# Spring 2012 Final

## CS184 - Foundations of Computer Graphics

## University of California at Berkeley

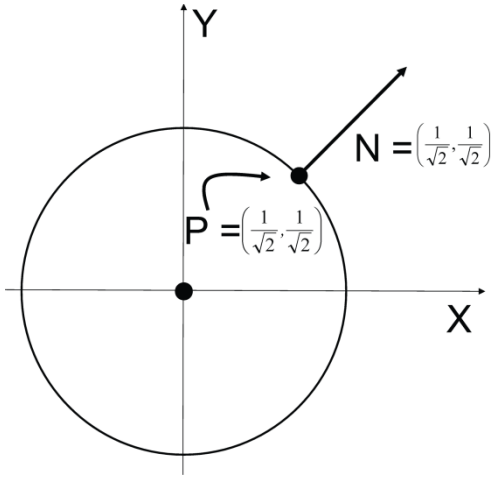
- **Write your name HERE:_____**
- **Write your login HERE:_____**
- **Closed book.**
- **You may not use any notes or printed/electronic materials.**
- **No calculators.**
- **In class, 3 hours.**
- **22 pages: 1 cover, 21 question pages.**

| Question | Points | Score |
|---|---|---|
| 1. Transformations | 20 | |
| 2. OpenGL Shading | 20 | |
| 3. Bezier Curves | 20 | |
| 4. Ray Tracing | 20 | |
| 5. Basic Illumination and Radiometry Concepts | 20 | |
| 6. Additional Topics | 20 | |

**You are encouraged to first look over all of the questions, in order to budget your time appropriately. If you are unsure of your answer (or even otherwise) please show your work so you can get partial credit. Additional worksheets if needed are on the final pages.**

## 1. Transformations [20]

Consider a unit radius circle in the Z= 0 plane centered at the origin. (i.e. it is a unit circle in the X-Y plane) Consider a particular point **P** at an angle of 45 degrees in polar coordinates, i.e., at $(1/\sqrt{2}, 1/\sqrt{2}, 0)$. Since it is a circle, the unit normal N at that location is also $(1/\sqrt{2}, 1/\sqrt{2}, 0)$.




We go through a number of transformations. For each, please draw the resulting shape and the new location **P'** of point **P**, as well as the new normal **N'** at **P'**. (That is, draw the transformed shape, and mark **P'**, **N'**).

Also, please write down the formulae for

*(i) The 4x4 transformation Matrix for Positions,*

*(ii) The new point P',*

*(iii) the new normal N'.*

The answers to parts a-d are all separate; the transforms are not applied in order (except that c combines a and b).

**a) Translation by +3 units along the X axis.**

| (i) Transformation matrix for positions | | | |
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| (ii) **P'** | (iii) **N'** |
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| (iv) Your transformed shape | | | | | | | | | |
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**b) Rotation by 45 degrees (about the Z axis)**

| (i) Transformation matrix for positions | | | |
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| (ii) **P'** | (iii) **N'** |
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| (iv) Your transformed shape | | | | | | | | | |
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# Work sheet

**c) First the translation (a) followed by the rotation (b)**

| (i) Transformation matrix for positions | | | |
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| (ii) **P'** | (iii) **N'** |
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| (iv) Your transformed shape | | | | | | | | | |
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**d) A scale by a factor of 2 along the x axis (unit scale along y and z).**

| (i) Transformation matrix for positions | | | |
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| (ii) **P'** | (iii) **N'** |
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| (iv) Your transformed shape | | | | | | | | | |
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## 2. OpenGL and Shading [20]

**a) There are three basic shading modes in OpenGL: Flat Shading, Gouraud Shading and Phong Shading. Briefly explain each of these and the differences between them. What are the advantages and disadvantages of each (just a brief explanation; formulas are not required but may help to make the point).**

**For Gouraud shading, what operations are typically performed in the vertex shader, OpenGL rasterizer, and fragment shader respectively?**

**For Phong shading, what operations are typically performed in the vertex shader, OpenGL rasterizer, and fragment shader respectively?**

**b) What is the Lambertian or Diffuse Illumination Model? What is the formula that is typically implemented in OpenGL?**

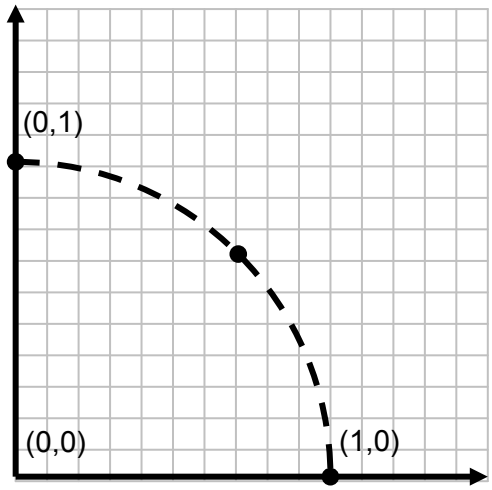
**c) What is the Phong Illumination Model for specular highlights?**
**Please explain how the Phong Illumination Model is different from the Phong shading model in part (a), and how they are usually combined?**

**What is the formula for specular highlights (Phong Illumination Model) that is typically implemented in OpenGL?**

**d) Name at least one other reflection model besides Lambertian and Phong above, and give at least an outline of the relevant shading formula.**
(This is not implemented by default in old-style OpenGL but can be implemented with programmable shaders).

## 3. Bezier Curves [20]  (Extra Credit for cubic curve)

Bezier curves are commonly used to approximate shapes. In this question, we consider quadratic and cubic Bezier curves used to approximate a quarter circle, i.e. an arc that starts at (0,1) and ends at (1,0). Remember that the Bezier curve interpolates the end-points of the control polygon and the control polygon is also tangent to the curve at these end points.




First consider a quadratic Bezier approximation to the quarter circle.

**a) What are the control points of the Bezier curve?**

**b) What is the mid-point?**

**c) What is the maximum error?**

(i.e. the difference between the mid-point and the mid-point of the quarter circle).

# Work sheet

**d) What is the corresponding quadratic B-Spline curve that best approximates the quarter-circle, i.e. what are its control points? What is the error for this curve? Is it the same or different from the Bezier curve?**

**e) [Extra Credit]**

**Now, consider a cubic Bezier curve. In this case, besides position/tangency constraints at the end-points, we also require the curve position and tangent to match at the mid-point.**

**What are the control points of the cubic Bezier curve that approximates the quarter circle?**

**f) [Extra Credit]**

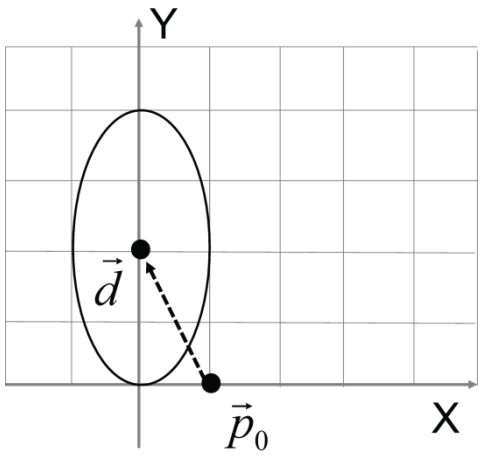
**What is the maximum error of the cubic Bezier approximation and how does it compare to the quadratic?**

# Work sheet

# Work sheet

## 4. Ray Tracing [20]

We step through the various stages of intersecting a ray with a transformed sphere. Consider an ellipsoid centered a (0,2,0) on the Y-axis, and with radius along Y being 2 units and along X and Z being one unit. That is, we took the unit sphere, scaled it by a factor of 2 in Y and then translated by 2 units in Y. We have a ray that starts at the location (1,0,0) and has direction (-1, 2, 0).




**a) Write down the 4x4 transformation matrix for this transform to the sphere.**

| **Transformation Matrix** | | | |
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**b) How would you intersect the ray against the transformed sphere by using a standard ray-surface intersection test? Write down the transformed ray.**

# Work sheet

**c) What is the general equation for ray-sphere intersection, given the transformed ray and the canonical sphere?** Note that the form of the ray is $\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_{\boldsymbol{0}} + t\vec{\boldsymbol{d}}$ where $\vec{\boldsymbol{p}}_{\boldsymbol{0}}$ is the origin and d the direction, while t is a parameter along the ray. The equation for the sphere is $(\vec{\boldsymbol{p}} - \vec{\boldsymbol{p}}_{\boldsymbol{c}}) \cdot (\vec{\boldsymbol{p}} - \vec{\boldsymbol{p}}_{\boldsymbol{c}}) = r^2$, where $\vec{\boldsymbol{p}}_{\boldsymbol{c}}$ is the center of the sphere and r is the radius.

**d) What is the intersection point in (i) the transformed space and (ii) the original (untransformed object: ellipsoid) space?**

| (i) | (ii) |
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# Work sheet

**e) What is the surface normal in (i) the transformed and (ii) the original (untransformed object: ellipsoid) space?**

| (i) | (ii) |
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**f) [Extra Credit] What is the reflected ray direction in (i) the transformed and (ii) the original (untransformed object: ellipsoid) space?**

| (i) | (ii) |
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# Work sheet

## 5. Basic Illumination and Radiometry Concepts [20]

**a) Define the terms Radiance, Irradiance and BRDF, and give the units for each.**

**b) Write down the local reflectance equation, i.e. express the net reflected radiance in a given direction as an integral over the incident illumination.** Prominently label the main terms of the equation such as the BRDF.

**c) If the radiance from every point in the upper hemisphere is 1 W/($m^2$ sr), what is the irradiance at a point?** (Use correct units)

**d) What is the rendering equation? Give a brief description, and write down a formula for it, labeling each of the individual terms.** (No derivation is needed, just the final form)

## 6. Additional Topics [20]

**a) What is standard (whitted-style) recursive ray-tracing? What visual effects does it allow for, that are hard to achieve in OpenGL?**

**b) What is aliasing? Are the jaggies when rasterizing (or ray tracing) the sharp edge of a triangle related to aliasing? What is one way of reducing aliasing or antialiasing?**

**c) Texture mapping can be used for many visual effects besides a simple color modulation. Name three possible uses for textures, or other visual effects that can be obtained from texture maps.**

**d) Briefly explain the concept of keyframe animation. In your answer, address the following questions: What are keyframes, who specifies them, and what is the task of the computer?**

**e) Briefly explain how to magnify or minify an image (with the final size being a non-integer multiple of the original) using proper reconstruction filters.**

**f) [Extra Credit]**
**Give the final convolution formula used in (e), and discuss the types of filters one may use.**

# Work sheet

# Work sheet