

UC Berkeley  
Computer Science  
Lecturer SOE  
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# CS39N

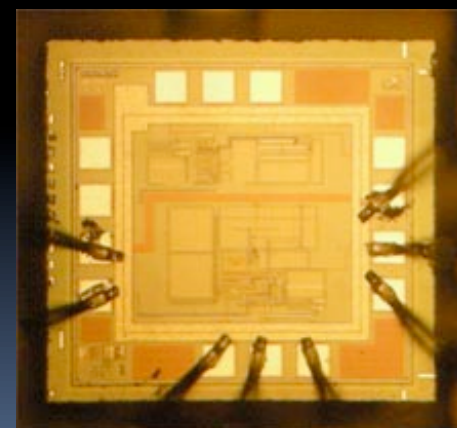
## The Beauty and Joy of Computing

Lecture #11  
Recursion III

2009-11-09

### BATTERY-FREE IMPLANTABLE SENSOR

It has been a challenge to power electronic components implanted within a body. Researchers at UW have developed a new chip that needs less power, and can be powered from outside the body (1 meter away)

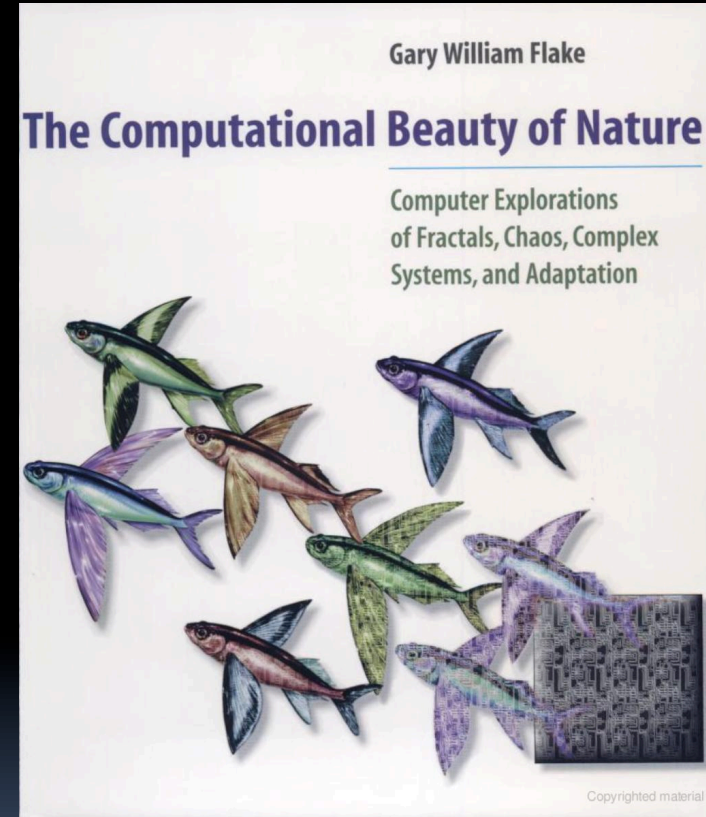


[technologyreview.com/computing/23878](http://technologyreview.com/computing/23878)

# Lecture Overview

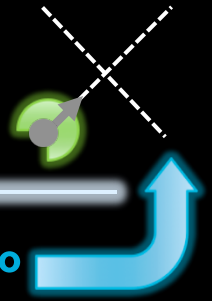
- Local vs Global drawing
- Example: Twig
- Example: C-Curve
- Lindenmayer (L) systems
  - Fibonacci & the golden mean
- Panspermia
- Summary

Great reference text!



# Local vs Global Drawing

(394,27)  
pointing 48°



Goal: Draw a 100-length "+" from point (394,27) at 48°

## Local (turtle) Drawing

- Concept of pen, sprite facing some direction
- State! (where sprite ends)
- Specify via local coords
- Example

- Move turtle to (394,27)
- Face 48°
- Pen down
- Move 100
- Pen up
- Move -50
- Turn right 90°
- Move -50
- Pen down
- Move 100

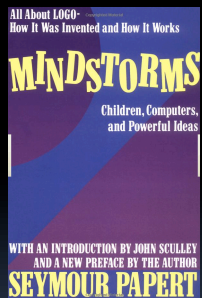


Scratch

← Where sprite ends matters!

## Global (god) Drawing

- No pen, just geometric primitives (DrawLine etc)
- Stateless
- Specify via global coords
- Example
  - SW corner = (394,27)
  - Calculate NE corner
    - How? Simple trig, function of (394,27), 100, 48°
  - DrawLine (SW,NE)
  - Calculate NW corner
  - Calculate SE corner
  - DrawLine (NW,SE)

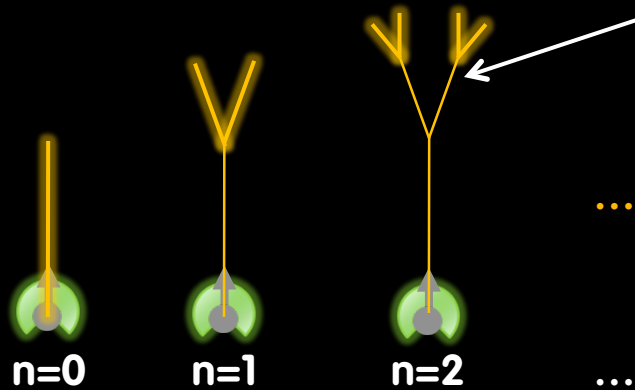


"Turtle"  
Graphics  
invented by  
Papert



# Example: Twig

- Want to draw a twig



Two copies of last recursion level rotated 20°

```
Draw fractal twig size size level level
pen down
move size / 2 steps
pen up
if not level = 0
  turn 20 degrees
  Draw fractal twig size size / 2 level level - 1
  turn 40 degrees
  Draw fractal twig size size / 2 level level - 1
  turn 20 degrees
move 0 - size / 2 steps
```

Notice drawing happens at every level

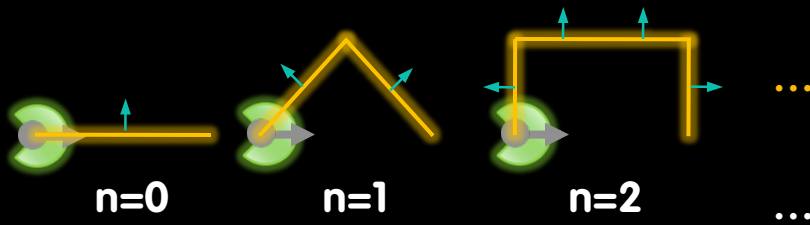
- Where does sprite end?
- Could add randomness
- Lots of variations
  - # branches
  - Angle



# Example: C-curve, Dragon curve

## ■ C-curve

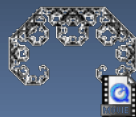
- break a straight line up to form 90° angle
- Left out, Right out



```

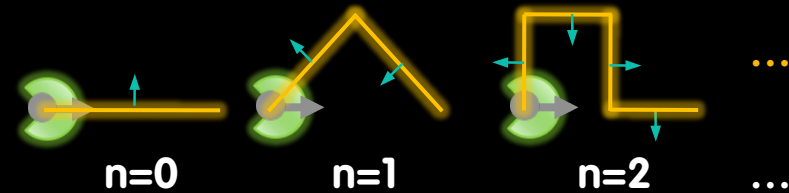
Draw fractal ccurve side side level level
if level = 0
  pen down
  move side steps
  pen up
else
  turn 45 degrees
  Draw fractal ccurve side side / sqrt of 2 level level - 1
  turn 90 degrees
  Draw fractal ccurve side side / sqrt of 2 level level - 1
  turn 45 degrees
  
```

Notice drawing only happens in base case



## ■ Dragon curve

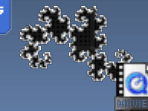
- break a straight line up to form 90° angle
- Left out, Right in



```

Draw fractal dragon side side level level
if level = 0
  pen down
  move side steps
  pen up
else
  turn 45 degrees
  Draw fractal ccurve side side / sqrt of 2 level level - 1
  turn 90 degrees
  move 0 - side / sqrt of 2 steps
  Draw fractal ccurve side side / sqrt of 2 level level - 1
  move 0 - side / sqrt of 2 steps
  turn 135 degrees
  
```

Notice drawing only happens in base case



# Example: Peano Curve

- Simple rewrite rule
  - Replace every straight line w/9 smaller lines, each a third the size (like an 8 with side lines)



- As  $n \rightarrow \infty$ , notice...
  - Space-filling!
  - Converging shape?

```
Draw fractal peano side side level level
if level = 0
  pen down
  move side steps
  pen up
else
  Draw fractal peano side side / 3 level level - 1
  repeat 4
    Draw fractal peano side side / 3 level level - 1
    turn 90 degrees
  turn 90 degrees
  repeat 3
    Draw fractal peano side side / 3 level level - 1
    turn 90 degrees
    turn 180 degrees
  Draw fractal peano side side / 3 level level - 1
```

Notice drawing only happens in base case





# Lindenmayer (L) systems

- **Rewriting system for**
  - Modeling plant growth
  - Generating fractals, languages, ...
- **Basics**
  - Variables (replaced each level)
  - Constants (not replaced)
  - Start (i.e., base case)
  - Rewrite rules (Recursive case)

## ▪ Example: Dragon Curve

- Variables : X Y
- Constants : F + -
- Start: FX
- Rules (angle : 90°)
  - $(X \rightarrow X+YF)$
  - $(Y \rightarrow FX-Y)$
  - F means "draw forward"
  - - means "turn left 90°"
  - + means "turn right 90°".
  - X and Y do not correspond to any drawing action and are only used to control the evolution of the curve

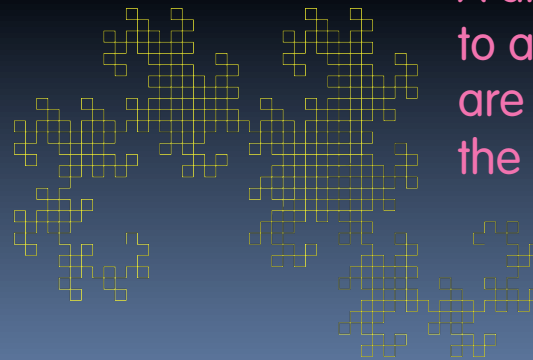
## ▪ Example: Fibonacci

Variables: A B

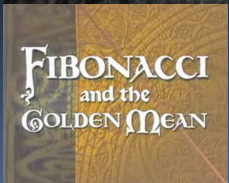
Start: A

Rules:  $(A \rightarrow B, B \rightarrow AB)$

- n = 0 : A (1)
- n = 1 : B (1)
- n = 2 : AB (2)
- n = 3 : BAB (3)
- n = 4 : ABBAB (5)
- n = 5 : BABABBAB (8)
- n = 6 : ABBABBABABBAB (13)
- n = 7 : BABABBABABBABBABABBAB (21)



Leonardo de Pisa  
aka, Fibonacci



# Panspermia

- **panspermia**  $\text{pan}^1\text{sp}^{\text{ə}}\text{m}^{\text{ē}}\text{ə}$ 
  - the theory that life on the earth originated from microorganisms or chemical precursors of life present in outer space and able to initiate life on reaching a suitable environment
- **Also, 1990 film by Karl Sims (famous computer graphics research and artist)**
  - "Attempts were made to bring together several concepts: chaos, complexity, evolution, self propagating entities, and the nature of life itself."



Scenes from Panspermia



L-systems plant growth





# Summary

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- Fractals can model coastlines, clouds, plants, trees, natural growth
  - Fibonacci 1<sup>st</sup> to see this
- When authoring fractals, make sure you're clear when pen goes up/down and where begins/ends
  - Scratch has Turtle graphics
- Infinite recursion = fun

