

# CS 184: Problems and Questions on Rendering

Ravi Ramamoorthi

## Problems

1. Define the terms Radiance and Irradiance, and give the units for each. Write down the formula (integral) for irradiance at a point in terms of the illumination  $L(\omega)$  incident from all directions  $\omega$ . Write down the local reflectance equation, i.e. express the net reflected radiance in a given direction as an integral over the incident illumination.
2. Match the surface material to the formula (and goniometric diagram shown in class). Also, give an example of a real material that reasonably closely approximates the mathematical description. Not all materials need have a corresponding diagram. The materials are *ideal mirror*, *dark glossy*, *ideal diffuse*, *retroreflective*. The formulae for the BRDF  $f_r$  are  $k_a(\vec{R} \cdot \vec{V})$ ,  $k_b(\vec{R} \cdot \vec{V})^4$ ,  $k_c/(\vec{N} \cdot \vec{V})$ ,  $k_d\delta(\vec{R})$ ,  $k_e$ .
3. Consider a simplified skylight model, so the radiance along any direction is given by  $A + B \sin \alpha$  where  $A$  and  $B$  are positive constants, and  $\alpha$  is the elevation angle (i.e. the angle to the horizontal, being 0 degrees toward the horizontal and 90 degrees toward the zenith or top of the sky). That is, the radiance is more higher up in the sky. The lighting is isotropic; there is no variation with azimuthal angle ( $\phi$ ). Assume for this problem that there is no occlusion by trees, buildings etc., the sky hemisphere is the only source of illumination [no ground lighting, direct sunlight etc.], the surfaces are Lambertian with albedo 1, and the sky can be assumed to be a distant source. What is the irradiance on the ground, assumed to be a horizontal surface?. Now, assume we have a sphere suspended (or on the ground, if that makes things more logical for you). Remember the assumptions, i.e. lighting only from the (distant) sky, no occlusions etc. Which point on the sphere will be brightest? What will be the reflected radiance at this point? Which point will be the dimmest? What will be the reflected radiance at that point? Qualitatively, how will the brightness on the sphere vary as a function of location (parameterized by spherical coordinates for instance)? Extra credit for deriving an analytic quantitative formula for the brightness of the sphere as a function of the surface normal.

## Answers

**1. Radiance and Irradiance** For the final, you should be prepared to know basic definitions and units.

- *Radiance* is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray. Its units are  $Wm^{-2}sr^{-1}$ .
- *Irradiance* is the radiant power per unit area. Its units are  $Wm^{-2}$ .

The irradiance is a cosine-weighted integral of the incoming radiance over the visible hemisphere,

$$E = \int L(\omega)(\omega \cdot n) d\omega, \tag{1}$$

where  $L$  is the radiance,  $E$  is the irradiance,  $n$  is the surface normal, and the integral is over the hemisphere. If we use spherical angles  $\theta$  and  $\phi$ ,

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L(\theta, \phi) \cos \theta \sin \theta d\theta d\phi, \quad (2)$$

where the  $\sin \theta d\theta d\phi$  is the solid angle measure.

For the reflectance equation, we must also consider the BRDF  $f$  of the surface, that is a function of incident and outgoing directions,

$$L_r(\omega_r) = \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_r) (\omega_i \cdot n) d\omega_i, \quad (3)$$

where the terms are respectively the reflected radiance, the incident radiance, BRDF and cosine term.

**2. Materials** An ideal mirror is something like a normal reflective mirror, and the BRDF corresponds to  $k_d \delta(\vec{R})$ . A dark glossy surface is close to glossy plastic with a formula like  $k_b (\vec{R} \cdot \vec{V})^4$ . An ideal diffuse surface is Lambertian, close to wall paint or ideally, spectralon, with BRDF a constant  $k_e$ . A retroreflective surface is something like a highway reflector, that reflects light back toward the viewer and would have a formula using  $\delta(\vec{L})$  instead of  $\vec{R}$  as in an ordinary reflector.

**3. Irradiance** The irradiance is given by

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L(\theta, \phi) \cos \theta \sin \theta d\theta, \quad (4)$$

where  $\theta$  stands for the angle with respect to the zenith, or normal to the ground plane. In this coordinate frame, the elevation angle  $= \pi/2 - \theta$ , and so the incident radiance  $L = A + B \cos \theta$ . Removing the azimuthal dependence in the above integral, we obtain

$$E = 2\pi \int_0^{\pi/2} (A + B \cos \theta) \cos \theta \sin \theta d\theta. \quad (5)$$

To evaluate this integral, we put  $u = \cos \theta$ , to obtain

$$E = 2\pi \int_0^1 Au + Bu^2 du = \pi \left( A + \frac{2B}{3} \right). \quad (6)$$

The brightest point is clearly the top of the sphere, or the point facing directly upward, which receives the same irradiance as the ground. Since the albedo is 1, the BRDF is  $1/\pi$  and the reflected radiance is  $A + 2B/3$ . The dimmest point will be at the bottom of the sphere, which will be completely dark (since it sees only the lower hemisphere that is dark, since we assume no interreflection from the ground). The radiance will decrease from top to bottom, remaining azimuthally symmetric.

Finding an analytic formula for the radiance as a function of angle is an advanced topic, well outside the scope of this course. One can try to use the spherical harmonic irradiance formula in my thesis.