

## Schema Refinement and Normalization

Nobody realizes that some people  
expend tremendous energy  
merely to be normal.

Albert Camus



## Functional Dependencies (FDs) (Review)

- A **functional dependency**  $X \rightarrow Y$  holds over relation schema  $R$  if, for every **allowable instance**  $r$  of  $R$ :

$$t1 \in r, t2 \in r, \pi_X(t1) = \pi_X(t2)$$

$$\text{implies } \pi_Y(t1) = \pi_Y(t2)$$

(where  $t1$  and  $t2$  are tuples;  $X$  and  $Y$  are sets of attributes)

- In other words:  $X \rightarrow Y$  means  
Given any two tuples in  $r$ , if the  $X$  values are the same, then the  $Y$  values must also be the same. (but not vice versa)
- Read " $\rightarrow$ " as "determines"



## Reasoning About FDs (Review)

- Given some FDs, we can usually infer additional FDs:

$title \rightarrow studio, star$  implies  $title \rightarrow studio$  and  $title \rightarrow star$   
 $title \rightarrow studio$  and  $title \rightarrow star$  implies  $title \rightarrow studio, star$   
 $title \rightarrow studio, studio \rightarrow star$  implies  $title \rightarrow star$

But,

$title, star \rightarrow studio$  does NOT necessarily imply that  
 $title \rightarrow studio$  or that  $star \rightarrow studio$

- An FD  $f$  is **implied by** a set of FDs  $F$  if  $f$  holds whenever all FDs in  $F$  hold.
- $F^+$  = **closure of  $F$**  is the set of all FDs that are implied by  $F$ . (includes "trivial dependencies")



## Rules of Inference (Review)

- **Armstrong's Axioms** ( $X, Y, Z$  are sets of attributes):

- **Reflexivity**: If  $X \supseteq Y$ , then  $X \rightarrow Y$
- **Augmentation**: If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any  $Z$
- **Transitivity**: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$

- These are **sound** and **complete** inference rules for FDs!

- i.e., using AA you can compute all the FDs in  $F^+$  and only these FDs.

- Some additional rules (that follow from AA):

- **Union**: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
- **Decomposition**: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$



## Attribute Closure

- Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)
- Typically, we just want to check if a given FD  $X \rightarrow Y$  is in the closure of a set of FDs  $F$ . An efficient check:
  - Compute **attribute closure** of  $X$  (denoted  $X^+$ ) wrt  $F$ .  
 $X^+$  = Set of all attributes  $A$  such that  $X \rightarrow A$  is in  $F^+$ 
    - $X^+ := X$
    - Repeat until no change: if there is an fd  $U \rightarrow V$  in  $F$  such that  $U$  is in  $X^+$ , then add  $V$  to  $X^+$
  - Check if  $Y$  is in  $X^+$
  - Approach can also be used to find the keys of a relation.
    - If all attributes of  $R$  are in the closure of  $X$  then  $X$  is a superkey for  $R$ .
    - Q: How to check if  $X$  is a "candidate key"?



## Normal Forms

- Back to schema refinement...

- **Q1: is any refinement needed??!**

- If a relation is in a **normal form** (BCNF, 3NF etc.):

- we know that certain problems are avoided/minimized.
- helps decide whether decomposing a relation is useful.

- **Role of FDs in detecting redundancy:**

- Consider a relation  $R$  with 3 attributes, ABC.
  - **No (non-trivial) FDs hold**: There is no redundancy here.
  - **Given  $A \rightarrow B$** : If  $A$  is not a key, then several tuples could have the same  $A$  value, and if so, they'll all have the same  $B$  value!

- **1<sup>st</sup> Normal Form – all attributes are atomic**

- i.e. the relational model

- **1<sup>st</sup>  $\supset$  2<sup>nd</sup> (of historical interest)  $\supset$  3<sup>rd</sup>  $\supset$  Boyce-Codd  $\supset$  ...**



## Boyce-Codd Normal Form (BCNF)

- Reln R with FDs F is in **BCNF** if, for all  $X \rightarrow A$  in  $F^+$ 
  - $A \in X$  (called a *trivial* FD), or
  - X is a superkey for R.
- In other words: "R is in BCNF if the only non-trivial FDs over R are key constraints."
- If R in BCNF, then every field of every tuple records information that **cannot be inferred** using FDs alone.
  - Say we know FD  $X \rightarrow A$  holds this example relation:

• Can you guess the value of the missing attribute?

X	Y	A
x	y1	A
x	y2	?

• Yes, so relation is not in BCNF



## Decomposition of a Relation Schema

- If a relation is not in a desired normal form, it can be *decomposed* into multiple relations that each are in that normal form.
- Suppose that relation R contains attributes  $A_1 \dots A_n$ . A *decomposition* of R consists of replacing R by two or more relations such that:
  - Each new relation scheme contains a *subset* of the attributes of R, and
  - Every attribute of R appears as an attribute of at least one of the new relations.



## Example (same as before)

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Hourly\_Emps

- SNLRWH has FDs  $S \rightarrow SNLRWH$  and  $R \rightarrow W$
- Q: Is this relation in BCNF?

No, The second FD causes a violation; W values repeatedly associated with R values.



## Decomposing a Relation

- Easiest fix is to create a relation RW to store these associations, and to remove W from the main schema:

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

R	W
8	10
5	7

Wages

Hourly\_Emps2

- Q: Are both of these relations are now in BCNF?
- **Decompositions should be used only when needed.**
- Q: potential problems of decomposition?



## Problems with Decompositions

- There are three potential problems to consider:
  - 1) May be *impossible* to reconstruct the original relation! (*Lossy Decomposition*)
    - Fortunately, not in the SNLRWH example.
  - 2) Dependency checking may require joins (*not Dependency Preserving*)
    - Fortunately, not in the SNLRWH example.
  - 3) Some queries become more expensive.
    - e.g., How much does Guldu earn?

**Tradeoff:** Must consider these issues vs. redundancy. (Well, not usually #1)



## Lossless Decomposition (example)

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

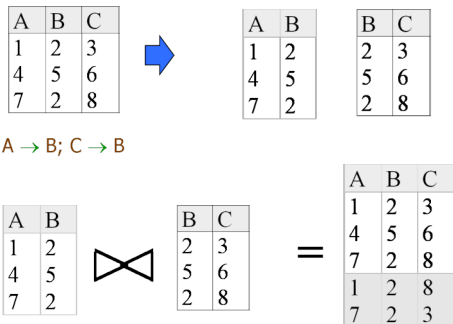


R	W
8	10
5	7

=

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
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434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

## Lossy Decomposition (example)



## Lossless Join Decompositions

- **Decomposition of R into X and Y is *lossless* w.r.t. a set of FDs F if, for every instance r that satisfies F:**

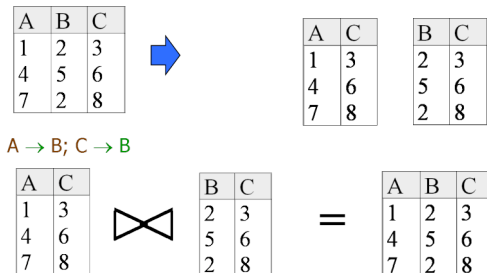
$$\pi_X(r) \bowtie \pi_Y(r) = r$$
- **It is always true that  $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$** 
  - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- **Definition extended to decomposition into 3 or more relations in a straightforward way.**
- **It is *essential* that all decompositions used to deal with redundancy be *lossless!* (Avoids Problem #1)**

## More on Lossless Decomposition

- The decomposition of R into X and Y is **lossless with respect to F** if and only if the **closure** of F contains:
 

$X \cap Y \rightarrow X$ , or i.e. the common attributes form a superkey for one side or the other  
 $X \cap Y \rightarrow Y$
- in example: decomposing ABC into AB and BC is lossy, because intersection (i.e., "B") is not a key of either resulting relation.
- **Useful result:** If  $W \rightarrow Z$  holds over R and  $W \cap Z$  is empty, then decomposition of R into R-Z and WZ is loss-less.

## Lossless Decomposition (example)



But, now we can't check  $A \rightarrow B$  without doing a join!

## Dependency Preserving Decomposition

- **Dependency preserving decomposition (Intuitive):**
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold individually on X, on Y and on Z, then all FDs that were given to hold on R must also hold. (Avoids Problem #2 on our list.)
    - Why do we care??
- **Projection of set of FDs F:** If R is decomposed into X and Y the projection of F on X (denoted  $F_X$ ) is the set of FDs  $U \rightarrow V$  in  $F^+$  (closure of F, not just F) such that all of the attributes U, V are in X. (same holds for Y of course)

## Dependency Preserving Decompositions (Contd.)

- **Decomposition of R into X and Y is *dependency preserving* if  $(F_X \cup F_Y)^+ = F^+$** 
  - i.e., if we consider only dependencies in the closure  $F^+$  that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in  $F^+$ .
- **Important to consider  $F^+$  in this definition:**
  - ABC,  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $C \rightarrow A$ , decomposed into AB and BC.
  - Is this dependency preserving? Is  $C \rightarrow A$  preserved????
    - note:  $F^+$  contains  $F \cup \{A \rightarrow C, B \rightarrow A, C \rightarrow B\}$ , so...
- $F_{AB}$  contains  $A \rightarrow B$  and  $B \rightarrow A$ ;  $F_{BC}$  contains  $B \rightarrow C$  and  $C \rightarrow B$
- So,  $(F_{AB} \cup F_{BC})^+$  contains  $C \rightarrow A$



## Decomposition into BCNF

- Consider relation  $R$  with FDs  $F$ . If  $X \rightarrow Y$  violates BCNF, decompose  $R$  into  $R - Y$  and  $XY$  (guaranteed to be loss-less).
  - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
  - e.g., CSJDPQV, key  $C$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$ 
    - {contractid, supplierid, projectid, deptid, partid, qty, value}
    - To deal with  $SD \rightarrow P$ , decompose into  $SDP$ , CSJDQV.
    - To deal with  $J \rightarrow S$ , decompose CSJDQV into  $JS$  and  $CJDQV$
    - So we end up with:  $SDP$ ,  $JS$ , and  $CJDQV$
- Note: several dependencies may cause violation of BCNF. The order in which we deal with them could lead to very different sets of relations!



## BCNF and Dependency Preservation

- In general, there may not be a dependency preserving decomposition into BCNF.
  - e.g.,  $CSZ$ ,  $CS \rightarrow Z$ ,  $Z \rightarrow C$
  - Can't decompose while preserving 1st FD; not in BCNF.
- Similarly, decomposition of CSJDPQV into  $SDP$ ,  $JS$  and  $CJDQV$  is not dependency preserving (w.r.t. the FDs  $JP \rightarrow C$ ,  $SD \rightarrow P$  and  $J \rightarrow S$ ).
  - {contractid, supplierid, projectid, deptid, partid, qty, value}
    - However, it is a lossless join decomposition.
    - In this case, adding  $JPC$  to the collection of relations gives us a dependency preserving decomposition.
      - but  $JPC$  tuples are stored only for checking the f.d. (Redundancy!)



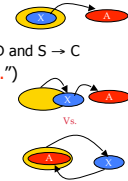
## Third Normal Form (3NF)

- Reln  $R$  with FDs  $F$  is in 3NF if, for all  $X \rightarrow A$  in  $F^+$ 
  - $A \in X$  (called a *trivial* FD), or
  - $X$  is a superkey of  $R$ , or
  - $A$  is part of some *candidate* key (not superkey!) for  $R$ . (sometimes stated as " $A$  is *prime*")
- **Minimality** of a key is crucial in third condition above!
- If  $R$  is in BCNF, obviously in 3NF.
- If  $R$  is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no "good" decomp, or performance considerations).
  - Lossless-join, dependency-preserving decomposition of  $R$  into a collection of 3NF relations always possible.



## What Does 3NF Achieve?

- If 3NF violated by  $X \rightarrow A$ , one of the following holds:
  - $X$  is a subset of some key  $K$  ("partial dependency")
    - We store  $(X, A)$  pairs redundantly.
    - e.g. Reserves SBDC ( $C$  is for credit card) with key  $SBD$  and  $S \rightarrow C$
  - $X$  is not a proper subset of any key. ("transitive dep.")
    - There is a chain of FDs  $K \rightarrow X \rightarrow A$
    - So we can't associate an  $X$  value with a  $K$  value unless we also associate an  $A$  value with an  $X$  value (different  $K$ 's, same  $X$  implies same  $A$ )
      - problem with initial SNLRWH example.
- But: even if  $R$  is in 3NF, these problems could arise.
  - e.g., Reserves SBDC (note: " $C$ " is for credit card here),  $S \rightarrow C$ ,  $C \rightarrow S$  is in 3NF (why?)
  - Even so, for each reservation of sailor  $S$ , same  $(S, C)$  pair is stored.
- Thus, 3NF is indeed a compromise relative to BCNF.
  - You have to deal with the partial and transitive dependency issues in your application code!



## An Aside: Second Normal Form

- Like 3NF, but allows transitive dependencies:
  - Reln  $R$  with FDs  $F$  is in 2NF if, for all  $X \rightarrow A$  in  $F^+$ 
    - $A \in X$  (called a *trivial* FD), or
    - $X$  is a superkey of  $R$ , or
    - $X$  is not part of any *candidate* key for  $R$ . (i.e. " $X$  is *not prime*")
- There's no reason to use this in practice
  - And we won't expect you to remember it



## Decomposition into 3NF

- Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier) but does not ensure dependency preservation.
- To ensure dependency preservation, one idea:
  - If  $X \rightarrow Y$  is not preserved, add relation  $XY$ .
- Problem is that  $XY$  may violate 3NF! e.g., consider the addition of  $CJP$  to 'preserve'  $JP \rightarrow C$ . What if we also have  $J \rightarrow C$ ?
- Refinement: Instead of the given set of FDs  $F$ , use a *minimal cover* for  $F$ .



## Minimal Cover for a Set of FDs

- **Minimal cover**  $G$  for a set of FDs  $F$ :
  - Closure of  $F$  = closure of  $G$ .
  - Right hand side of each FD in  $G$  is a single attribute.
  - If we modify  $G$  by deleting an FD or by deleting attributes from an FD in  $G$ , the closure changes.
- **Intuitively, every FD in  $G$  is needed, and ``as small as possible'' in order to get the same closure as  $F$ .**
- **e.g.,  $A \rightarrow B$ ,  $ABCD \rightarrow E$ ,  $EF \rightarrow GH$ ,  $ACDF \rightarrow EG$  has the following minimal cover:**
  - $A \rightarrow B$ ,  $ACD \rightarrow E$ ,  $EF \rightarrow G$  and  $EF \rightarrow H$
- **M.C. implies Lossless-Join, Dep. Pres. Decomp!!!**
  - (in book, p. 627)



## Summary of Schema Refinement

- **BCNF: each field contains information that cannot be inferred using only FDs.**
  - ensuring BCNF is a good heuristic.
- **Not in BCNF? Try decomposing into BCNF relations.**
  - Must consider whether all FDs are preserved!
- **Lossless-join, dependency preserving decomposition into BCNF impossible? Consider 3NF.**
  - Same if BCNF decomp is unsuitable for typical queries
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.
- **Note: even more restrictive Normal Forms exist (we don't cover them in this course, but some are in the book.)**