CS10
The Beauty and Joy of Computing

UC Berkeley EECS
Summer Instructor
Ben Chun

Lecture #7
Algorithmic Complexity

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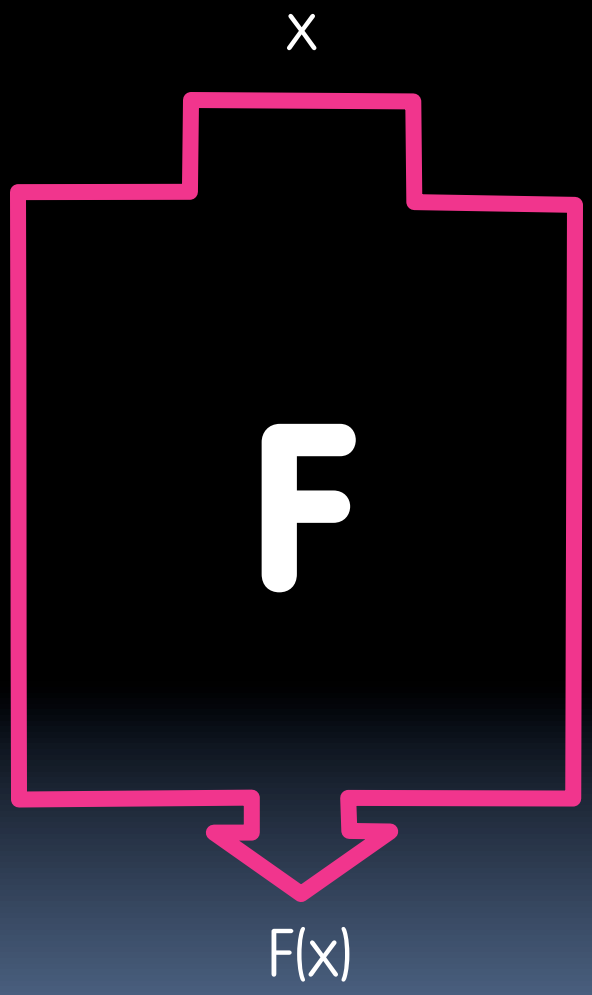
SEVEN MINUTES OF TERROR

The Curiosity Mars rover will use a "sky crane" system to slow down after it enters the atmosphere – and software will be flying. It takes 14 minutes for a radio signal to travel from Mars to Earth, but the whole landing will be over in just 7 minutes.

http://www.youtube.com/watch?v=pzqdoXwLBT8

Functional Abstraction (review)

- A **function** has inputs & outputs
  - Possibly no inputs
  - Must have outputs (or else the block is a command, probably with side effects)
- The contract describing what that block does is called a **specification** or **spec**



What IS in a spec?

- Typically they all have
  - NAME
  - INPUT(s)
    - (and types, if appropriate)
    - Requirements
  - OUTPUT (or NONE)
  - (SIDE-EFFECTS)
  - EXAMPLES
- Example
  - NAME : Double
  - INPUT : n (a number)
  - OUTPUT: n + n

What IS NOT in a spec?

- **How!**
  - That's the beauty of a functional abstraction; it doesn't say how it will do its job.
- **Example: Double**
  - Could be n * 2
  - Could be n + n
  - Could be n+1 (n times)
    - if n is a positive integer
- **This gives great freedom to author!**
  - You choose Algorithm(s)!

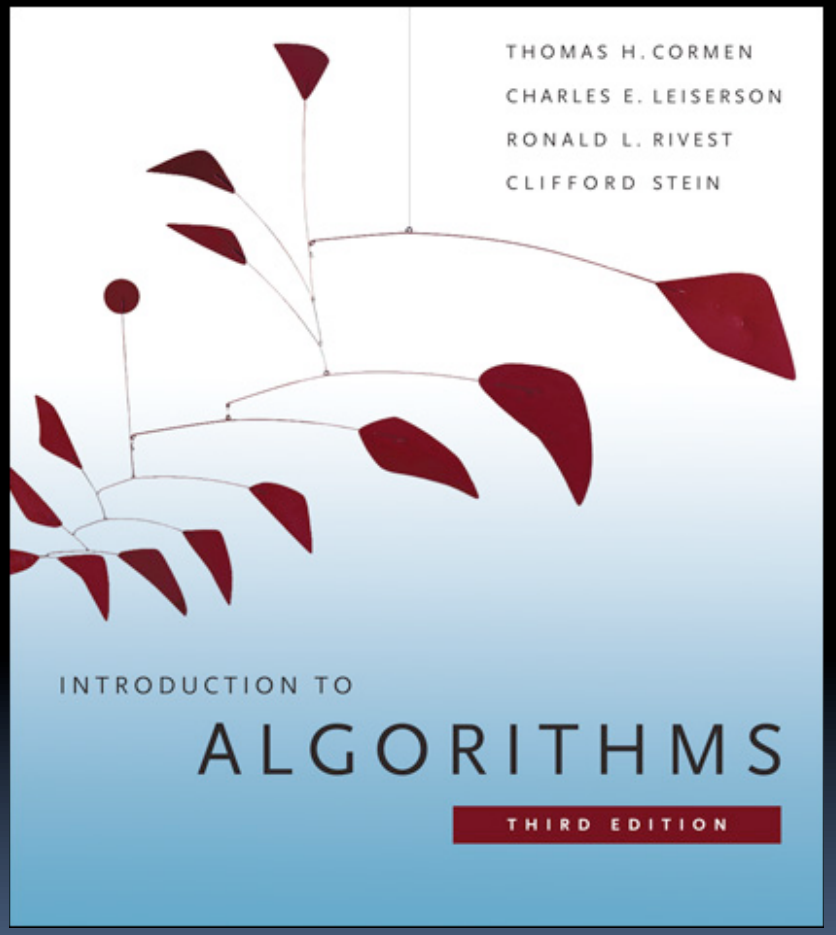
What do YOU think?

Which factor below is the most important in choosing the algorithm to use?

A. Simplest?
B. Easiest to implement?
C. Takes less time?
D. Uses up less space (memory)?
E. Gives a more precise answer?

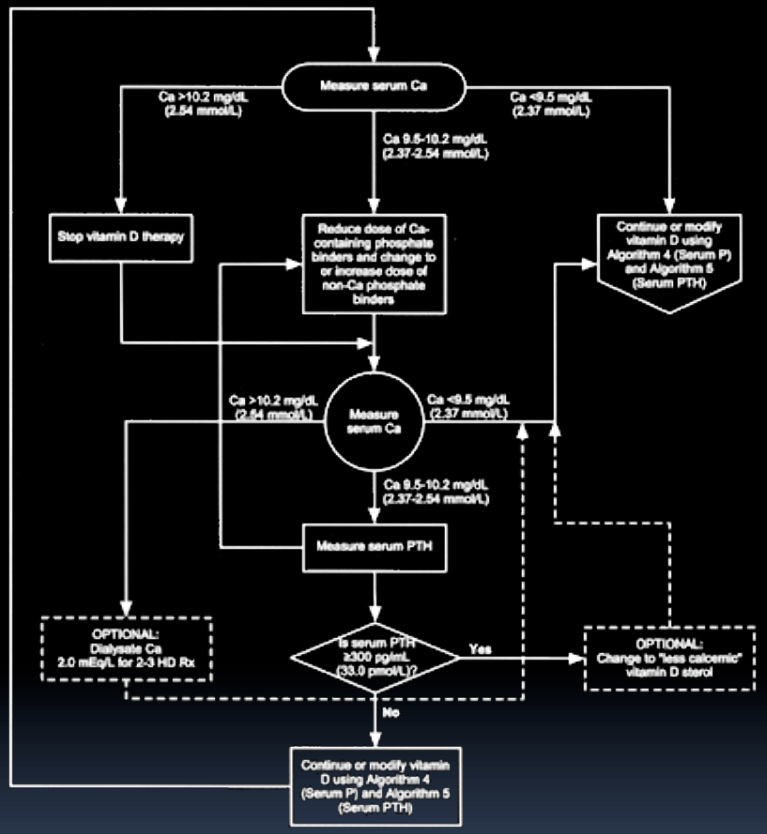
Reference text

- **This book launched a generation of CS students into Algorithm Analysis**
  - It's on everyone's shelf
  - It might be hard to grok now, but if you go on in CS, remember it & own it!
  - Get the most recent version



Algorithm analysis : the basics

- **An algorithm is correct if, for every input, it reports the correct output and doesn't run forever or cause an error.**
  - Incorrect algorithms may run forever, or may crash, or may not return the correct answer.
    - They could still be useful!
    - Consider an approximation…
  - For now, we'll only consider correct algorithms



Algorithm for managing Vitamin D sterols based on serum calcium levels.

www.kidney.org/professionals/kdoqi/guidelines_bone/guide8b.htm

Algorithm analysis : running time

- One commonly used criterion in making a decision is **running time**
  - How long does the algorithm take to run and finish its task?
- How do we measure it?

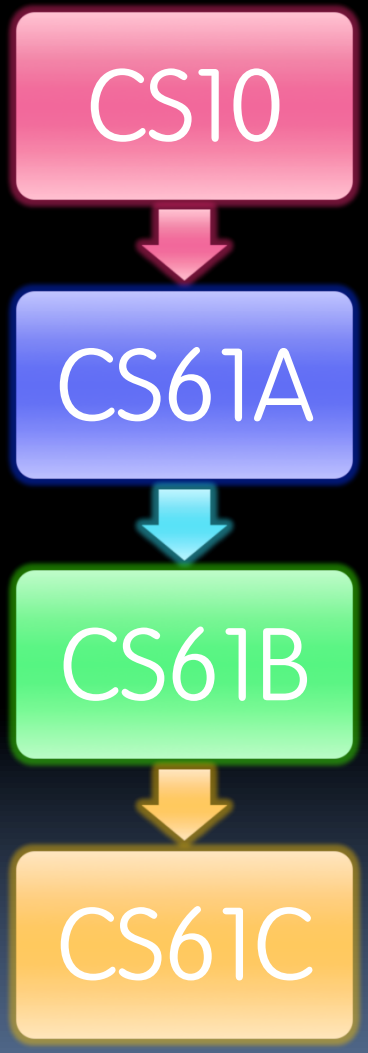
Runtime analysis problem & solution

- **Time w/stopwatch, but…**
  - Different computers may have different runtimes. ☹
  - Same computer may have different runtime on the same input. ☹
  - Need to implement the algorithm first to run it. ☹

- ***Solution*: Count the number of "steps" involved, not time!**
  - Each operation = 1 step
  - When we say "running time" we mean number of steps, not time on the clock!

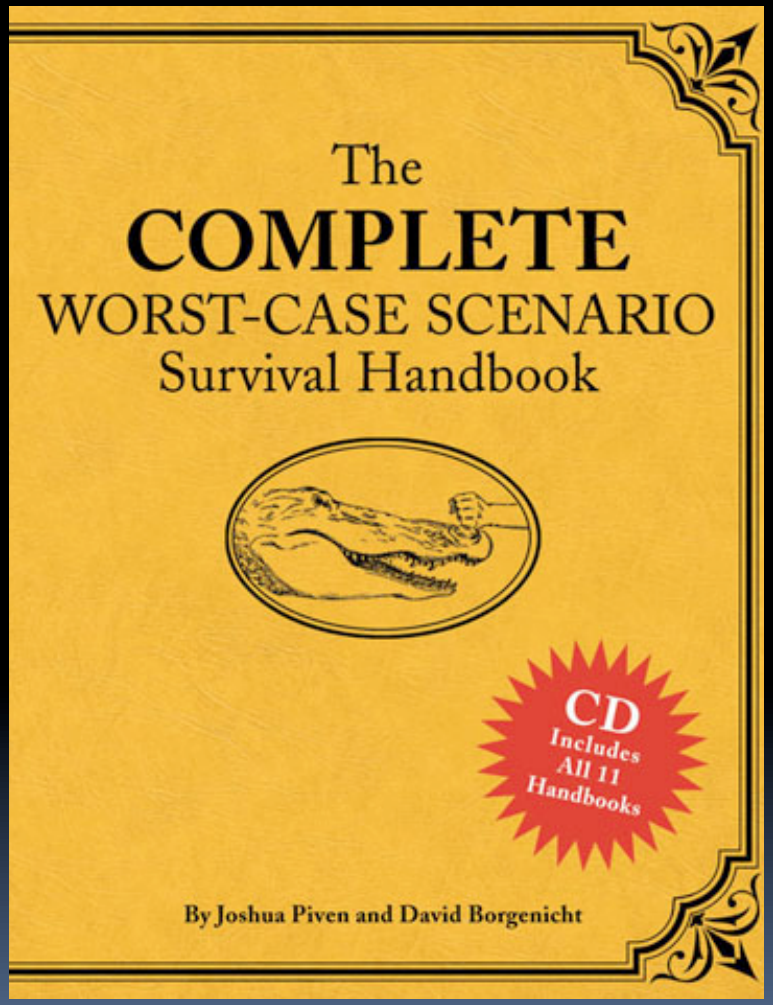
Runtime analysis : input size & efficiency

- **Definition**
  - Input size: the # of things in the input.
  - E.g., # of things in a list
  - Running time as a function of input size
  - Measures efficiency
- **Important!**
  - In CS10 we won't care about the efficiency of your solutions!
  - …in CS61B we will



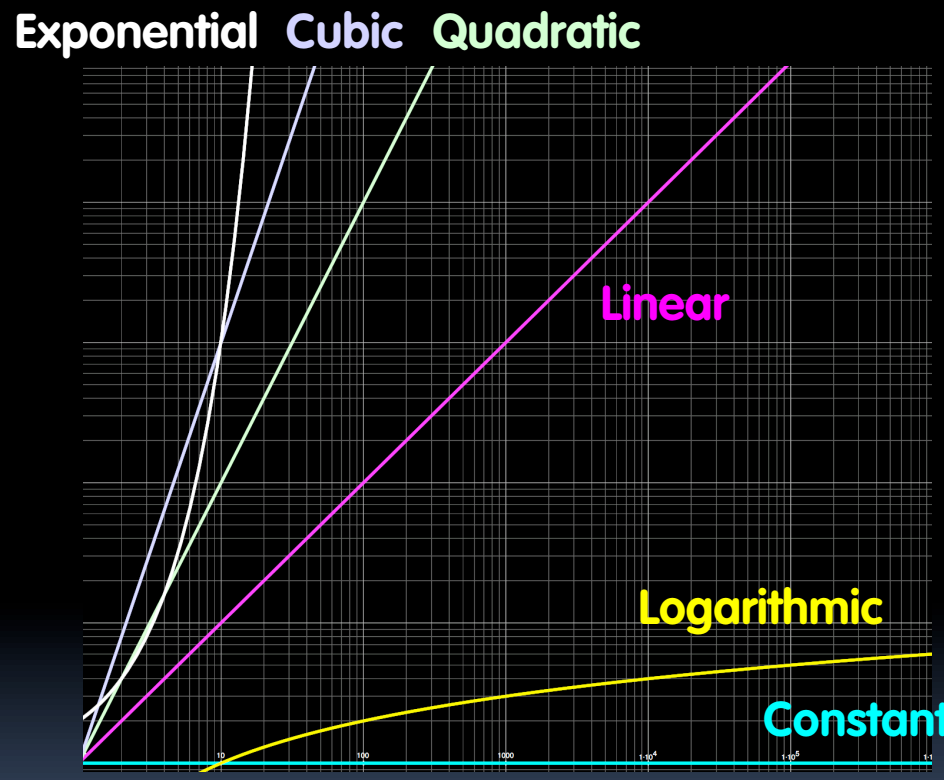
Runtime analysis : worst or avg case?

- **Could use avg case**
  - Average running time over a vast # of inputs
- **Instead: use worst case**
  - Consider running time as input grows
- **Why?**
  - Nice to know most time we'd ever spend
  - Worst case happens often
  - Avg is often ~ worst



Runtime analysis: Final abstraction

- Instead of an exact number of operations we'll use abstraction
  - Want order of growth, or dominant term
- In CS10 we'll consider
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- E.g. $10\ n^2 + 4 \log n + n$
  - …is quadratic



Graph of order of growth curves on log-log plot

Example: Finding a student (by ID)

- **Input**
  - Unsorted list of students L
  - Particular student S
- **Output**
  - True if S is in L, else False
- **Pseudocode Algorithm**
  - Go through one by one, checking for match.
  - If match, true
  - If exhausted L and didn't find S, false

- **Worst-case running time as function of the size of L?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Example: Finding a student (by ID)

- **Input**
  - Sorted list of students L
  - Particular student S
- **Output : same**
- **Pseudocode Algorithm**
  - Start in middle
  - If match, report true
  - Else throw away half of L and check again in the middle of remaining part of L
  - If nobody left, report false

- **Worst-case running time as function of the size of L?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Example: Finding a student (by ID)

- Same problem, with a new twist
- What if L were given to you in advance and you had infinite storage?
- What's the best you could do?

- Worst-case running time as function of the size of L?

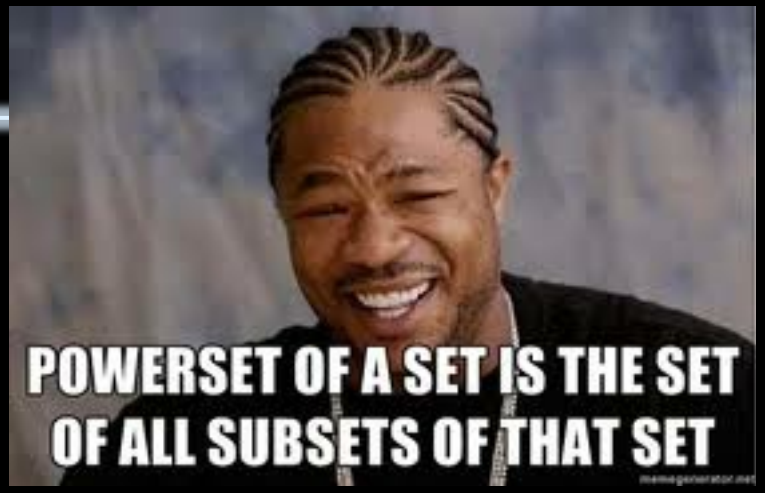
1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Example: Shared birthday?

- **Input**
  - Unsorted list L (of size n) of birthdays of team
- **Output**
  - True if any two people shared birthday, else False
- **Think about the algorithm you would use and how many steps it will take**
- **Worst-case running time as function of n?**
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Example: Power set

- **Input:**
  - Unsorted list L (of size n) of people
- **Output**
  - All the subsets
- **E.g., for 3 people (a,b,c):**
  - 1x empty: { }
  - 3x 1-person: {a, b, c}
  - 3x 2-person: {ab, bc, ac}
  - 1x 3-person: {abc}



- **Worst-case running time as function of n?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Summary

- **When choosing algorithm, could optimize for**
  - Simplest
  - Easiest to implement?
  - Most efficient
  - Uses up least resources
  - Gives most precision
  - …
- **In CS10 we'll consider**
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential

How does the goalie choose how to block the ball?