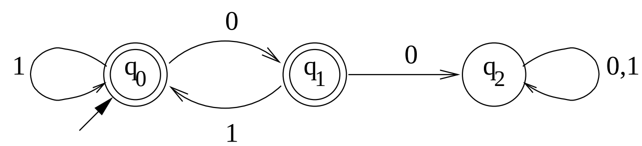
# Homework 4

**Out:** 19 Feb, 2009
**Due:** 26 Feb., 2009

*Note: Questions marked with an asterisk (*) are to be handed in. The others are for practice and will not be graded. Put your solutions to the (*) problems in the (now unique) homework box on Soda level 2 by 4pm on the due date. The usual remarks about clear answers and the collaboration policy still hold. Depending on grading resources, we may grade only a random subset of the problems and check off the rest; so you are advised to attempt all questions.*

1. This problem consists of four parts, three of which are designed to improve your understanding of the Myhill-Nerode Theorem and the DFA minimization algorithm.

   (a) (*) Recall the following DFA $M$, which we saw in Homework 1:

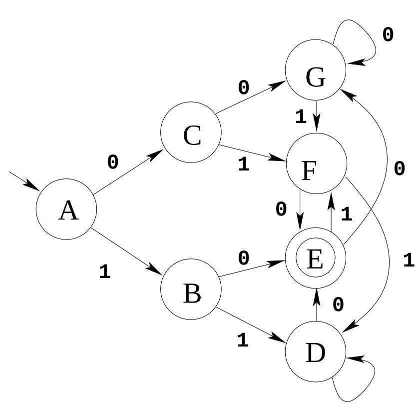



   Determine the equivalence relation $R_M$ for this DFA. You should specify the equivalence relation by writing down each of its equivalence classes in the form of a regular expression. [NOTES: (i) Recall that the relation $R_M$ is defined by $xR_My$ iff $M$ ends up in the same state on inputs $x$ and $y$. (ii) Recall also from HW1 that you have already described the set of strings that take $M$ to each of its states.]

   (b) (*) In class we showed that if $L$ is a regular language, then the equivalence relation $R_L$ (indistinguishability) has only finitely many equivalence classes (these correspond to the states of the minimal DFA for $L$). Now consider the language $L = \{0^n1^n : n \geq 0\}$. By describing the equivalence classes of $R_L$, prove that $L$ is *not* regular. [NOTES: (i) Recall that the relation $R_L$ is defined by $xR_Ly$ iff $\forall z(xz \in L \Leftrightarrow yz \in L)$. Of course, we could also prove that $L$ is not regular using the pumping lemma; the point of this problem is to get you to use a different method based on the Myhill-Nerode Theorem.]

   (c) Now use the pumping lemma to show that $L$ is not regular.

   (d) (*) Apply the minimization procedure discussed in class to construct a minimal DFA that is equivalent to the following DFA. Show clearly the steps you used to arrive at your answer; you should consider the pairs of states in lexicographic order.



2. Consider the language $L = \{w = a^i b^j c^k : i, j, k \geq 0$ and if $i = 1$ then $j = k\}$.

   (a) Show that $L$ "acts like" a regular language with respect to the pumping lemma. Specifically, give a pumping length $p$ and show that $L$ satisfies the conditions of the lemma for this $p$.

   (b) Now show that $L$ is *NOT* regular.

   (c) Why is this not a contradiction?

3. (*) Show that for any positive integer $n$ there is a language $L_n$ for which both of the following statements hold

   (a) There is a DFA with $n$ states that recognizes $L_n$ and

   (b) No DFA with fewer than $n$ states recognizes $L_n$.

4. The pumping lemma says that every regular language $L$ has a pumping length $p$, and that any string $\underline{w} = w_1 w_2 \cdots w_n \in L$ with $n \geq p$ can be pumped. Clearly if $p$ is a pumping length for $L$, so is $p'$ with $p' \geq p$. The **minimum pumping length** for $L$ is the smallest $p$ that is a pumping length for $L$. [e.g., when $L = 01^*$, the minimum pumping length is 2: the string $\underline{w} = 0$ is in $L$, has length 1, but cannot be pumped (why?), but *any* string in $L$ of length $\geq 2$ must contain a 1 and can be pumped (why?, how?)].

   For each of the following languages, find the minimum pumping length and justify your answer.

   (a) $L = 0001^*$

   (b) (*) $L = 1011$

   (c) $L = 0^*1^*$

   (d) (*) $L = 001 \cup 0^*1^*$

   (e) (*) $L = 10(11^*0)0$

   (f) $L = 0^*1^*0^*1^* \cup 10^*1$

   (g) (*) $L = (01)^*$

   (h) (*) $L = \varepsilon$

   (i) (*) $L = 1^*01^*01^*$