Announcements

- Midterm prep page is up
- Tell us NOW about midterm conflicts
- Project 2.1 up soon, due after midterm
- Project 1.4 (Learning Pacman) and the Pacman contest will be after the midterm
- Review session this Sunday, details on web
Recap: MDPs

- Markov decision processes:
  - States $S$
  - Actions $A$
  - Transitions $P(s'|s,a)$ (or $T(s,a,s')$)
  - Rewards $R(s,a,s')$
  - Start state $s_0$

- Quantities:
  - Returns = sum of discounted rewards
  - Values = expected future returns from a state (optimal, or for a fixed policy)
  - Q-Values = expected future returns from a q-state (optimal, or for a fixed policy)

Reinforcement Learning

- Reinforcement learning:
  - Still have an MDP:
    - A set of states $s \in S$
    - A set of actions (per state) $A$
    - A model $T(s,a,s')$
    - A reward function $R(s,a,s')$
  - Still looking for a policy $\pi(s)$

- New twist: don’t know $T$ or $R$
  - I.e. don’t know which states are good or what the actions do
  - Must actually try actions and states out to learn
Example: Animal Learning

- RL studied experimentally for more than 60 years in psychology
  - Rewards: food, pain, hunger, drugs, etc.
  - Mechanisms and sophistication debated

- Example: foraging
  - Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
  - Bees have a direct neural connection from nectar intake measurement to motor planning area

Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to V(s) using a neural network
- Combined with depth 3 search, one of the top 3 players in the world

- You could imagine training Pacman this way...

- … but it’s tricky!
Passive Learning

- Simplified task
  - You don't know the transitions $T(s,a,s')$
  - You don't know the rewards $R(s,a,s')$
  - You are given a policy $\pi(s)$
  - **Goal:** learn the state values (and maybe the model)

- In this case:
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - We'll get to the general case soon

Example: Direct Estimation

- Episodes:

<table>
<thead>
<tr>
<th>Episode</th>
<th>Reward</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1) up -1</td>
<td>-1</td>
</tr>
<tr>
<td>(1,2) up -1</td>
<td>-1</td>
</tr>
<tr>
<td>(1,2) up -1</td>
<td>-1</td>
</tr>
<tr>
<td>(1,3) right -1</td>
<td>-1</td>
</tr>
<tr>
<td>(2,3) right -1</td>
<td>-1</td>
</tr>
<tr>
<td>(3,3) right -1</td>
<td>-1</td>
</tr>
<tr>
<td>(3,2) up -1</td>
<td>-1</td>
</tr>
<tr>
<td>(3,2) up -1</td>
<td>-1</td>
</tr>
<tr>
<td>(3,3) right -1</td>
<td>-1</td>
</tr>
<tr>
<td>(4,3) exit +100</td>
<td>-100</td>
</tr>
<tr>
<td>(done)</td>
<td></td>
</tr>
</tbody>
</table>

$\gamma = 1, R = -1$

$U(1,1) \sim \frac{(92 + -106)}{2} = -7$

$U(3,3) \sim \frac{(99 + 97 + -102)}{3} = 31.3$
Model-Based Learning

- **Idea:**
  - Learn the model empirically (rather than values)
  - Solve the MDP as if the learned model were correct

- **Empirical model learning**
  - Simplest case:
    - Count outcomes for each s,a
    - Normalize to give estimate of $T(s,a,s')$
    - Discover $R(s,a,s')$ the first time we experience $(s,a,s')$
  - More complex learners are possible (e.g. if we know that all squares have related action outcomes, e.g. "stationary noise")

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Example: Model-Based Learning

- **Episodes:**
  - $(1,1)$ up -1
  - $(2,3)$ right -1
  - $(3,2)$ up -1
  - $(3,3)$ right -1
  - $(4,3)$ exit +100

  $T(<3,3>, \text{right}, <4,3>) = 1 / 3$

  $T(<2,3>, \text{right}, <3,3>) = 2 / 2$
Model-Based Learning

- In general, want to learn the optimal policy, not evaluate a fixed policy

- Idea: adaptive dynamic programming
  - Learn an initial model of the environment:
  - Solve for the optimal policy for this model (value or policy iteration)
  - Refine model through experience and repeat
  - Crucial: we have to make sure we actually learn about all of the model

Example: Greedy ADP

- Imagine we find the lower path to the good exit first
- Some states will never be visited following this policy from (1,1)
- We'll keep re-using this policy because following it never collects the regions of the model we need to learn the optimal policy
What Went Wrong?

- Problem with following optimal policy for current model:
  - Never learn about better regions of the space if current policy neglects them

- Fundamental tradeoff: exploration vs. exploitation
  - Exploration: must take actions with suboptimal estimates to discover new rewards and increase eventual utility
  - Exploitation: once the true optimal policy is learned, exploration reduces utility
  - Systems must explore in the beginning and exploit in the limit

Model-Free Learning

- Big idea: why bother learning $T$?
  - Update each time we experience a transition
  - Frequent outcomes will contribute more updates (over time)
- Temporal difference learning (TD)
  - Policy still fixed!
  - Move values toward value of whatever successor occurs

$$V^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, a, s') + \gamma V^\pi(s')]$$

$sample = R(s, a, s') + \gamma V^\pi(s')$

$$V^\pi(s) \leftarrow V^\pi(s) + \alpha (sample - V^\pi(s))$$
Example: Passive TD

\[ V^\pi(s) \leftarrow V^\pi(s) + \alpha \left[ R(s, a, s') + \gamma V^\pi(s') - V^\pi(s) \right] \]

(1,1) up -1  
(1,2) up -1  
(1,2) up -1  
(1,3) right -1  
(1,3) right -1  
(2,3) right -1  
(3,3) right -1  
(3,2) up -1  
(3,3) right -1  
(4,3) exit +100  
(done)  

Take \( \gamma = 1, \alpha = 0.5 \)

Problems with TD Value Learning

- TD value leaning is model-free for policy evaluation
- However, if we want to turn our value estimates into a policy, we’re sunk:

\[ \pi(s) = \arg \max_a Q^*(s, a) \]

\[ Q^*(s, a) = \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V^*(s') \right] \]

- Idea: learn Q-values directly
- Makes action selection model-free too!
Q-Learning

- Learn $Q^*(s,a)$ values
  - Receive a sample $(s,a,s',r)$
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:
    $$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V^*(s') \right]$$
    $$\text{sample} = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$
  - Nudge the old estimate towards the new sample:
    $$Q(s,a) \leftarrow Q(s,a) + \alpha [\text{sample} - Q(s,a)]$$

Q-Learning Example

- [DEMO]
Q-Learning Properties

- Will converge to optimal policy
  - If you explore enough
  - If you make the learning rate small enough

- Neat property: does not learn policies which are optimal in the presence of action selection noise

Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions ($\varepsilon$-greedy)
    - Every time step, flip a coin
    - With probability $\varepsilon$, act randomly
    - With probability $1-\varepsilon$, act according to current policy

- Problems with random actions?
  - You do explore the space, but keep thrashing around once learning is done
  - One solution: lower $\varepsilon$ over time
  - Another solution: exploration functions
Exploration Functions

- **When to explore**
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established

- **Exploration function**
  - Takes a value estimate and a count, and returns an optimistic utility, e.g. $f(u, n) = u + k/n$ (exact form not important)

$$Q_{i+1}(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q_i(s', a')$$

$$Q_{i+1}(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q_i(s', a'), N(s', a'))$$