Relational Calculus

CS 186, Fall 2005 R&G, Chapter 4

We will occasionally use this arrow notation unless there is danger of no confusion.

Ronald Graham

Elements of Ramsey Theory





Relational Calculus

- Comes in two flavors: <u>Tuple relational calculus</u> (TRC) and <u>Domain</u> relational calculus (DRC).
- Calculus has variables, constants, comparison ops, logical connectives and quantifiers.
 - <u>TRC</u>: Variables range over (i.e., get bound to) tuples.
 Like SQL.
 - <u>DRC</u>: Variables range over domain elements (= field values).
 - Like Query-By-Example (QBE)
 Both TRC and DRC are simple subsets of first-order logic.
 - We'll focus on TRC here
- Expressions in the calculus are called formulas.
- Answer tuple is an assignment of constants to variables that make the formula evaluate to true.



Tuple Relational Calculus

- Query has the form: {T | p(T)}
 - p(T) denotes a formula in which tuple variable T appears.
- <u>Answer</u> is the set of all tuples T for which the formula p(T) evaluates to true.
- Formula is recursively defined:
 - start with simple atomic formulas (get tuples from relations or make comparisons of values)
 - build bigger and better formulas using the logical connectives.



TRC Formulas

• An Atomic formula is one of the following:

 $R \in Rel$

R.a op S.b

R.a op constant

op is one of $<,>,=,\leq,\geq,\neq$

- A formula can be:
 - an atomic formula
 - ¬p, p∧q, p∨q where p and q are formulas
 - ∃R(p(R)) where variable *R* is a tuple variable
- $\forall R(p(R))$ where variable R is a tuple variable



Free and Bound Variables

- The use of quantifiers $\exists X$ and $\forall X$ in a formula is said to *bind* X in the formula.
 - A variable that is not bound is <u>free</u>.
- Let us revisit the definition of a query:
 - $-\{T\mid p(T)\}$
- · There is an important restriction
 - the variable *T* that appears to the left of `|' must be the *only* free variable in the formula *p*(7).
 - in other words, all other tuple variables must be bound using a quantifier.



Selection and Projection

Find all sailors with rating above 7

 $\{S \mid S \in Sailors \land S.rating > 7\}$

- Modify this query to answer: Find sailors who are older than 18 or have a rating under 9, and are called 'Bob'.
- Find names and ages of sailors with rating above 7.

 $\{S \mid \exists S1 \in Sailors(S1.rating > 7)\}$

 \land S.sname = S1.sname

 $\land S.age = S1.age$)

- Note: S is a tuple variable of 2 fields (i.e. {S} is a projection of Sailors)
 - \bullet only 2 fields are ever mentioned and ${\cal S}$ is never used to range over any relations in the query.



Joins

Find sailors rated > 7 who've reserved boat #103

 $\{S \mid S \in Sailors \land S.rating > 7 \land \exists R(R \in Reserves \land R.sid = S.sid \land R.bid = 103)\}$

Note the use of ∃ to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.



Joins (continued)

 $\{S \mid S \in Sailors \land S.rating > 7 \land \\ \exists R(R \in Reserves \land R.sid = S.sid \\ \land \exists B(B \in Boats \land B.bid = R.bid \\ \land B.color = 'red'))\}$

Find sailors rated > 7 who've reserved a red boat

- Observe how the parentheses control the scope of each quantifier's binding.
- This may look cumbersome, but it's not so different from SQL!



Division (makes more sense here???)

Find sailors who've reserved all boats (hint, use ♥)

 Find all sailors S such that for all tuples B in Boats there is a tuple in Reserves showing that sailor S has reserved B.



Division – a trickier example...

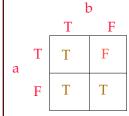
Find sailors who've reserved all Red boats

 $\{S \mid S \in Sailors \land \\ \forall B \in Boats (B.color = 'red' \Rightarrow \\ \exists R(R \in Reserves \land S.sid = R.sid \\ \land B.bid = R.bid))\}$ Alternatively...

 $\{S \mid S \in Sailors \land \\ \forall B \in Boats (B.color \neq 'red' \lor \\ \exists R(R \in Reserves \land S.sid = R.sid \land B.bid = R.bid))\}$



$\mathbf{a} \Rightarrow \mathbf{b}$ is the same as $\neg \mathbf{a} \lor \mathbf{b}$



- If a is true, b must be true!
 - If a is true and b is false, the implication evaluates to false.
- If a is not true, we don't care about b
 - The expression is always true.



Unsafe Queries, Expressive Power

• \exists syntactically correct calculus queries that have an infinite number of answers! <u>Unsafe</u> queries.

- e.g.,
$$\{S | \neg \{S \in Sailors\}\}$$

- Solution???? Don't do that!
- Expressive Power (Theorem due to Codd):
 - every query that can be expressed in relational algebra can be expressed as a *safe* query in DRC / TRC; the converse is also true.
- <u>Relational Completeness</u>: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus. (actually, SQL is more powerful, as we will see...)



Summary

- The relational model has rigorously defined query languages — simple and powerful.
- · Relational algebra is more operational
 - useful as internal representation for query evaluation plans.
- Relational calculus is non-operational
 - users define queries in terms of what they want, not in terms of how to compute it. (*Declarative*)
- Several ways of expressing a given query
 - a *query optimizer* should choose the most efficient version.
- Algebra and safe calculus have same expressive power
 - leads to the notion of *relational completeness*.



Addendum: Use of \forall

- $\forall x (P(x))$ is only true if P(x) is true for *every* x in the universe
- Usually:

```
\forall x ((x \in Boats) \Rightarrow (x.color = "Red")
```

- ⇒ logical implication,
 - a ⇒ b means that if a is true, b must be
 - $a \Rightarrow b$ is the same as $\neg a \lor b$



Find sailors who've reserved all boats

 $\{S \mid S \in Sailors \land \\ \forall B((B \in Boats) \Rightarrow \\ \exists R(R \in Reserves \land S.sid = R.sid \\ \land B.bid = R.bid))\}$

• Find all sailors S such that for each tuple B either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor S has reserved it.

 $\{S \mid S \in Sailors \land \\ \forall B(\neg(B \in Boats) \lor \\ \exists R(R \in Reserves \land S.sid = R.sid \\ \land B.bid = R.bid))\}$



... reserved all red boats

 $\begin{cases} S \mid S \in Sailors \land \\ \forall B ((B \in Boats \land B.color = "red") \Rightarrow \\ \exists R (R \in Reserves \land S.sid = R.sid \\ \land B.bid = R.bid)) \end{cases}$

• Find all sailors S such that for each tuple B either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor S has reserved it.

SQL: The Query Language Part 1

CS186, Fall 2005 R&G, Chapter 5

Life is just a bowl of queries.

-Anon (not Forrest Gump)



Relational Query Languages

- A major strength of the relational model: supports simple, powerful querying of data.
- Two sublanguages:
- DDL Data Definition Language
 - define and modify schema (at all 3 levels)
- DML Data Manipulation Language
 Queries can be written intuitively.
- The DBMS is responsible for efficient evaluation.
 - The key: precise semantics for relational queries.
 - Allows the optimizer to re-order/change operations, and ensure that the answer does not change.
 - Internal cost model drives use of indexes and choice of access paths and physical operators.



The SQL Query Language

- The most widely used relational guery language.
 - Current standard is SQL-1999
 - · Not fully supported yet
 - Introduced "Object-Relational" concepts (and lots more) Many of which were pioneered in Postgres here at Berkeley!
 - SQL-200x is in draft
 - SQL-92 is a basic subset
 - Most systems support a medium
 - PostgreSQL has some "unique" aspects
 - as do most systems.
 - XML support/integration is the next challenge for SQL (more on this in a later class).



DDL - Create Table

CREATE TABLE table_name

({ column_name data_type [DEFAULT default_expr] [column_constraint [, ...]] | table_constraint } [, ...])

Data Types (PostgreSQL) include:

character(n) – fixed-length character string character varying(n) - variable-length character string smallint, integer, bigint, numeric, real, double precision date, time, timestamp, .. serial - unique ID for indexing and cross reference

PostgreSQL also allows OIDs, arrays, inheritance, rules...

conformance to the SQL-1999 standard is variable so we won't use these in the project.



Create Table (w/column constraints)

CREATE TABLE table_name

({ column_name data_type [DEFAULT default_expr] [column_constraint [, ...]] | table_constraint } [, ...])

[CONSTRAINT constraint_name] NOT NULL | NULL | UNIQUE | PRIMARY KEY | CHECK (expression) |

REFERENCES reftable [(refcolumn)] [ON DELETE action] [ON UPDATE action] }

action is one of:

NO ACTION, CASCADE, SET NULL, SET DEFAULT

expression for column constraint must produce a boolean result and reference the related column's value only.



Create Table (w/table constraints)

CREATE TABLE table_name ({ column_name data_type [DEFAULT default_expr] [column_constraint [, ...]] | table_constraint } [, ...])

Table Constraints:

• [CONSTRAINT constraint_name] { UNIQUE (column_name [, ...]) | PRIMARY KEY (column_name [, ...]) | CHECK (expression) | FOREIGN KEY ($column_name[, ...]$) REFERENCES reftable[(refcolumn [, ...])][ON DELETE action] [ON UPDATE action 1 }

Here, expressions, keys, etc can include multiple columns



Create Table (Examples)

CREATE TABLE films (

CHAR(5) PRIMARY KEY, code

VARCHAR(40), title DECIMAL(3), did date_prod DATE, kind VARCHAR(10),

CONSTRAINT production UNIQUE(date_prod) FOREIGN KEY did REFERENCES distributors

ON DELETE NO ACTION

CREATE TABLE distributors (

did DECIMAL(3) PRIMARY KEY,

name VARCHAR(40)

CONSTRAINT con1 CHECK (did > 100 AND name <> ' ')



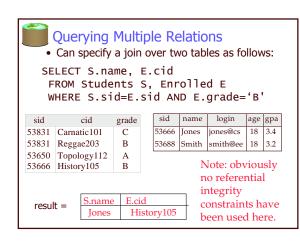
The SQL DML

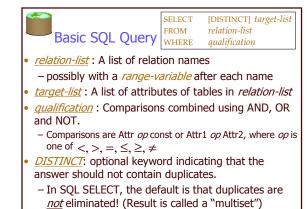
- Single-table queries are straightforward.
- To find all 18 year old students, we can write:

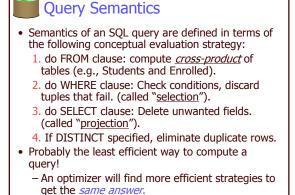
SELECT * FROM Students S WHERE S.age=18

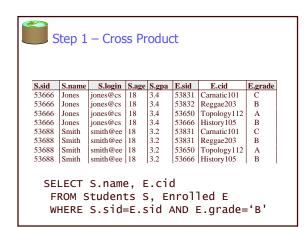
sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@ee	18	3.2

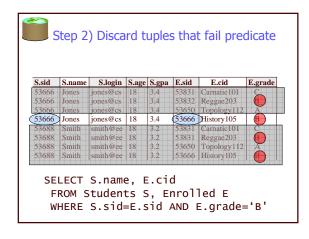
• To find just names and logins, replace the first line: SELECT S.name, S.login

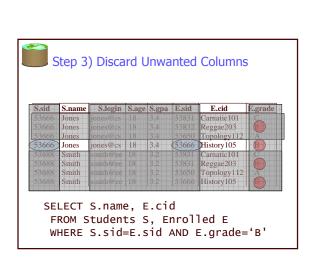










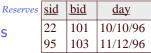




We will use these Sailors

instances of relations in our examples.

(Question: If the key for the Reserves relation contained only the attributes sid and bid, how would the semantics differ?)



<u>sid</u>	sname	rating	age
22	Dustin	7	45.0
31	Lubber	8	55.5
95	Bob	3	63.5

<u>bid</u>	bname	color
101	Interlake	
102	Interlake	red
	Clipper	green
104	Marine	red



Example Schemas

CREATE TABLE Sailors (sid INTEGER PRIMARY KEY, sname CHAR(20), rating INTEGER, age REAL)

CREATE TABLE Boats (bid INTEGER PRIMARY KEY, bname CHAR (20), color CHAR(10))

CREATE TABLE Reserves (
sid INTEGER REFERENCES Sailors,
bid INTEGER, day DATE,
PRIMARY KEY (sid, bid, day),
FOREIGN KEY (bid) REFERENCES Boats)



Another Join Query

Boats

SELECT sname FROM Sailors, Reserves WHERE Sailors.sid=Reserves.sid AND bid=103

	(sid)	sname	rating	age	(sid)	bid	day
ĺ	22	dustin	7	45.0	22	101	10/10/96
	22	dustin	7	45.0	95	103	11/12/96
	31	lubber	8	55.5	22	101	10/10/96
	31	lubber	8	55.5	95	103	11/12/96
	95	Bob	3	63.5	22	101	10/10/96
	95	Bob	3	63.5	95	103	11/12/96



Some Notes on Range Variables

- Can associate "range variables" with the tables in the FROM clause.
 - saves writing, makes queries easier to understand
- · Needed when ambiguity could arise.
 - for example, if same table used multiple times in same FROM (called a "self-join")

SELECT sname FROM Sailors,Reserves WHERE Sailors.sid=Reserves.sid AND bid=103

Can be rewritten using range variables as: SELECT S.sname FROM Sailors S, Reserves R WHERE S.sid=R.sid AND bid=103



More Notes

• Here's an example where range variables are required (self-join example):

SELECT x.sname, x.age, y.sname, y.age FROM Sailors x, Sailors y WHERE x.age > y.age

 Note that target list can be replaced by "*" if you don't want to do a projection:

SELECT *
FROM Sailors x
WHERE x.age > 20



Find sailors who've reserved at least one boat

SELECT S.sid FROM Sailors S, Reserves R WHERE S.sid=R.sid

- Would adding DISTINCT to this query make a difference?
- What is the effect of replacing S.sid by S.sname in the SELECT clause?
 - Would adding DISTINCT to this variant of the query make a difference?



Expressions

- Can use arithmetic expressions in SELECT clause (plus other operations we'll discuss later)
- Use AS to provide column names

```
SELECT S.age, S.age-5 AS age1, 2*S.age AS age2
   FROM Sailors S
WHERE S.sname = 'Dustin'
```

• Can also have expressions in WHERE clause:

```
SELECT S1.sname AS name1, S2.sname AS name2
FROM Sailors S1, Sailors S2
WHERE 2*S1.rating = S2.rating - 1
```



String operations

- •SQL also supports some string operations
- •"LIKE" is used for string matching.

```
SELECT S.age, S.age-5 AS age1, 2*S.age AS age2
  FROM Sailors S
WHERE S.sname LIKE 'B_%b'
```

' stands for any one character and `%' stands for 0 or more arbitrary characters.



Find sid's of sailors who've reserved a red or a areen boat

• UNION: Can be used to compute the union of any two union-compatible sets of tuples (which are themselves the result of SQL queries).

> SELECT R.sid FROM Boats B, Reserves R WHERE R.bid=B.bid AND (B.color='red'OR B.color='green')

SELECT R.sid FROM Boats B, Reserves R WHERE R.bid=B.bid AND B.color='red' UNTON SELECT R.sid

FROM Boats B, Reserves R WHERE R.bid=B.bid AND B.color='green'

Find sid's of sailors who've reserved a red and a green boat

- If we simply replace OR by AND in the previous query, we get the wrong answer. (Why?)
- · Instead, could use a self-join:

```
SELECT R1.sid
   FROM Boats B1, Reserves R1,
               Boats B2, Reserves R2
WHERE R1.sid=R2.sid
      AND R1.bid=B1.bid
      AND R2.bid=B2.bid
      AND (B1.color='red' AND B2.color='green')
```



AND Continued...

INTERSECT: discussed in book. Can be used to compute the intersection of any two unioncompatible sets of tuples.

- Also in text: EXCEPT (sometimes called MINUS)
- Included in the SQL/92 standard, but many systems don't support them.
 - But PostgreSQL does!

Key field! SELECT S.sid

FROM Sailors S, Boats B, Reserves R WHERE S.sid=R.sid AND R.bid=B.bid AND B.color='red' INTERSECT

SELECT S.sid

FROM Sailors S, Boats B, Reserves R WHERE S.sid=R.sid

AND R.bid=B.bid AND B.color='green'

Nested Queries

- Powerful feature of SQL: WHERE clause can itself contain an SQL query!
 - Actually, so can FROM and HAVING clauses.

Names of sailors who've reserved boat #103:

SELECT S.sname FROM Sailors S WHERE S.sid IN (SELECT R.sid FROM Reserves R WHERE R.bid=103)

- To find sailors who've not reserved #103, use NOT IN.
- To understand semantics of nested queries:
 - think of a nested loops evaluation: For each Sailors tuple, check the qualification by computing the subquery.



Nested Queries with Correlation

Find names of sailors who've reserved boat #103:

```
SELECT S.sname
FROM Sailors S
WHERE EXISTS (SELECT *
FROM Reserves R
WHERE R.bid=103 AND S.sid=R.sid)
```

- EXISTS is another set comparison operator, like IN.
- Can also specify NOT EXISTS
- If UNIQUE is used, and * is replaced by R.bid, finds sailors with at most one reservation for boat #103.
 - UNIQUE checks for duplicate tuples in a subquery;
- · Subquery must be recomputed for each Sailors tuple.
 - Think of subquery as a function call that runs a query!



More on Set-Comparison Operators

- We've already seen IN, EXISTS and UNIQUE. Can also use NOT IN, NOT EXISTS and NOT UNIQUE.
- Also available: op ANY, op ALL
- Find sailors whose rating is greater than that of some sailor called Horatio:

```
SELECT *
FROM Sailors S
WHERE S.rating > ANY (SELECT S2.rating
FROM Sailors S2
WHERE S2.sname='Horatio')
```



Rewriting INTERSECT Queries Using IN

Find sid's of sailors who've reserved both a red and a green boat:

```
SELECT R.sid
FROM Boats B, Reserves R
WHERE R.bid=B.bid
AND B.color='red'
AND R.sid IN (SELECT R2.sid
FROM Boats B2, Reserves R2
WHERE R2.bid=B2.bid
AND B2.color='green')
```

- Similarly, EXCEPT queries re-written using NOT IN.
- How would you change this to find *names* (not *sid's*) of Sailors who've reserved both red and green boats?



Division in SQL

Find sailors who've reserved all boats.

FROM Sailors S Sailors S such that ...

WHERE NOT EXISTS (SELECT B.bid there is no boat B without FROM Boats B WHERE NOT EXISTS (SELECT R.bid a Reserves tuple showing S reserved B

where R.bid=B.bid
AND R.sid=S.sid))



Basic SQL Queries - Summary

- An advantage of the relational model is its welldefined query semantics.
- SQL provides functionality close to that of the basic relational model.
 - some differences in duplicate handling, null values, set operators, etc.
- Typically, many ways to write a query
 - the system is responsible for figuring a fast way to actually execute a query regardless of how it is written.
- Lots more functionality beyond these basic features. Will be covered in subsequent lectures.



Aggregate Operators

 Significant extension of relational algebra.

SELECT COUNT (*)
FROM Sailors S

SELECT AVG (S.age) FROM Sailors S WHERE S.rating=10

SELECT COUNT (DISTINCT S.rating) FROM Sailors S WHERE S.sname='Bob'

COUNT (*)
COUNT ([DISTINCT] A)
SUM ([DISTINCT] A)
AVG ([DISTINCT] A)
MAX (A)
MIN (A)

\single column



Aggregate Operators

COUNT (*)
COUNT ([DISTINCT] A)
SUM ([DISTINCT] A)
AVG ([DISTINCT] A)
MAX (A)
MIN (A)

single column

SELECT S.sname FROM Sailors S

 $\begin{array}{c} \text{WHERE S.rating= (SELECT MAX(S2.rating)} \\ \text{FROM Sailors S2)} \end{array}$

SELECT AVG (DISTINCT S.age) FROM Sailors S WHERE S.rating=10



Find name and age of the oldest sailor(s)

 The first query is incorrect!

 Third query equivalent to second query

 allowed in SQL/92 standard, but not supported in some systems.

PostgreSQL seems to run it

SELECT S.sname, MAX (S.age) FROM Sailors S

SELECT S.sname, S.age FROM Sailors S WHERE S.age =

(SELECT MAX (S2.age) FROM Sailors S2)

SELECT S.sname, S.age FROM Sailors S

WHERE (SELECT MAX (S2.age) FROM Sailors S2) = S.age



GROUP BY and HAVING

- So far, we've applied aggregate operators to all (qualifying) tuples.
 - Sometimes, we want to apply them to each of several groups of tuples.
- Consider: Find the age of the youngest sailor for each rating level.
 - In general, we don't know how many rating levels exist, and what the rating values for these levels are!
 - Suppose we know that rating values go from 1 to 10; we can write 10 queries that look like this (!):

For i = 1, 2, ..., 10:

SELECT MIN (S.age) FROM Sailors S WHERE S.rating = *i*

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