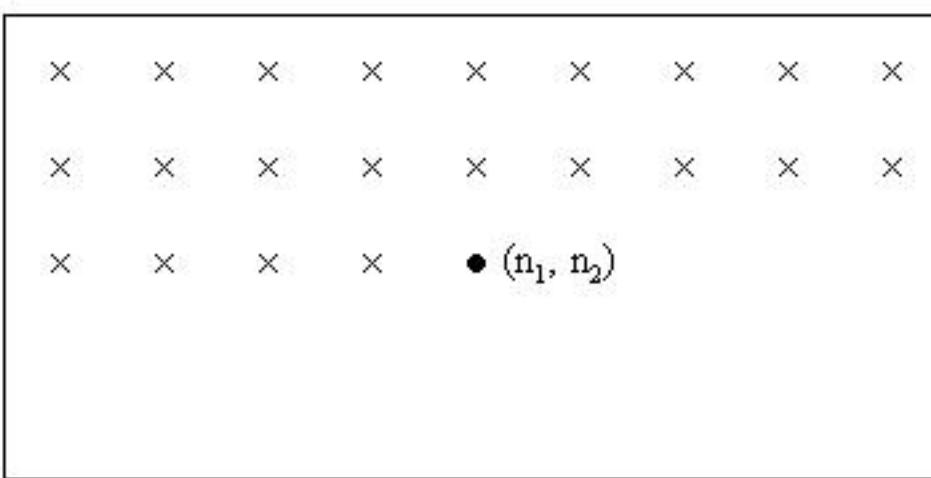


DPCM (Differential Pulse Code Modulation)



K-L Transform

- “Best” linear transform.
 - All transform coefficients are completely uncorrelated
 - “Best” compaction properties. On average, the first M coefficients have more energy than any other transform.
- What is KL?
 - Assume images are samples of a stationary R.P.
 - Covariance function:

$$K_f(n_1, n_2; l_1, l_2) = E[(x(n_1, n_2) - \bar{x}(n_1, n_2))(x(l_1, l_2) - \bar{x}(l_1, l_2))]$$



K-L Transform

- Eigenvectors: A

$$\lambda(k_1, k_2)A(n_1, n_2; k_1, k_2) = \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} K_f(n_1, n_2; l_1, l_2)A(l_1, l_2; k_1, k_2)$$

$$F_k(k_1, k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} f(n_1, n_2)A(l_1, l_2; k_1, k_2)$$

- Assuming a first order Markov model for ρ , one can show:
 - $\rho \rightarrow 1$
 - KLT \rightarrow DCT



1-D DCT

- $x(n) \rightarrow N$ points
- $y(n) = x(n) + x(2N-1-n)$
- $Y(k) = 2N\text{-point DFT } \{y(n)\}$
- $C_x(k) = \begin{cases} W_{2N}^{\frac{k}{2}} Y(k) & 0 \leq k \leq N-1 \\ 0 & \text{otherwise} \end{cases}$

$$W_{2N}^{\frac{k}{2}} = e^{-\frac{j2\pi k/2}{2N}}$$

$$C_x(k) \equiv DCT\{x\}$$



Inverse 1-D DCT

- Step 1

$$Y(k) = \begin{cases} W_{2N}^{-\frac{k}{2}} C_x(k) & 0 \leq k \leq N-1 \\ 0 & k = N \\ W_{2N}^{-\frac{k}{2}} C_x(2N-k) & N+1 \leq k \leq 2N-1 \end{cases}$$

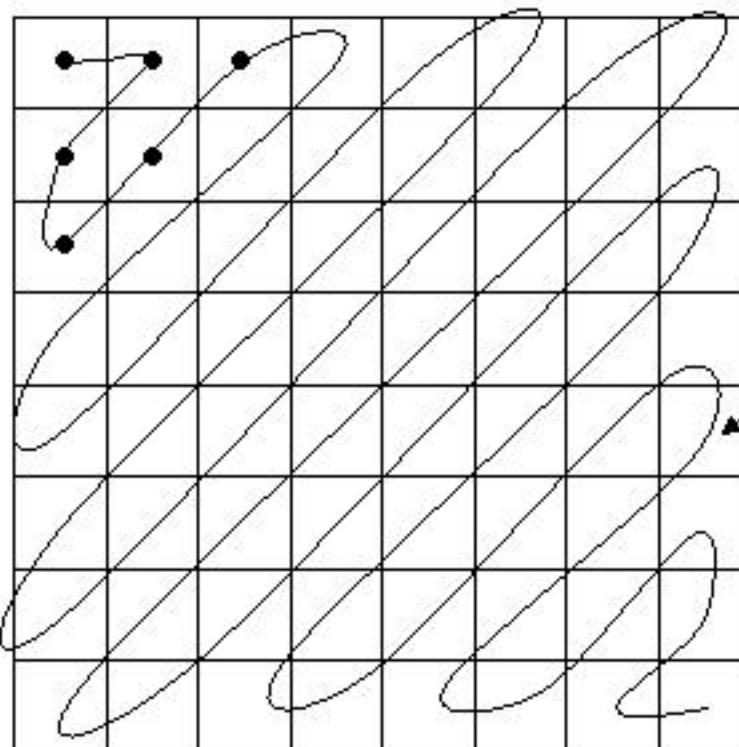
- Step 2 $y(n) = 2N$ -point IDFT $\{Y(k)\}$

- Step 3

$$x(n) = \begin{cases} y(n) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$



Inverse 1-D DCT



Run length
coding

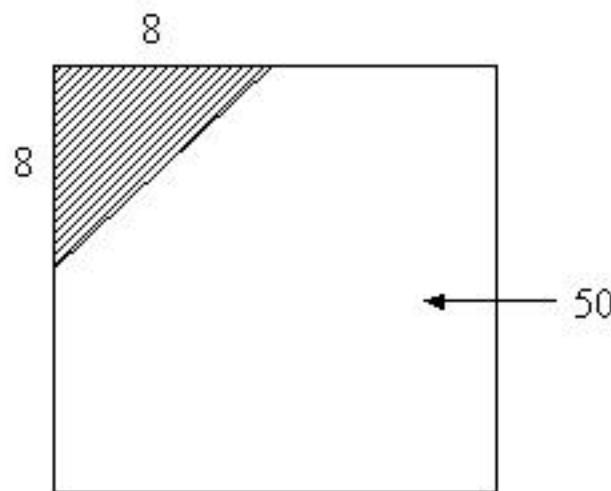
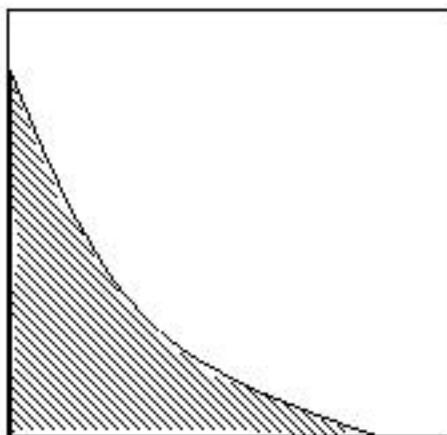
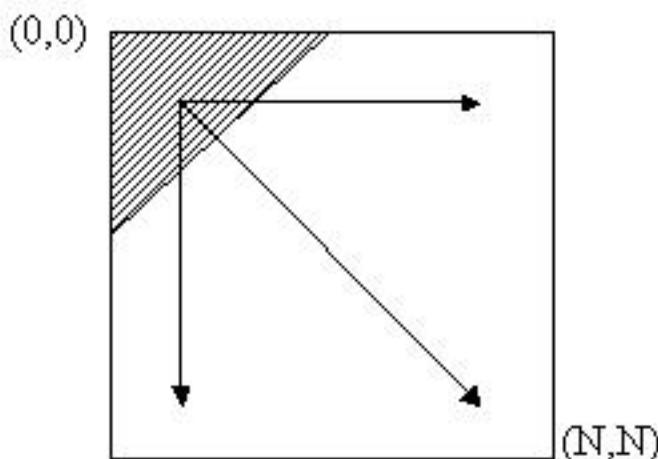
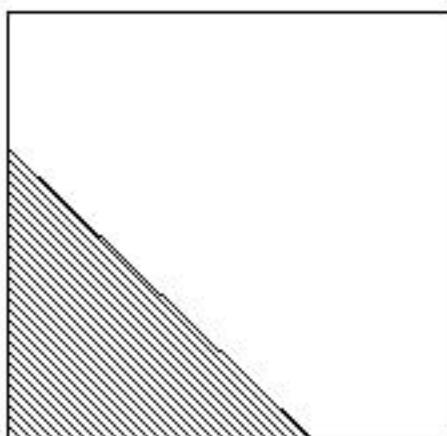
1 1 1 1 0 0 0 1 1 1 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0

3 5

8



Zonal Coding



Zonal Coding

- Assume N scalars $f_i, 1 \leq i \leq N$
- Assume f_i is quantized to L_i reconstruction levels
- Total of B bits are used
- All f_i have some pdf but different variances σ_i^2

$$MSE = \sum_{i=1}^N E[(\hat{f}_i - f_i)^2]$$

$$B_i = \frac{B}{N} + \frac{1}{2} \log_2 \frac{\sigma_i^2}{\left(\prod_{j=1}^N \sigma_j^2 \right)^{1/N}} \quad L_i = \frac{\sigma_i}{\left(\prod_{j=1}^N \sigma_j \right)^{1/N}} 2^{B/N}$$

Jayant and Noll

