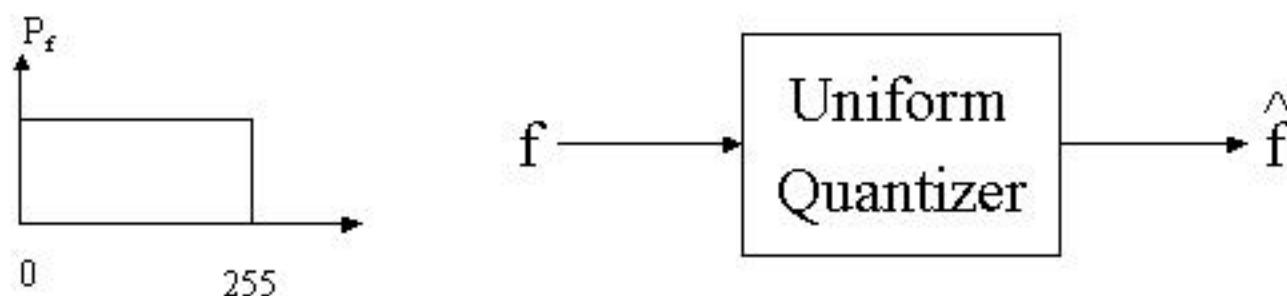
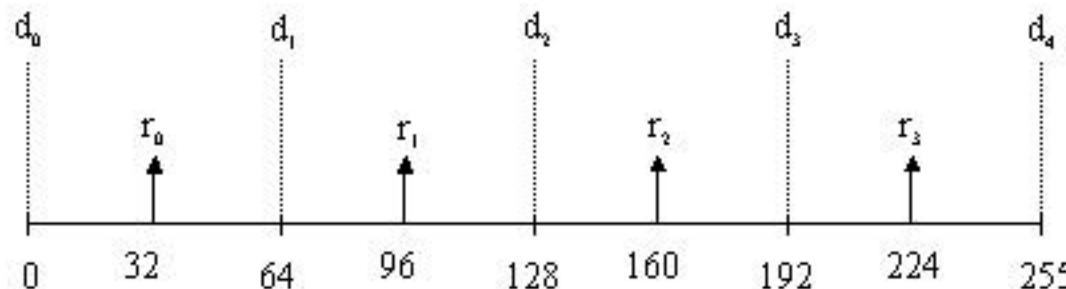


# Quantization

- Assume a coding image intensity value.
- $f$  pixels can be assigned values  $0 \rightarrow 255$ , so 8 bits are used to code each pixel.

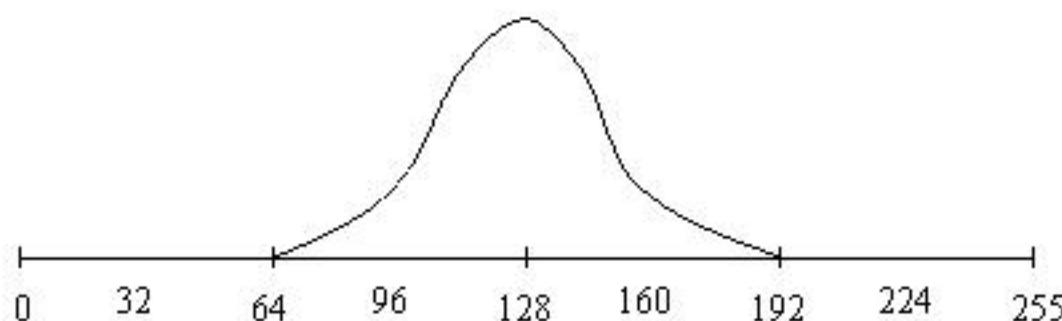


- There are 4 bins in this uniform quantizer.



# Quantization

- Non-uniform pdf:



## Generalization of Quantization Problem

- $f_{\min} \leq f \leq f_{\max}$
- $J = \text{number of reconstruction levels}$
- $P(f) = \text{pdf for } f$



# Generalization of Quantization Problem

- Choose  $r_i, d_i$  so that they minimize:

$$\varepsilon = E[(f - \hat{f})^2]$$

$$\varepsilon = \int_{f_{\min}}^{f_{\max}} (f - \hat{f})^2 p(f) df$$

$$\varepsilon = \sum_{j=0}^{J-1} \int_{d_j}^{d_{j+1}} (f - r_j)^2 p(f) df$$

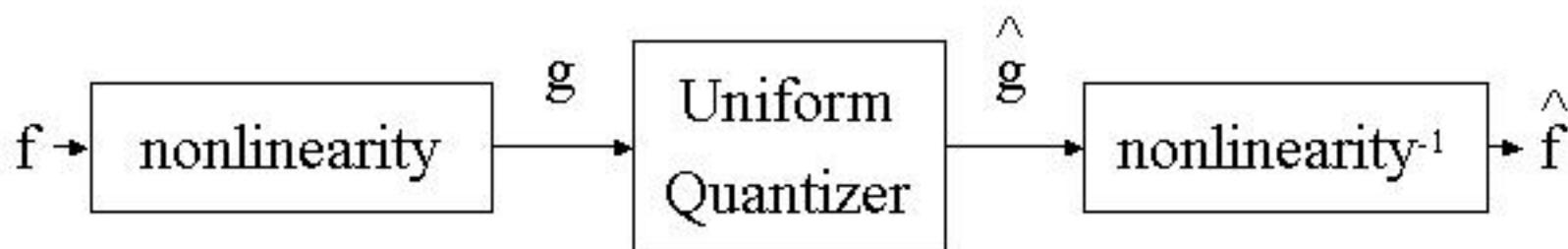
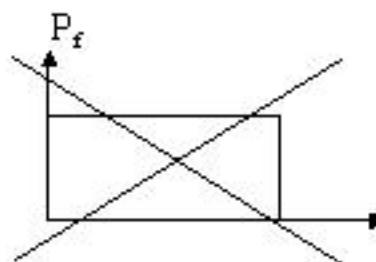
$$\frac{\partial \varepsilon}{\partial r_j} = 0 \rightarrow r_j = 2d_j - r_{j-1}$$

$$\frac{\partial \varepsilon}{\partial d_j} = 0 \rightarrow \frac{\int_{d_j}^{d_{j+1}} f \cdot p(f) df}{\int_{d_j}^{d_{j+1}} p(f) df}$$



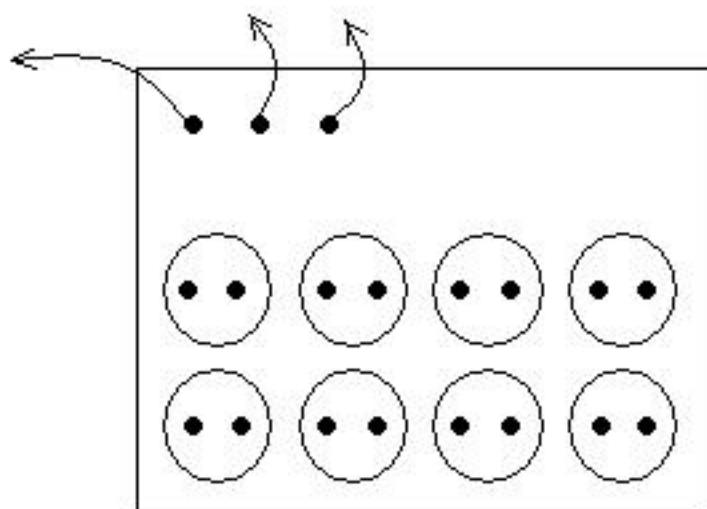
# Approach for Avoiding Non-Uniform Quantization

- $f$  has a non-uniform pdf:



# Vector Quantization

- $f$  ranges from  $0 \rightarrow 255$ .
- There is a level assignment for 2 pixels at a time, one from  $f_1$  and one from  $f_2$ .



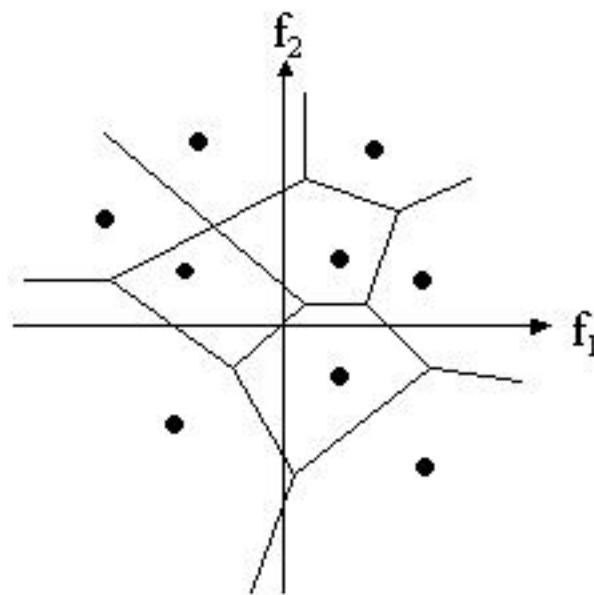
Scalar quantization

Vector quantization  
“dimension 2”

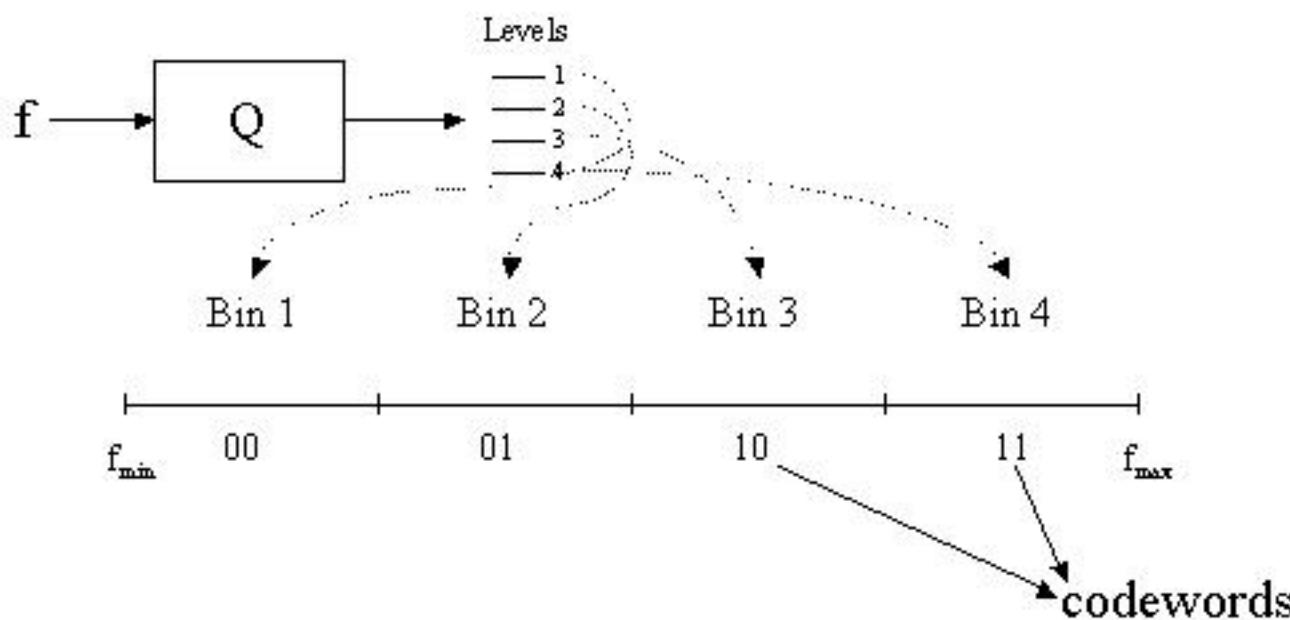
$$\varepsilon = \iint_{f_1 f_2} \left[ (f_1 - \hat{f}_1)^2 + (f_2 - \hat{f}_2)^2 \right] p(f_1, f_2) df_1 df_2$$



# Vector Quantization



# Entropy Coding, Codeword Design, Bit Allocation



- Shorter codewords are assigned to more probable symbols.

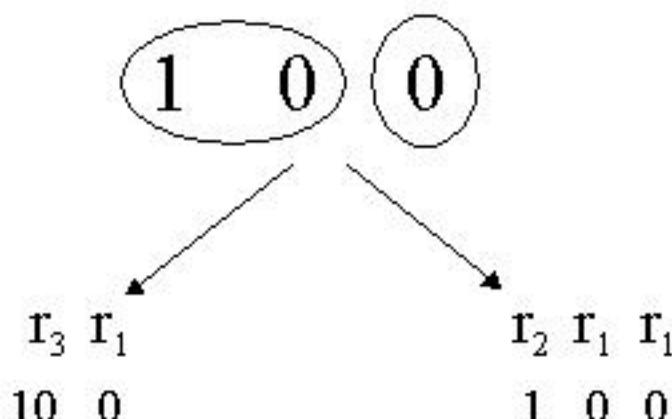


# Entropy Coding, Codeword Design, Bit Allocation

- Example

4 symbols

- $r_1 \rightarrow 0$
- $r_2 \rightarrow 1$
- $r_3 \rightarrow 10$
- $r_4 \rightarrow 11$



Not uniquely decodable

- Problem One codeword is a prefix of others.
- Solution Use a “prefix coder”, where no codeword is a prefix of any other codeword  $\Rightarrow$  unique decodability



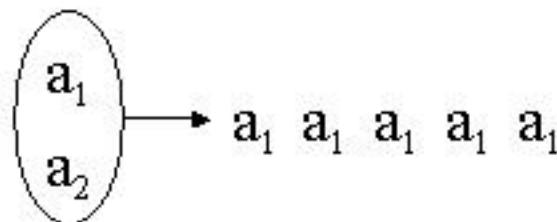
# Entropy Coding

- Goal: Design variable length codewords such that:
  - the average bit rate is minimized
  - the codewords are uniquely decodable

$$\text{Entropy } H = \sum_{i=1}^L p_i \log_2 p_i$$

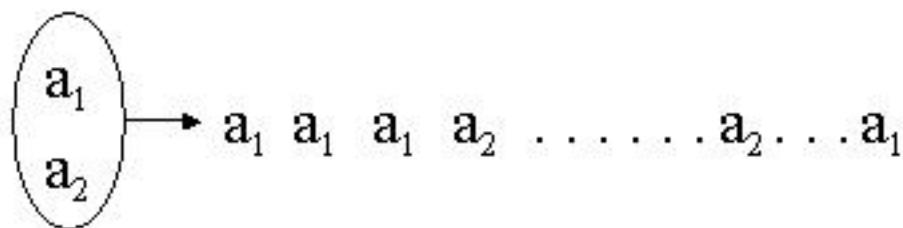
$p_i$  = "probability of symbol i"

- Entropy is the average amount of information in a message.
- 2 symbols with  $p_1 = 1$  and  $p_2 = 0 \Rightarrow H = 0 \Rightarrow$  no information

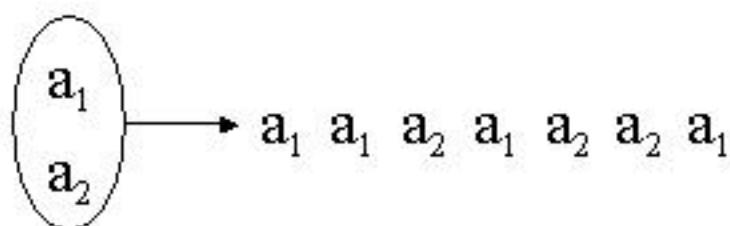


# Entropy Coding

- 2 symbols with  $p_1 = 99\%$  and  $p_2 = 1\%$

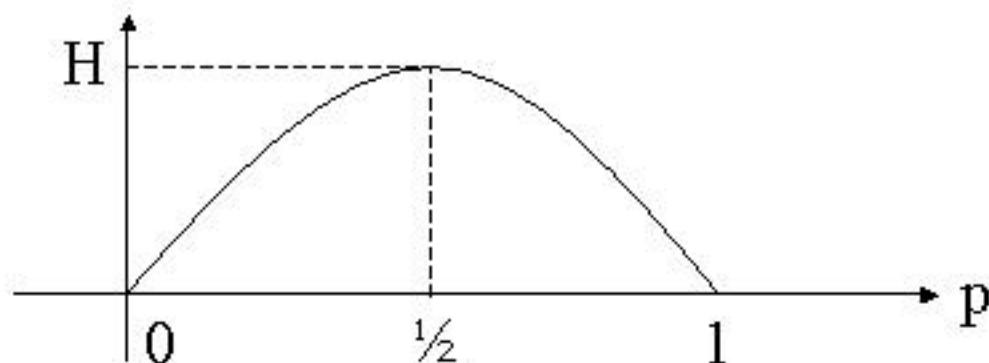


- 2 symbols with  $p_1 = 50\%$  and  $p_2 = 50\%$



# Entropy Coding

- Source with 2 letters with probabilities  $p$  and  $(1-p)$



- Source with  $L$  letters

$$0 \leq H \leq \log_2 L$$

$$\sum_{i=1}^L p_i = 1$$

