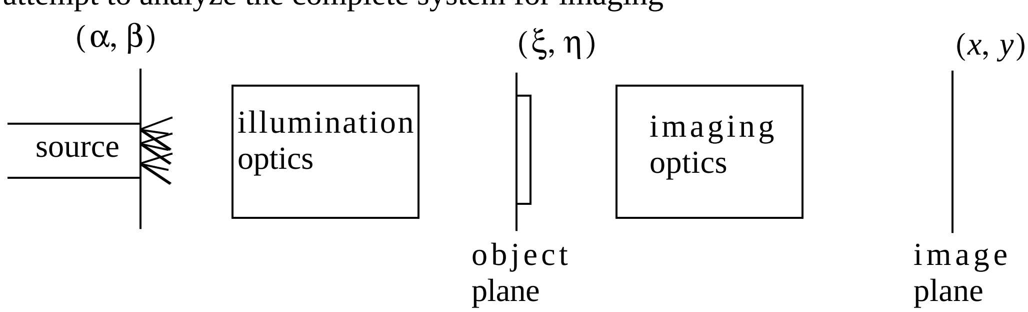
It's easy to verify that for incoherent light coming from the object, we recover our previous result that the image intensity is given by a convolution of the object intensity with the square magnitude of the point spread function.

Note that the quasi-monochromatic requirement here is:

$\dfrac{L_o^2}{4z_o} + \dfrac{L_i^2}{4z_i} \ll l_c$ The object field is $L_o \times L_o$; the image field is $L_i \times L_i$.

For L_o, L_i is in the centimeter range; z_o, z_i is in the tens of centimeters range, and l_c is in the millimeter range.

We now attempt to analyze the complete system for imaging



This model of an imaging system is applicable to practical applications:

- Microlithography
- Slide and vugraph projector

- Microscope
- Projection TV
- Photo-enlarger

We need to combine our understanding of propagation of coherence from source to object, with the object-image coherence formulation just completed to get the full theory for this system. We will develop a very powerful linear systems approach for this, which is also very general:

- The coherence of the source illumination will be simply characterized by the illuminator pupil function.

- The imaging system is characterized by the point spread function, or the imaging pupil function.

First - a more conceptually intuitive method, suitable for spatially incoherent source. If the source is incoherent, we can consider the source as a superposition of point sources. Calculate the image *intensity* for each source point, then add the *image intensities.*

Define $f(\xi, \eta; \alpha, \beta)$: the point spread function for the illumination optics. This tells us the field at (ξ, η) due to the point source at (α, β).

Also, $h(u, v; \xi, \eta)$: the point spread function of imaging optics.

Assume quasi-monochromatic conditions. The source point at (α, β) emits a field phasor $U_S(\alpha, \beta; t)$.

Then the field after the object is (We can use coherent optics, since we assume a point source!):

$$U_o'(\xi, \eta; \alpha, \beta, t) = f(\xi, \eta; \alpha, \beta) t_o(\xi, \eta) U_S(\alpha, \beta; t - \delta) \tag{12.15}$$

Here $t_o(\xi, \eta)$ is the object amplitude transmittance, and δ is the time delay from (α, β) to (ξ, η).

The image plane field is

$$U_i(u, v; \alpha, \beta, t) = \int_{-\infty}^{\infty}\int h(u, v; \xi, \eta) t_o(\xi, \eta) f((\xi, \eta; \alpha, \beta) U_S(\alpha, \beta; t - \delta_1 - \delta_2)) d\xi d\eta \tag{12.16}$$

Where δ_2 is the delay from (ξ, η) to (u,v).

The *partial* image intensity at (u, v) is due to the point α, β on source is:

$$\boxed{} \tag{12.17}$$

$$I_i(u, v;\alpha, \beta) = \int\int\int\int_{-\infty}^{\infty} h(u, v;\xi_1, \eta_1)h^*(u, v;\xi_2, \eta_2)t_o(\xi_1, \eta_1)t_o^*(\xi_2, \eta_2) \times f(\xi_1, \eta_1;\alpha, \beta)f^*(\xi_2, \eta_2;\alpha, \beta) \times \langle U_S(\alpha, \beta, t - \delta_1 - \delta_2)U_S^*(\alpha, \beta, t - \delta_1' - d_2')\rangle d\xi_1 d\eta_1 d\xi_2 d\eta_2 \tag{12.18}$$

if the quasi-monochromatic condition applies $|\delta_1 + \delta_2 - \delta_1' - \delta_2| \ll \tau_c$, the time average inside the integral is simply $I_S(\alpha, \beta)$. To get the full image, we have to *sum* the contributions from all the source points <u>adding intensities</u>

(12.19)

$$= \int\int_{-\infty}^{\infty} I_S(\alpha, \beta)\int\int\int\int_{-\infty}^{\infty} h(u, v;\xi_1, \eta_1)h^*(u, v;\xi_2, \eta_2)f(\xi_1, \eta_1;\alpha, \beta)f^*(\xi_2, \eta_2;\alpha, \beta) \times t_o(\xi_1, \eta_1)t_o^*(\xi_2, \eta_2)d\xi_1 d\eta_1 d\xi_2 d\eta_2 d\alpha d\beta \tag{12.20}$$