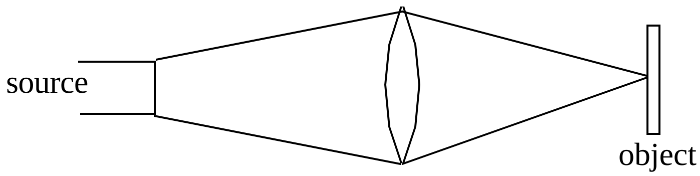
Chapter 13
Partially Coherent Imaging, continued

As an example, a common illuminator design is one in which the source is imaged onto the object. This is known as "critical illumination"



- source has to be quite uniform
- could be magnified or de-magnified

Review: In the above approach to analyzing imaging systems, the source is modeled as a collection of independent point sources. The image due to each point source is separately calculated using coherent optics, and then the final image is found by adding the intensities of the individual images from all the source points. This method is known as Abbe's method.

An alternative approach is to first find the mutual intensity of the object illumination: $J_o(\xi_1, \eta_1; \xi_2, \eta_2)$.

Under certain conditions, a detailed knowledge of the source is not needed in order to find the mutual intensity of the object illumination.

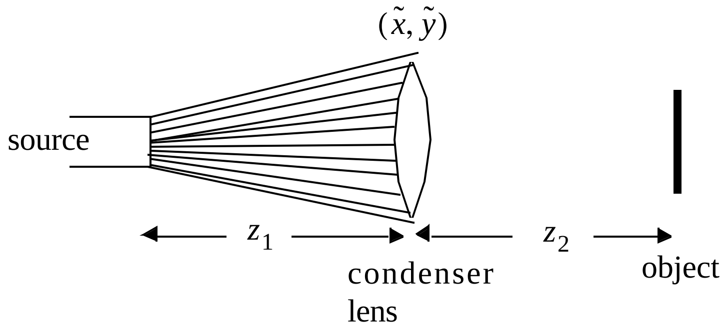
If we know $J_o(\xi_1, \eta_1; \xi_2, \eta_2)$, then:

(13.1)

In this case the image intensity is

$$I_i(u, v) = \iint\int\int_{-\infty}^{\infty} h(u, v; \xi_1, \eta_1) h^*(u, v; \xi_2, \eta_2) t_o(\xi_1, \eta_1) t_o^*(\xi_2, \eta_2) \times J_o(\xi_1, \eta_1; \xi_2 \eta_2) d\xi_1 d\eta_1 d\xi_2 d\eta_2 \quad (13.2)$$

Illuminator



Assume that the source coherence is very low, so that the coherence area on the source is small compared to the source size: $(A_c^{(S)} \ll A_s)$.

We further assume that the lens collects a large angle such that the light coherence area on the lens is much smaller than the lens area:

$$A_l \gg \frac{(\lambda z_1)^2}{A_s} \tag{13.3}$$

The lens aperture can then be viewed as an incoherent source as well.

Let's examine this in detail. The source is assumed to be incoherent, so that the Van Cittert-Zernike theorem applies. The mutual intensity incident on the condenser lens is:

$$\begin{aligned} J_l(\tilde{x}_1, \tilde{y}_1; \tilde{x}_1, \tilde{y}_2) = {} & \frac{\kappa}{(\bar{\lambda} z_1)^2} \exp\left\{-j\frac{\pi}{\bar{\lambda} z_1}[(\tilde{x}_2^2 + \tilde{y}_2^2) - (\tilde{x}_1^2 + \tilde{y}_1^2)]\right\} \\ & \times \int_{-\infty}^{\infty}\int I_s(\alpha, \beta) \exp\left\{j\frac{2}{\bar{\lambda} z_1}(\Delta\tilde{x}\alpha + \Delta\tilde{y}\beta)\right\} d\alpha d\beta \end{aligned} \tag{13.4}$$

$$\begin{aligned} J_l'(\tilde{x}_1, \tilde{y}_1; \tilde{x}_1, \tilde{y}_2) = {} & \tilde{P}_c(\tilde{x}_1, \tilde{y}_1)\tilde{P}_c^*(\tilde{x}_2, \tilde{y}_2) \\ & \exp\left\{-j\frac{\pi}{\lambda f}[(\tilde{x}_1^2 + \tilde{y}_1^2) - (\tilde{x}_2^2 + \tilde{y}_2^2)]\right\} J_l(\tilde{x}_1, \tilde{y}_1; \tilde{x}_2, \tilde{y}_2) \end{aligned} \tag{13.5}$$

Here, $\tilde{P}_c(\tilde{x}, \tilde{y})$ is the condenser pupil function, such that:

(13.6)

and $W(\tilde{x}, \tilde{y})$ represents the pupil aberrations in the condenser.

We assume that J_l is very narrow in $\Delta\tilde{x}, \Delta\tilde{y}$.

Then

$$\exp\left\{-j\frac{\pi}{\bar{\lambda}}\left(\frac{1}{z_1}-\frac{1}{f}\right)[(\tilde{x}_2^2+\tilde{y}_2^2)-(\tilde{x}_1^2+\tilde{y}_1^2)]\right\}\cong 1 \tag{13.7}$$

$$\boxed{} \tag{13.8}$$

So we can write:

$$J_c'(\tilde{x}_1,\tilde{y}_1;\tilde{x}_2,\tilde{y}_2)\cong\frac{\kappa}{(\bar{\lambda}z_1)^2}\left|\tilde{P}_c(\tilde{x},\tilde{y})\right|^2 I_S\left(\frac{\Delta\tilde{x}}{\lambda z_1},\frac{\Delta\tilde{y}}{\lambda z_1}\right) \tag{13.9}$$

where

$$I_S(\nu_x,\nu_y) = F[I_S(\alpha,\beta)]\ . \tag{13.10}$$

I_S is very narrow in $\Delta\tilde{x}, \Delta\tilde{y}$, so that J_l' describes an incoherent source with an intensity distribution of $\frac{\kappa}{(\bar{\lambda}z_1)^2}\left|\tilde{P}_c(\tilde{x},\tilde{y})\right|^2$. We use the Van Cittert-Zernike theorem to propagate to the object:

$$J_o(\xi_1,\eta_1,\xi_2,\eta_2) = \frac{\kappa'}{(\bar{\lambda}z_2)^2}\exp\left\{-j\frac{\pi}{(\bar{\lambda}z_2)}[(\xi_2^2+\eta_2^2)-(\xi_1^2+\eta_1^2)]\right\} \tag{13.11}$$
$$\times\int_{-\infty}^{\infty}\int\left|\tilde{P}_c(\tilde{x}_1,\tilde{y}_1)\right|^2\exp\left\{j\frac{2\pi}{\bar{\lambda}z_2}(\Delta\xi\tilde{x}_1+\Delta\eta\tilde{y}_1)\right\}d\tilde{x}_1 d\tilde{y}_1$$

From this result we can conclude that:

- the mutual intensity at the object is independent of aberrations in the illuminator

- the coherence diameter at the object, $d_c\sim\frac{\bar{\lambda}z_2}{D_c}\sim\left(\frac{\bar{\lambda}}{NA}\right)$, where D_c is the diameter of the condenser pupil. This will come up again later.

<u>Another approach: four dimensional linear systems approach</u>

- We will work in the spatial frequency domain.

- This will lead to the method used in the program SPLAT for partially coherent imaging.

The relation between the object and image mutual intensities

$$J_i(u_1,v_1,u_2,v_2) = \int_{-\infty}^{\infty}\int\iint J_o'(\xi_1,\eta_1,\xi_2,\eta_2)h(u_1,v_1,\xi_1,\eta_1) \tag{13.12}$$
$$h^*(u_2,v_2,\xi_2,\eta_2)d\xi_1 d\eta_1 d\xi_2 d\eta_2$$

This is a four dimensional superposition integral. If the impulse response is space-invariant, h depends only on the coordinate differences. Then it becomes a convolution integral.

The quadratic phase factors in the definition of h interfere with the space invariance of h. These can be dropped for small coherence areas at the object and image planes.

Four dimensional Fourier Transform

Given a function $J(x_1, x_2, x_3, x_4)$,

$$F[J(x_1, x_2, x_3, x_4)] \equiv \iiiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 dx_4 \exp[j2\pi(x_1\nu_1 + x_2\nu_2 + x_3\nu_3 + x_4\nu_4)] \quad (13.13)$$

$$\times J(x_1, x_2, x_3, x_4)$$

$$= J(\nu_1, \nu_2, \nu_3, \nu_4)$$

If space invariance holds, then h is a function of 2 variables, $h(u_1 - \xi_1, v_1 - \eta_1)$.

Define:

$$H(\nu_1, \nu_2) \equiv F[h(x_1, x_2)] \quad (13.14)$$

The transfer function involved in the imaging system is

(13.15)

It's easy to show then that

$$H(\nu_1, \nu_2, \nu_3, \nu_4) = H(\nu_1, \nu_2)H^*(-\nu_3, -\nu_4) \quad (13.16)$$

Define the Fourier Transforms of the object and image mutual intensities

(13.17)

(13.18)

Then, generalizing the convolution theorem to 4D gives

$$J_i(\nu_1, \nu_2, \nu_3, \nu_4) = H(\nu_1, \nu_2)H^*(-\nu_3, -\nu_4)J_o'(\nu_1, \nu_2, \nu_3, \nu_4) \quad (13.19)$$

Recall that H is the coherent transfer function for the imaging lens.

$$H(\nu_1, \nu_2) = P(\bar{\lambda} z_i \nu_1, \bar{\lambda} z_i \nu_2) \quad (13.20)$$

which is the imaging system pupil function. Then:

$$J_i(\nu_1, \nu_2, \nu_3, \nu_4) = P(\bar{\lambda} z_i \nu_1, \bar{\lambda} z_i \nu_2) P^*(-\bar{\lambda} z_i \nu_3, -\bar{\lambda} z_i \nu_4) J_o'(\nu_1, \nu_2, \nu_3, \nu_4) \tag{13.21}$$

Now, let's examine J_o' - this depends on both the object and illumination.

Recall that

(13.22)

Then

$$J_o'(\nu_1, \ldots \nu_4) = \iint\int_{-\infty}^{\infty}\int J_o(\Delta\xi, \Delta\eta) t_o(\xi_1, \eta_1) t_o^*(\xi_2, \eta_2) \tag{13.23}$$

$$\times \exp[j2\pi(\nu_1\xi_1 + \nu_2\eta_1 + \nu_3\xi_2 + \nu_4\eta_2)] d\xi_1 d\eta_1 d\xi_2 d\eta_2$$

Using $\xi_2 = \xi_1 + \Delta\xi \quad \eta_2 = \eta_1 + \Delta\eta$

$$J_o'(\nu_1, \ldots \nu_4) = \int_{-\infty}^{\infty}\int d\xi_1 d\eta_1 t_o(\xi_1, \eta_1) \exp\{j2\pi[(\nu_1 + \nu_3)\xi_1 + (\nu_2 + \nu_4)\eta_1]\} \tag{13.24}$$

$$\times \int_{-\infty}^{\infty}\int d\Delta\xi d\Delta\eta J_o(\Delta\xi, \Delta\eta) t_o^*(\xi_1 + \Delta\xi_1 \eta_1 + \Delta\eta) \exp\{j2\pi[\nu_3\Delta\xi + \nu_4\Delta\eta]\}$$

The second integral is a Fourier Transform of the product of two functions. It can be expressed as the convolution of transforms.

Define $T_o = F[t_o]$.

The second integral can then be written (using the shift theorem) as

$$\int_{-\infty}^{\infty}\int J_o(p_1, q) T_o^*(p - \nu_3, q - \nu_4) \exp\{-j2\pi[\xi_1(\nu_3 - p) + \eta_1(\nu_4 - q)]\} dp dq \tag{13.25}$$

The final result is now:

$$J_o'(\nu_1, \ldots \nu_4) = \int_{-\infty}^{\infty}\int\iint d\xi_1 d\eta_1 dp dq t_o(\xi_1, \eta_1) J_o(p_1, q) T_o^*(p - \nu_3, q - \nu_4) \tag{13.26}$$

$$\times \exp\{j2\pi[(\nu_1 + \nu_3)\xi_1 - (\nu_3 - p)\xi_1 + (\nu_2 + \nu_4)\eta_1 - (\nu_4 - q)\eta_2]\}$$

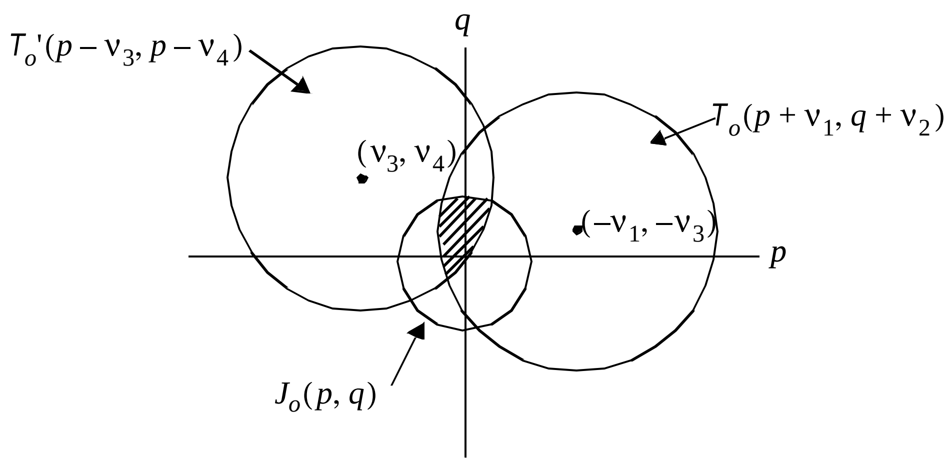
$$J_o'(\nu_1, \ldots \nu_4) = \int_{-\infty}^{\infty}\int dp dq T_o(p + \gamma_1, q + \gamma_2) T_o^*(p - \gamma_3, q - \gamma_4) J_o(p_1 q) \tag{13.27}$$

Recall that if the source is large and incoherent, the illumination of the condenser lens is also incoherent. In this case, $J_o(\Delta\xi, \Delta\eta)$ is proportional to the Fourier Transform of $|\tilde{P}_c|^2$. So

(13.28)

where z_2 is the distance from the condenser pupil to the object.

Assume that the object spectrum has a certain bandwidth. Then $J_o'(\nu_1, \ldots \nu_4)$ is obtained from the overlap shown graphically:



Now combine equations (13.19) and (13.27)

$$J_i(\nu_1 \ldots \nu_4) = H(\nu_1, \nu_2) H^*(-\nu_3, -\nu_4) \int_{-\infty}^{\infty}\int T_o(p + \nu_1, q + \nu_2) \tag{13.29}$$

$$T_o^*(p - \nu_3, q - \nu_4) J_o(p, q) dp dq$$

Recall H is the coherent transfer function of the imaging system, which is given by the pupil function of the imaging lens.

This gives us a nice result. Given the condenser pupil, the imaging system pupil function and the object spectrum, we get a four dimensional spectrum of the image mutual intensity.

Finally, we want the image intensity or a two dimensional image intensity spectrum.

One way: 1) Transform back to get $J_i(u_1, v_1; u_2, v_2)$

2) $I(u, v) = J_i(u_1, v_1; u_2, v_2)$

Alternative: obtain the image intensity spectrum, $I_i(\nu_u, \nu_v)$, from $J_i(\nu_1 \ldots \nu_4)$.

It is straightforward to show that

$$I_i(v_u, v_v) = \int_{-\infty}^{\infty}\int J_i(v_1, v_2, v_u - v_1, v_v - v_2)\, dv_1 dv_2 \qquad (13.30)$$

Now use equations (13.29) and (13.30): [let $\mu_1 \equiv p + v_1; \mu_2 = q + v_2$]

$$I_i(\gamma_u, \gamma_v) = \int_{-\infty}^{\infty}\int d\mu_1 d\mu_2 T_o(v_1, \mu_2) T_o^*(\mu_1 - v_u, \mu_2 - v_v) \underbrace{\times \left[\int_{-\infty}^{\infty}\int dp\, dq\, H(\mu_1 - p, \mu_2 - q) H^*(\mu_1 - p - v_u, \mu_2 - q - v_v) J_o(p, q) \right]}_{\text{Transmission cross coefficient (TCC)}} \qquad (13.31)$$

The transmission cross coefficient only depends on the coherence of the illumination and the imaging pupil function. This involves the integral over the overlap region of the three circles again.

This integral is the basis of SPLAT, originally written by K. Toh, M.S. 1988. The easiest way to use SPLAT is to go to:

http://cuervo.eecs.berkeley.edu/Volcano/

Click on Simulators, then on SPLAT. There is at least one commercialized version that has undergone some significant improvement in speed and ease of use called PROLITH from Finle Technologies.

One nice feature is that given pupil aberrations and illumination partial coherence, the transmission cross coefficients can be calculated and stored. A variety of mask patterns can be calculated rapidly after that.

Partial coherence in SPLAT

In common usage in lithography and other imaging system communities,is the partial coherence parameter:

$$\qquad (13.32)$$

How does σ relate to $J_o(p, q)$?

To see this, we go back to the real-space calculation of image intensity:

$$I_i(u, v) = \int\int_{-\infty}^{\infty}\int\int h(u-\xi, v-\eta)h^*(u-\xi-\Delta\xi, v-\eta-\Delta\eta) \qquad (13.33)$$

$$\times t_o(\xi, \eta)t_o^*(\xi+\Delta\xi, \eta+\Delta\eta)J_o(\Delta\xi, \Delta\eta)d\xi d\eta d\Delta\xi d\Delta\eta$$

Here we used a change of variables from an earlier form of this integral. The coherent limit occurs when $J_o(\Delta\xi, \Delta\eta)$ is constant, I_o. Then

(13.34)

This holds as long as $J_o(\Delta\xi, \Delta\eta)$ is essentially constant over the range of $(\Delta\xi, \Delta\eta)$ for which the integrand is appreciable.

When $\Delta\xi, \Delta\eta$ are greater than the object size, then

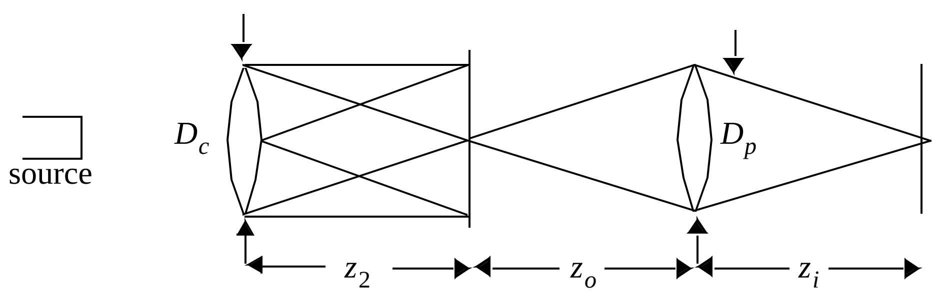
$$t_o(\xi, \eta)t_o^*(\xi+\Delta\xi, \eta+\Delta\eta) \sim 0 \qquad (13.35)$$

But *h* is much narrower. When $\Delta\xi, \Delta\eta$ exceed the width of *h*, then

$$h(u-\xi, v-\eta)h^*(u-\xi-\Delta\xi, v-\eta-\Delta\eta) \approx 0 \qquad (13.36)$$

Therefore, the illumination may be considered coherent when the illumination coherence area exceeds the area of the point spread function (scaled to the object plane).

From our previous result: $d_c \sim \frac{\bar{\lambda} z_2}{D_c} \sim \frac{\bar{\lambda}}{NA_{\text{illuminator}}}$ [this is valid when the condenser pupil is incoherent]



Imaging system resolution:

$$d_i \sim \frac{\bar{\lambda} z_i}{D_p} \sim \frac{\bar{\lambda}}{NA_{\text{image exit}}} \qquad (13.37)$$

When scaled to the object side:

$$d_o \sim \frac{\bar{\lambda} z_o}{D_p} \sim \frac{\bar{\lambda}}{NA_{\text{image entrance}}} \tag{13.38}$$

The coherent illumination condition is then:

$$d_c \gg d_o \text{ or } \frac{1}{NA_{\text{illumination}}} \gg \frac{1}{NA_{\text{image entrance}}} \tag{13.39}$$

We define the partial coherence factor:

$$\sigma \equiv \frac{NA_{\text{illumination}}}{NA_{\text{image input}}} . \tag{13.40}$$

Then the coherent illumination condition is:

$$\boxed{\qquad\qquad} \tag{13.41}$$

Note that in general, $\sigma \leq 1$. The maximum value of σ occurs when the illuminator completely fills the entrance pupil of the imaging system.

SPLAT assumes $J_o(p, q)$ is a constant inside a circle with a radius proportional to σ.

$\sigma \sim 0$: fully coherent $\sigma = 1$: incoherent

This completes our theoretical discussion of partial coherence in imaging. The ramifications will be played out when we discuss applications.