CS10: The Beauty and Joy of Computing

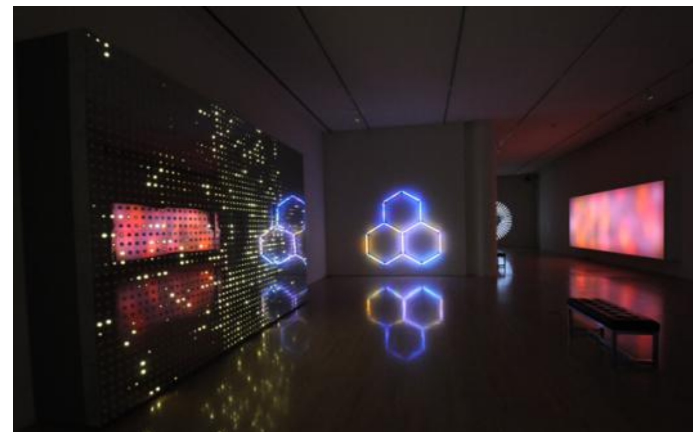
Lecture #7: Algorithm Complexity

TA Jon Kotker (2010-09-27)

LEDs + Math = Art

Leo Villareal combines modern LED control systems to produce contemporary modern art. The exhibit is on display at the San Jose Museum of Art.

http://news.cnet.com/8301-13772_3-20017310-52.html



BIG IDEA

- Many ways to do the same thing = many *algorithms* to accomplish the same task.
- Example: Distributing candy!
- Example: Searching through a list of numbers to find a specific number.
  - *Linear* search (list is unsorted): Go through the list number by number and check if each number is The One.
  - *Binary* search (list is sorted): Look at the middle of the list. If it is not The One, break the list into two smaller halves and ignore the irrelevant half.
  - Any other algorithms?

MAKING A DECISION

How do we decide which algorithm to use?

- Which is easier to implement?
- Which takes less time?
- Which uses up less space (memory)?
- Which gives a more precise answer?
- Which of the above questions even *matter*?

WHAT DO *YOU* THINK?

Which of the factors below is most important in making a choice between two algorithms?

A. Which is easier to implement?
B. Which takes less time?
C. Which uses up less space (memory)?
D. Which gives a more precise answer?
E. I don't know / I don't have an opinion.

RUNTIME ANALYSIS

One commonly used criterion in making a decision is **runtime** – how much time does the algorithm take to run and finish its task?

Computers are most useful for large inputs, so find the runtime of the algorithm on large inputs.

How do we do that?

RUNTIME ANALYSIS

Time the algorithm with a stopwatch! But…

- Different computers will have different runtimes. ☹
- Same computer can have different runtime on the same input. ☹
- Need to implement the algorithm first so that we can run it. o_O;

Solution: Need to somehow *abstract* the computer away from the algorithm.

RUNTIME ANALYSIS

Idea: Do not focus on how long the algorithm takes on one input. Instead, focus on how the **worst-case** runtime of the algorithm *scales* as we scale the input.

Why? Abstracts the computer out. A good algorithm should work well, no matter what the computer = a good recipe should produce a great dish, no matter what the kitchen.

RUNTIME ANALYSIS

Idea: Do not focus on how long the algorithm takes on one input. Instead, focus on how the **worst-case** runtime of the algorithm *scales* as we scale the input.

Why? Computers are mainly used for large sets of data. The runtime of an algorithm should scale "reasonably" as we make the dataset even larger, or else we need to improve/discard that algorithm.

Dangerous for Math majors.

A *LOT* OF APPROXIMATION AHEAD!

Courtesy http://t5bg.roblox.com/bdc77779cfe00dc3291623e373969193

ARITHMETIC OPERATIONS

Key Idea: As the input scales, arithmetic operations take approximately the same time. Arithmetic operations are **constant-time** operations.

Another Key Idea: We only care about how the runtime of the block scales as the input *scales*!

LAUNDRY

It Must Be Done

WASHING LOADS

If each load takes about the same time to launder, how does the runtime scale as the number of loads doubles? Triples?

WASHING LOADS

Key Idea: The runtime of the algorithm *scales by the same amount* as the size of its input scales.

Doing laundry is a **linear-time** operation *in the number of loads*.

FINDING CLOTHES TO WEAR

find a good pair to wear in (number) shirts and pants

How does the **worst-case** total time to find a good pair scale as the number of shirts and pants doubles?

1. Stays the same.
2. Doubles.
3. Halves.
4. Quadruples.
5. Buh?

Hint: For each shirt that I own, how many pants do I have to match against it?

FINDING CLOTHES TO WEAR

Key Idea: If I have 3 shirts and pants, there are 9 different combinations that I can try, because for *each* shirt, I can try on 3 pants to match with it.

Double it: If I have **6** shirts and pants, there are **36** different combinations that I can try.

If I double the number of shirts and pants that I have, then the number of different combinations that I can try **quadruples**.

FINDING CLOTHES TO WEAR

Key Idea: The runtime of the algorithm scales by the **square** of the amount that the input scales by.

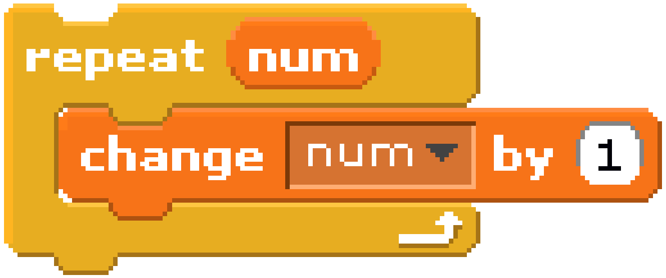
Finding a good pair of clothes to wear is a **quadratic-time** algorithm *in the number of shirts and pants*.

LAUNDRY

It Has Been Done

RUNTIME ANALYSIS

What is the runtime of this script?



1. Constant in `num`.
2. Linear in `num`.
3. Quadratic in `num`.
4. Buh?

RUNTIME ANALYSIS

What is the runtime of this script?

1. Constant in `num`.
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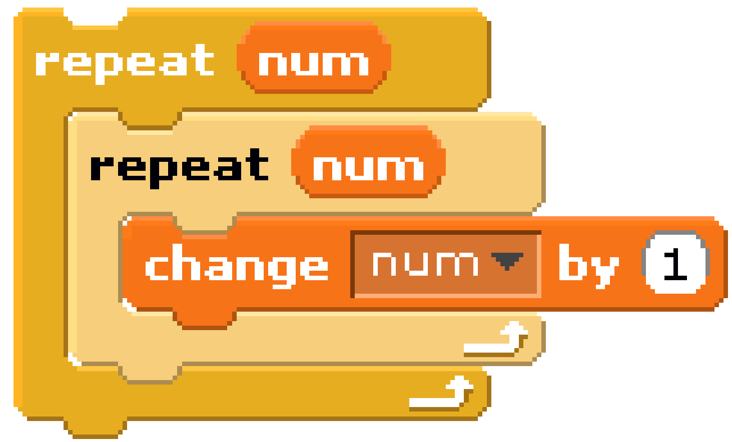
RUNTIME ANALYSIS

What is the runtime of this script?



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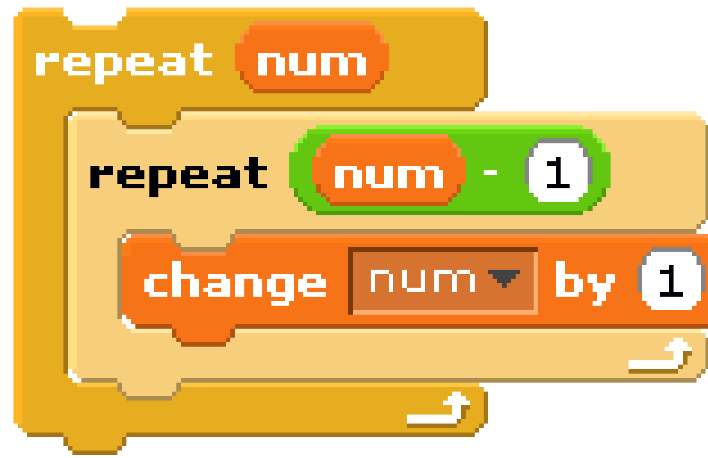
RUNTIME ANALYSIS

What is the runtime of this script?



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4. Buh?

IT'S ALL APPROXIMATE!

Which is better: a **linear-time** algorithm or a **quadratic-time** algorithm?

Input Size (N)	10	100	1000	10000	100000
Linear (msec)	C	10C	100C	1000C	10000C
Quadratic (msec)	C	100C	10000C	10^6C	10^8C

As the input size increases, the quadratic-time algorithm takes so much more time than the linear-time algorithm that the linear-time algorithm is *negligible* in comparison.

IT'S ALL APPROXIMATE!

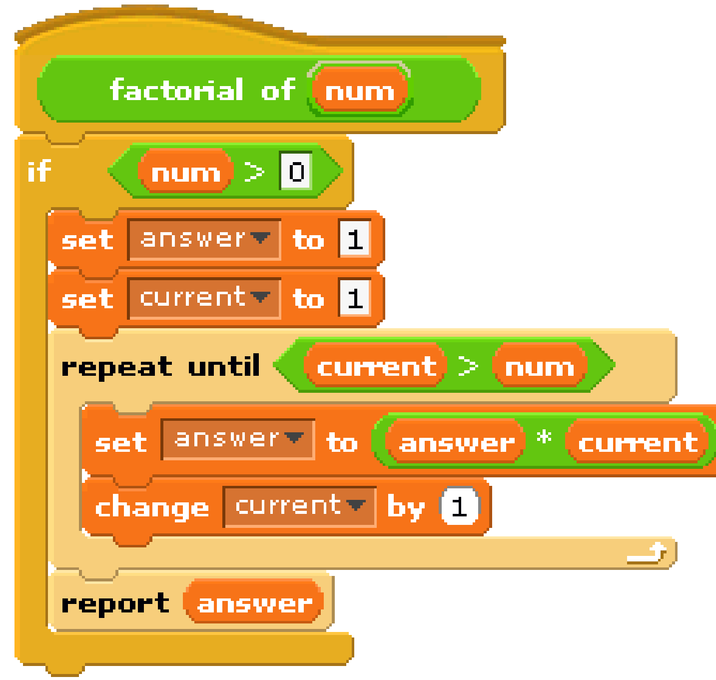
Which is better: a **linear-time** algorithm or a **quadratic-time** algorithm?

Input Size (N)	**10**	**100**	**1000**	**10000**	**100000**
Linear (msec)	C	10C	100C	1000C	10000C
Quadratic (msec)	C	100C	10000C	10^6C	10^8C

Since we only consider large sized inputs, expressions like $N^2 - N$ or $N^2 + N$ are considered approximately equal to N and thus **quadratic-time**; the linear-time part is ignored.

RUNTIME ANALYSIS (EXTRA)

What is the runtime of this algorithm to find the factorial of a number?



1. Constant in the number.
2. Linear in the number.
3. Quadratic in the number.
4. Buh?