CS-184: Computer Graphics

Lecture #14: Subdivision

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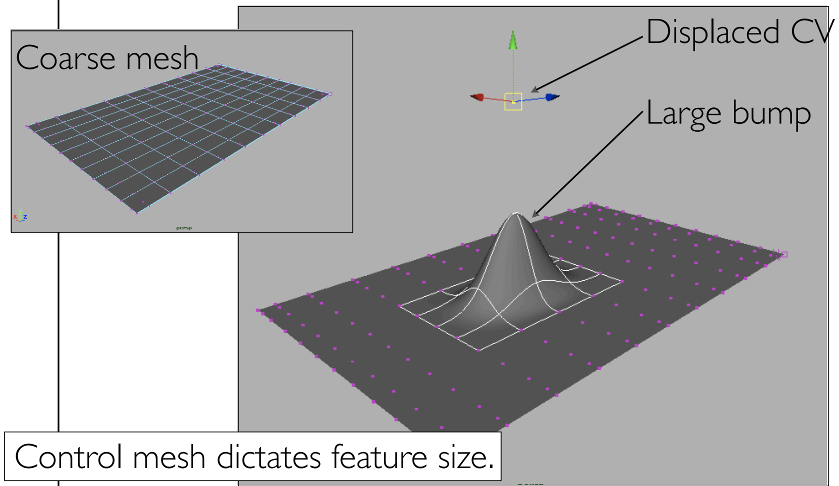
Subdivision

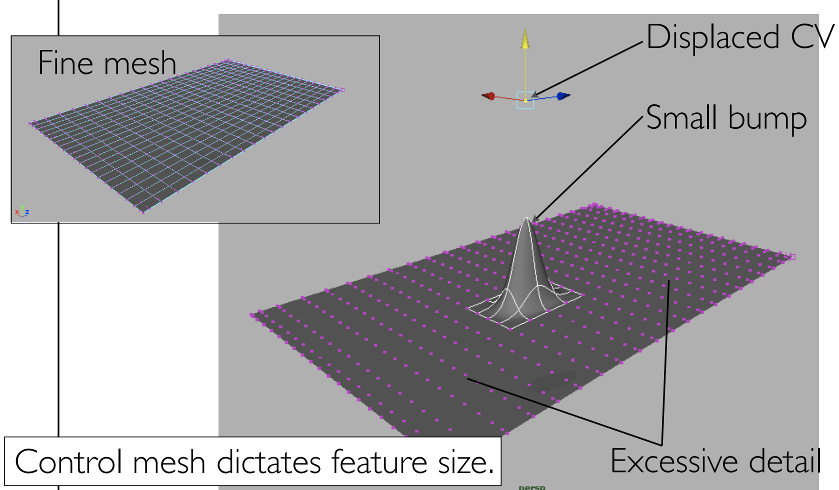
- Start with:
  - Given control points for a curve or surface, find new control points for a sub-section of curve/surface
- Key extension to basic idea:
  - Generalize to non-regular surfaces

Consider NURBS Surface



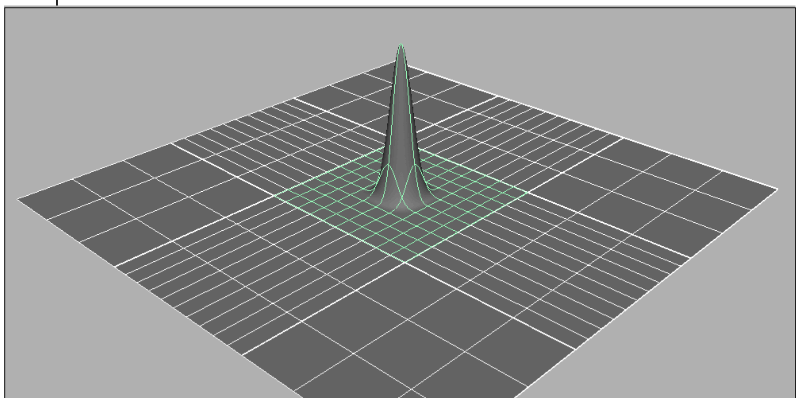
Control mesh dictates feature size.

Consider NURBS Surface



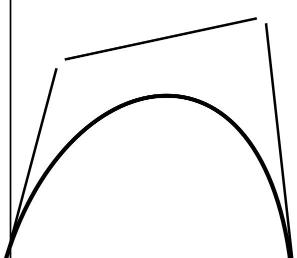
Control mesh dictates feature size.

Tensor Product Surface



Refinement must be constant across *u* or *v* directions

Bézier Subdivision



$$u \in [0..1]$$

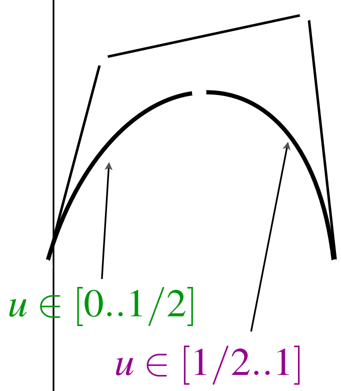
$$\mathbf{x}(u) = \sum_i b_i(u)\,\mathbf{p}_i$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\,\beta_Z\,\mathbf{P}$$

Vector of control points

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

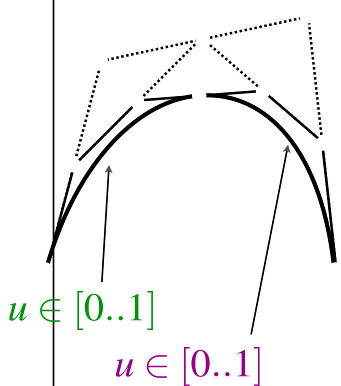
Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3]\, \beta_Z \mathbf{P}$$

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision

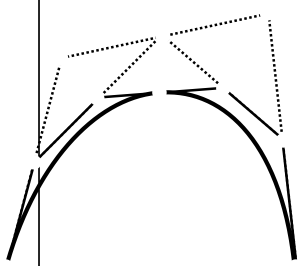


$$\mathbf{x}(u) = [1, u, u^2, u^3]\, \beta_Z \mathbf{P}$$

Can't change these...

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3]\, \beta_Z \mathbf{P} \quad u \in [0..\tfrac{1}{2}]$$

$$\mathbf{x}(u) = [1, \frac{u}{2}, \frac{u^2}{4}, \frac{u^3}{8}] \beta_Z \mathbf{P} \quad u \in [0..1]$$

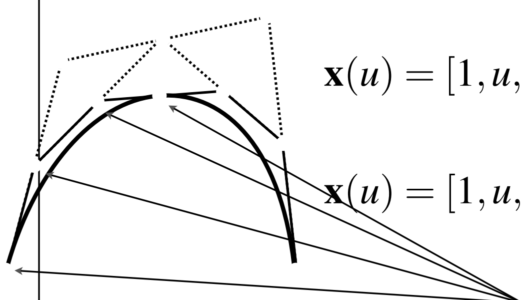
$$\mathbf{x}(u) = [1, u, u^2, u^3] \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/8 \end{bmatrix}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \beta_Z^{-1} \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

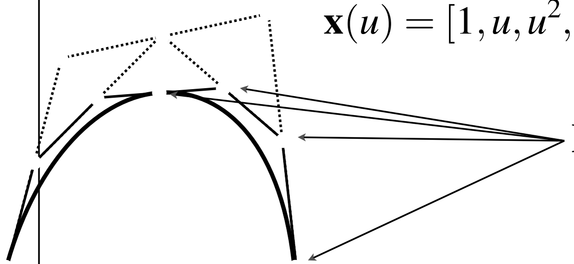
$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}_1$$

$$\mathbf{P}_1 = \mathbf{H}_{Z1} \mathbf{P}$$

$$\mathbf{H}_{Z1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix}$$

Bézier Subdivision

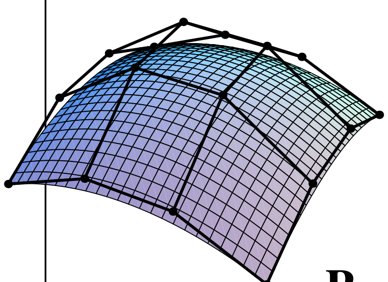
$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \mathbf{P}_2$$

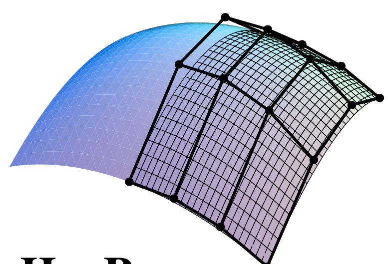


$$\mathbf{P}_2 = \mathbf{H}_{Z2}\mathbf{P}$$

$$\mathbf{H}_{Z2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{S}_2 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{8} \\ 0 & 0 & \frac{1}{4} & \frac{3}{8} \\ 0 & 0 & 0 & \frac{1}{8} \end{bmatrix}$$

Bézier Subdivision



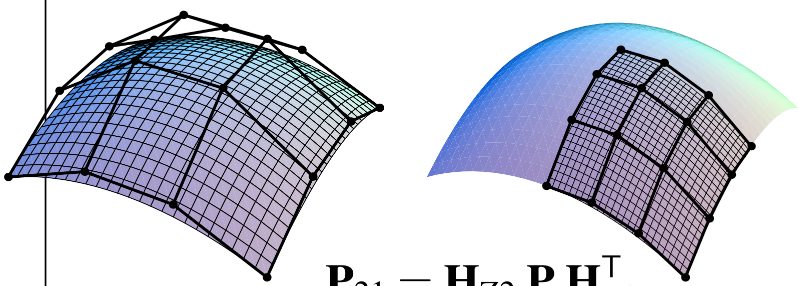


$$\mathbf{P}_{2\cdot} = \mathbf{H}_{Z2}\,\mathbf{P}$$

$$\mathbf{x}(u, v) = [1, u, u^2, u^3]\beta_Z \mathbf{P} \beta_Z^{\mathsf{T}}[1, v, v^2, v^3]^{\mathsf{T}}$$

4 x 4 matrix of control points

Bézier Subdivision

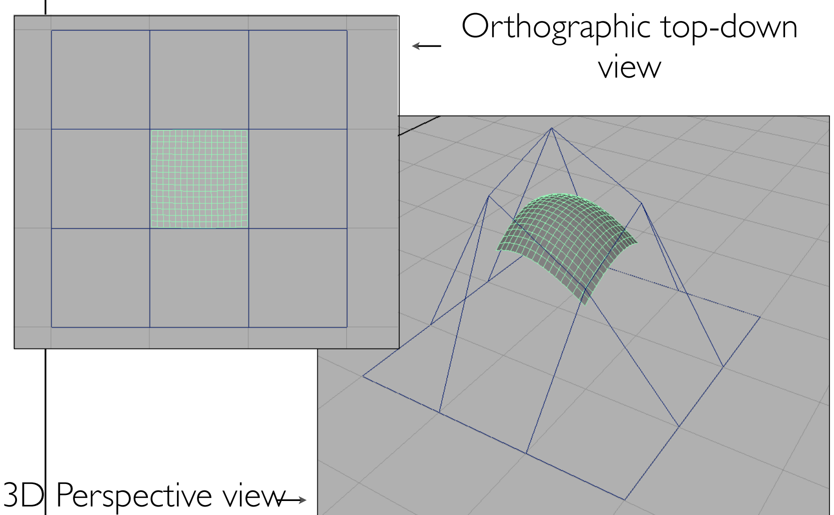


$$\mathbf{P}_{21} = \mathbf{H}_{Z2}\,\mathbf{P}\,\mathbf{H}_{Z1}^{\mathsf{T}}$$

$$\mathbf{x}(u,v) = [1,u,u^2,u^3]\beta_Z \mathbf{P} \beta_Z^{\mathsf{T}} [1,v,v^2,v^3]^{\mathsf{T}}$$

4 × 4 matrix of control points

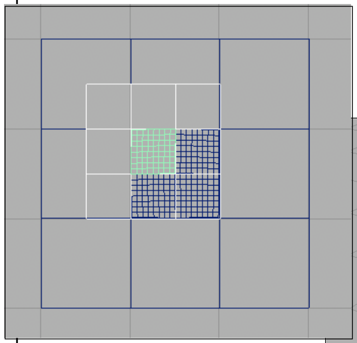
Regular B-Spline Subdivision

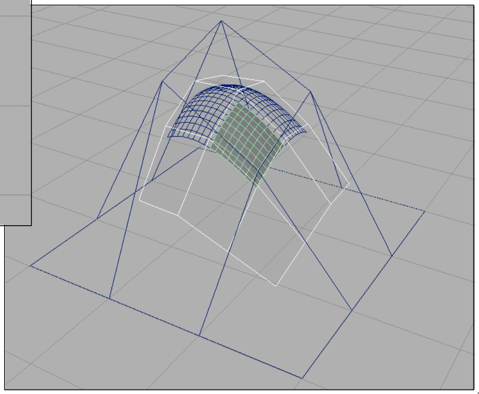


Orthographic top-down view

3D Perspective view

Regular B-Spline Subdivision

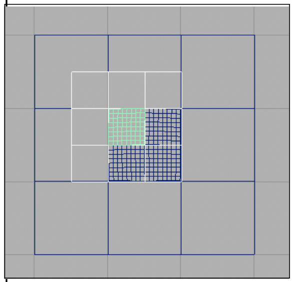


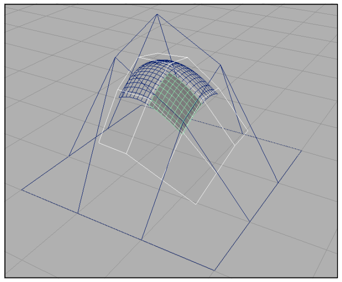


← Orthographic top-down view

3D Perspective view →

Regular B-Spline Subdivision

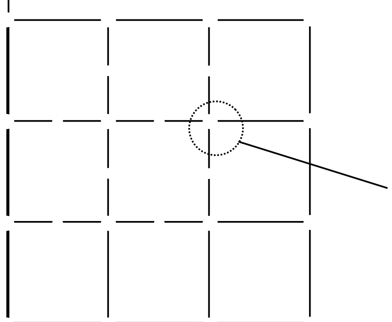




$$\mathbf{x}(u,v) = [1,u,u^2,u^3]\beta_\mathrm{B}\,\mathbf{P}\,\beta_\mathrm{B}^\mathsf{T}[1,v,v^2,v^3]^\mathsf{T}$$

$$\mathbf{P}_{11} = \mathbf{H}_{\mathrm{B}1}\,\mathbf{P}\,\mathbf{H}_{\mathrm{B}1}^\mathsf{T}$$

Regular B-Spline Subdivision

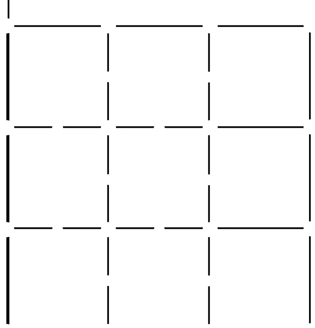


$$\mathbf{P}_{11} = \mathbf{H}_{\mathrm{B}1}\,\mathbf{P}\,\mathbf{H}_{\mathrm{B}1}^{\mathsf{T}}$$

In this parametric view these knot points are collocated.

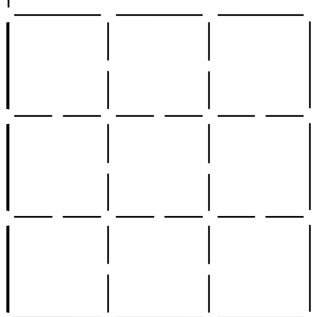
The 3D control points are not.

Regular B-Spline Subdivision



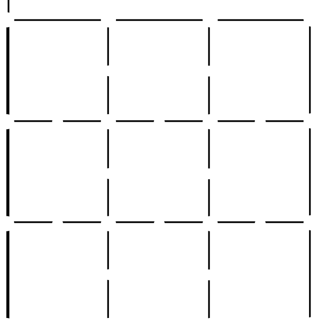
$$\mathbf{P}_{11} = \mathbf{H}_{\mathrm{B}1}\,\mathbf{P}\,\mathbf{H}_{\mathrm{B}1}^{\mathsf{T}}$$
$$\mathbf{P}_{12} = \mathbf{H}_{\mathrm{B}1}\,\mathbf{P}\,\mathbf{H}_{\mathrm{B}2}^{\mathsf{T}}$$

Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\top}$$
$$\mathbf{P}_{12} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B2}^{\top}$$
$$\mathbf{P}_{22} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B2}^{\top}$$

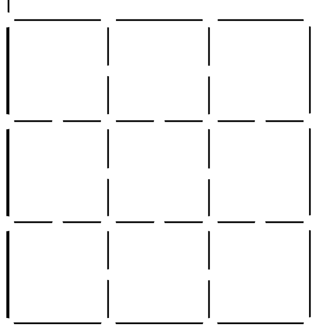
Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\top}$$
$$\mathbf{P}_{12} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B2}^{\top}$$
$$\mathbf{P}_{22} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B2}^{\top}$$
$$\mathbf{P}_{21} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B1}^{\top}$$

$$\mathbf{H}_{B1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \end{bmatrix} \quad \mathbf{H}_{B2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

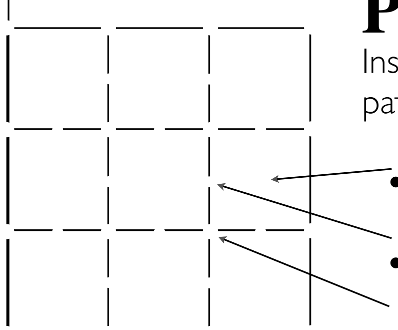
Regular B-Spline Subdivision



$$\mathbf{P}^{i+1} = \mathbf{H}\,\mathbf{P}^{i}$$

- $\mathbf{P}^{i+1}$: Length 25 vector of fine CPs
- $\mathbf{H}$: 25 × 16 subdivision matrix
- $\mathbf{P}^{i}$: Length 16 vector of coarse CP

Regular B-Spline Subdivision



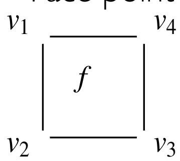
$$\mathbf{P}^{i+1} = \mathbf{H}\,\mathbf{P}^{i}$$

Inspection would reveal a pattern

- Face points
- Edge points
- Vertex points

Regular B-Spline Subdivision

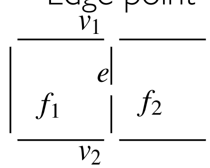
Face point

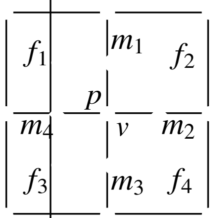


$$f = \frac{v_1 + v_2 + v_3 + v_4}{4}$$

$$e = \frac{v_1 + v_2 + f_1 + f_2}{4}$$

Edge point





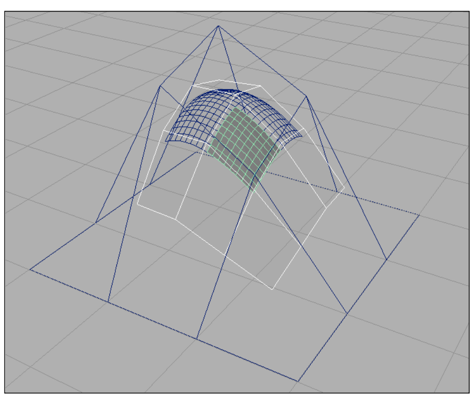
Vertex point

$$v = \frac{f_1 + f_2 + f_3 + f_4 + 2(m_1 + m_2 + m_3 + m_4) + 4p}{16}$$

m midpoint of edge, not "edge point"
p old "vertex point"

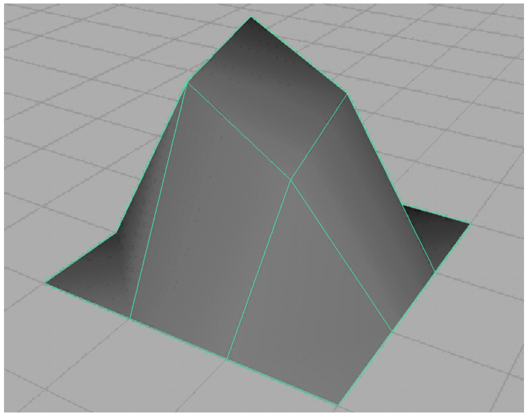
Regular B-Spline Subdivision

- Recall that control mesh approaches surface



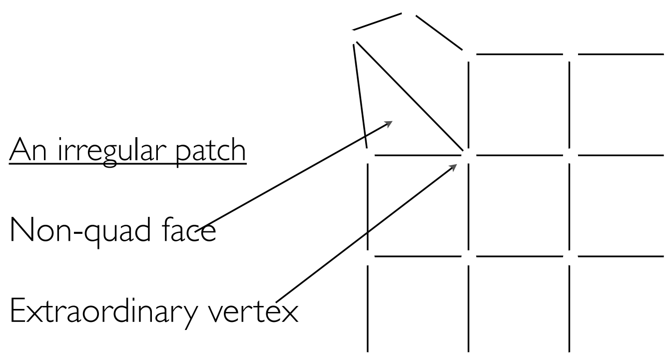
Regular B-Spline Subdivision

- Limit of subdivision is the surface



Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
  - Generalizes regular B-Spine subdivision



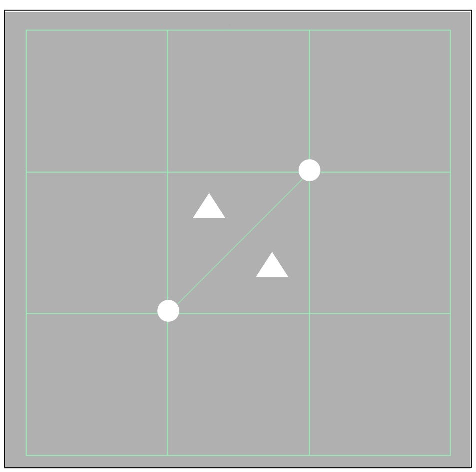
Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
  - Generalizes regular B-Spine subdivision
  - Rules reduce to regular for ordinary vertices/faces

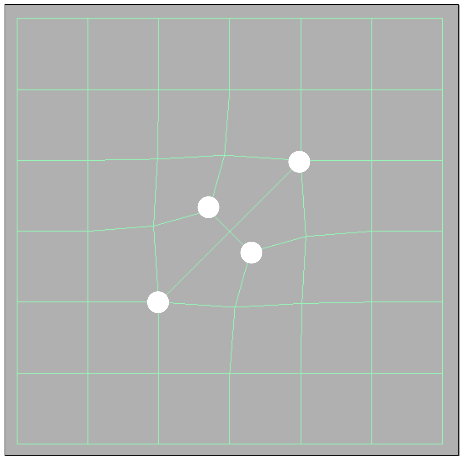
$$
\begin{aligned}
f &= \text{average of surrounding vertices} \\
e &= \frac{f_1 + f_2 + v_1 + v_2}{4} \\
v &= \frac{\bar{f}}{n} + \frac{2\bar{m}}{n} + \frac{p(n-3)}{n}
\end{aligned}
$$

$$
\begin{aligned}
\bar{m} &= \text{average of adjacent midpoints} \\
\bar{f} &= \text{average of adjacent face points} \\
n &= \text{valence of vertex}
\end{aligned}
$$

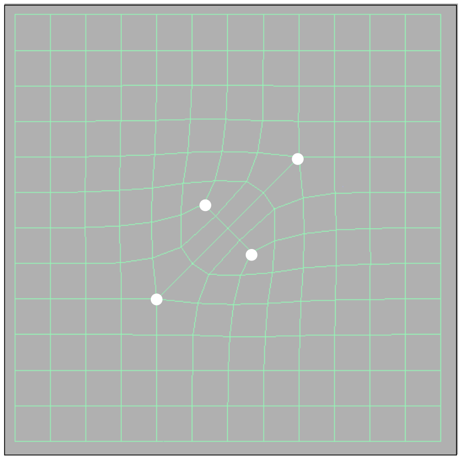
Catmull-Clark Subdivision



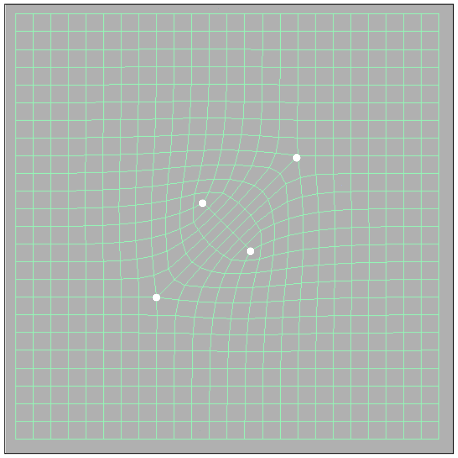
Catmull-Clark Subdivision



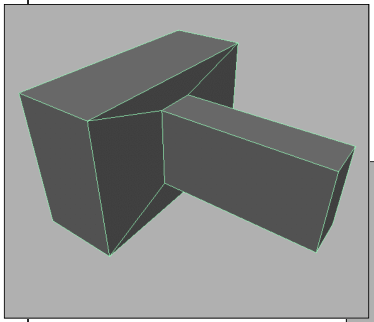
Catmull-Clark Subdivision

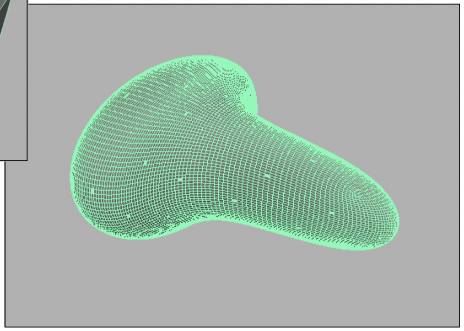


Catmull-Clark Subdivision

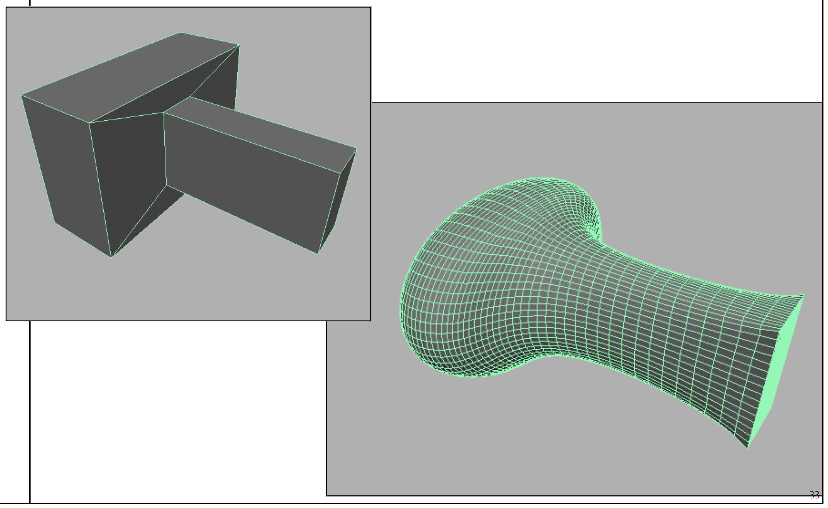


Catmull-Clark Subdivision

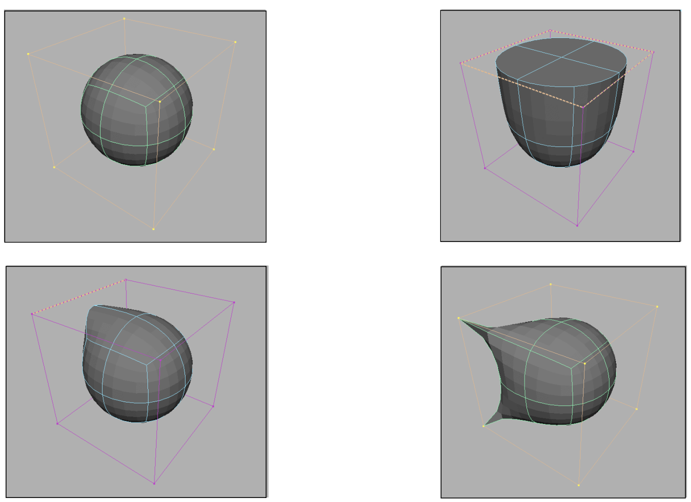




Catmull-Clark Subdivision



Catmull-Clark Subdivision



Catmull-Clark Subdivision