

CS-184: Computer Graphics

Lecture #22: Spring and Mass systems

Prof. James O'Brien
University of California, Berkeley

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Today

- Spring and Mass systems
 - Distance springs
 - Spring dampers
 - Edge springs

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Tuesday, November 24, 2009

A Simple Spring

- Ideal **zero**-length spring

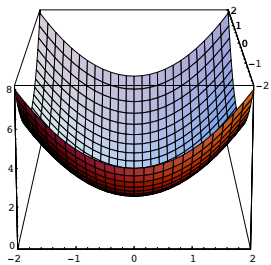
$$\text{---}\text{~}\text{---}\text{~}\text{---}\text{~}\text{---}\text{~}\text{---}\text{---} \quad \mathbf{f}_{a \rightarrow b} = k_s(\mathbf{b} - \mathbf{a})$$

- Force pulls points together $\mathbf{f}_{b \rightarrow a} = -\mathbf{f}_{a \rightarrow b}$
- Strength proportional to distance

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A Simple Spring

- Energy potential

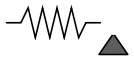


$$E = 1/2 k_s(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a})$$

$$\mathbf{f}_{a \rightarrow b} = k_s(\mathbf{b} - \mathbf{a})$$

$$\mathbf{f}_{b \rightarrow a} = -\mathbf{f}_{a \rightarrow b}$$

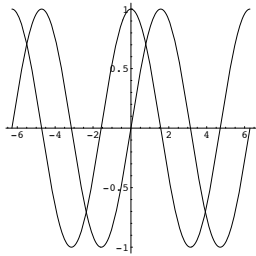
$$\mathbf{f}_a = -\nabla_a E = - \left[\frac{\partial E}{\partial a_x}, \frac{\partial E}{\partial a_y}, \frac{\partial E}{\partial a_z} \right]$$



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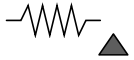
A Simple Spring

- Energy potential: kinetic vs elastic



$$E = 1/2 k_S (\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a})$$

$$E = 1/2 m (\dot{\mathbf{b}} - \dot{\mathbf{a}}) \cdot (\dot{\mathbf{b}} - \dot{\mathbf{a}})$$



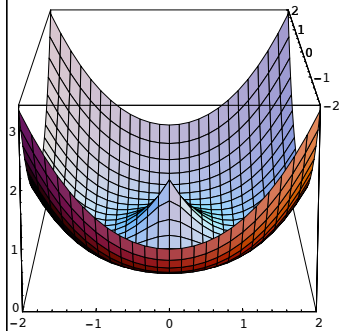
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Non-Zero Length Springs



$$\mathbf{f}_{a \rightarrow b} = k_S \frac{\mathbf{b} - \mathbf{a}}{\|\mathbf{b} - \mathbf{a}\|} (\|\mathbf{b} - \mathbf{a}\| - l)$$

Rest length



$$E = k_S (\|\mathbf{b} - \mathbf{a}\| - l)^2$$

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Comments on Springs

- Springs with zero rest length are linear
- Springs with non-zero rest length are nonlinear
 - Force *magnitude* linear w/ displacement (from rest length)
 - Force direction is non-linear
 - Singularity at

$$||\mathbf{b} - \mathbf{a}|| = 0$$

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Damping

- “Mass proportional” damping


$$\overleftarrow{\mathbf{f}} \quad \overrightarrow{\dot{\mathbf{a}}} \quad \mathbf{f} = -k_d \dot{\mathbf{a}}$$

- Behaves like viscous drag on all motion
- Consider a pair of masses connected by a spring
 - How to model rusty **vs** oiled spring
 - Should internal damping slow group motion of the pair?
- Can help stability... up to a point

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Damping

- “Stiffness proportional” damping

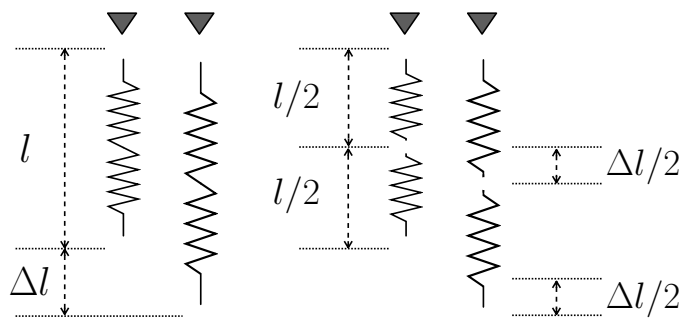

$$f_a = -k_d \frac{\mathbf{b} - \mathbf{a}}{\|\mathbf{b} - \mathbf{a}\|^2} (\mathbf{b} - \mathbf{a}) \cdot (\dot{\mathbf{b}} - \dot{\mathbf{a}})$$

- Behaves viscous drag on change in spring length
- Consider a pair of masses connected by a spring
 - How to model rusty vs oiled spring
 - Should internal damping slow group motion of the pair?

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Spring Constants

- Two ways to model a single spring



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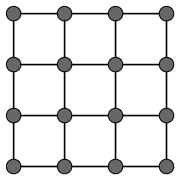
Spring Constants

- Constant k_S gives inconsistent results with different discretizations
- Change in length is not what we want to measure
- Strain: change in length as fraction of original length

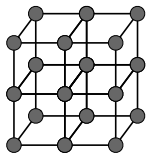
$$\epsilon = \frac{\Delta l}{l_0} \quad \text{Nice and simple for 1D...}$$

Structures from Springs

- Sheets



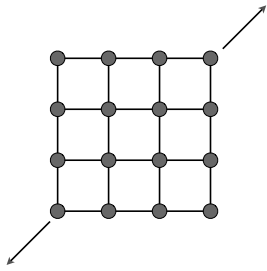
- Blocks



- Others

Structures from Springs

- They behave like what they are (obviously!)



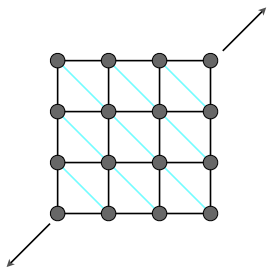
This structure will not resist shearing

This structure will not resist out-of-plane bending either...

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Structures from Springs

- They behave like what they are (obviously!)



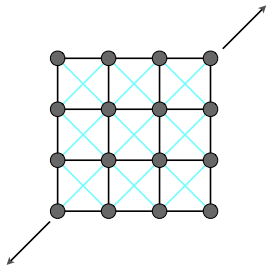
This structure will resist shearing but has anisotropic bias

This structure still will not resist out-of-plane bending

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Structures from Springs

- They behave like what they are (obviously!)



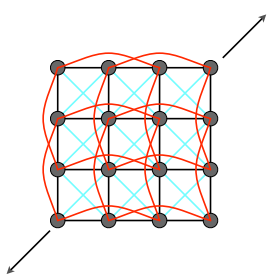
This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist
out-of-plane bending

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Structures from Springs

- They behave like what they are (obviously!)

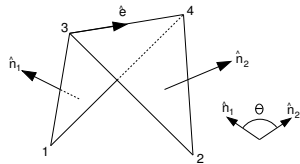


This structure will resist shearing
Less bias
Interference between spring sets

This structure will resist out-of-
plane bending
Interference between spring sets
Odd behavior

How do we set spring constants? 16

Edge Springs



$$u_1 = |E| \frac{N_1}{|N_1|^2} \quad u_2 = |E| \frac{N_2}{|N_2|^2}$$

$$u_3 = \frac{(x_1 - x_4) \cdot E}{|E|} \frac{N_1}{|N_1|^2} + \frac{(x_2 - x_4) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$$

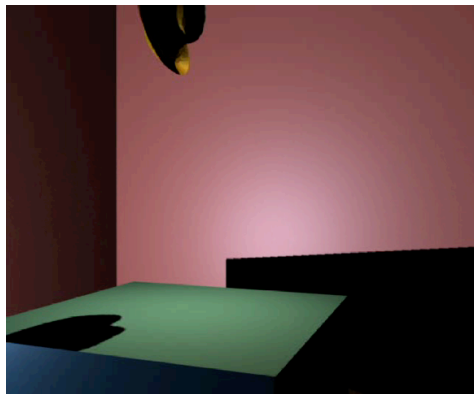
$$u_4 = -\frac{(x_1 - x_3) \cdot E}{|E|} \frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$$

$$F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|} \sin(\theta/2) u_i$$

From Bridson *et al.*, 2003, also see Grinspun *et al.*, 2003

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Example: Thin Material



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Suggested Reading

- Physically Based Modeling: Principles and Practice
 - Andy Witkin and David Baraff
 - <http://www-2.cs.cmu.edu/~baraff/sigcourse/index.html>
- Grinspun, Hirani, Desbrun, and Peter Schroder, "Discrete Shells," SCA 2003
- Bridson, Marino, and Fedkiw, "Simulation of Clothing with Folds and Wrinkles," SCA 2003
- O'Brien and Hodgins, "Graphical Modeling and Animation of Brittle Fracture," SIGGRAPH 99