# Automatic Image Alignment

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CS194: Image Manipulation & Computational Photography

Alexei Efros, UC Berkeley, Fall 2014

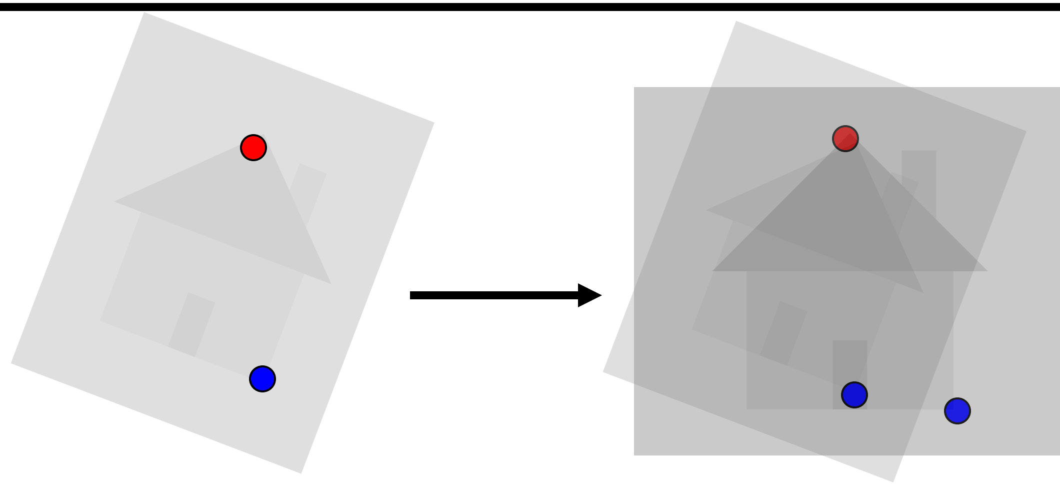
*with a lot of slides stolen from Steve Seitz and Rick Szeliski*

# Live Homography DEMO

Check out panoramio.com “Look Around” feature!

**Also see OpenPhoto VR: http://openphotovr.org/**

# Image Alignment



How do we align two images automatically?

Two broad approaches:

- Feature-based alignment
  - Find a few matching features in both images
  - compute alignment
- Direct (pixel-based) alignment
  - Search for alignment where most pixels agree

# Direct Alignment

The simplest approach is a brute force search (hw1)

- Need to define image matching function
  - SSD, Normalized Correlation, edge matching, etc.
- Search over all parameters within a reasonable range:

e.g. for translation:

```
for tx=x0:step:x1,
  for ty=y0:step:y1,
     compare image1(x,y) to image2(x+tx,y+ty)
  end;
end;
```

Need to pick correct `x0,x1` and `step`

- What happens if `step` is too large?

# Direct Alignment (brute force)

What if we want to search for more complicated transformation, e.g. homography?

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

```
for a=a0:astep:a1,
  for b=b0:bstep:b1,
    for c=c0:cstep:c1,
      for d=d0:dstep:d1,
        for e=e0:estep:e1,
          for f=f0:fstep:f1,
             for g=g0:gstep:g1,
                for h=h0:hstep:h1,
     compare image1 to H(image2)
end; end; end; end; end; end; end; end;
```

# Problems with brute force

## Not realistic

- Search in $O(N^8)$ is problematic
- Not clear how to set starting/stopping value and step

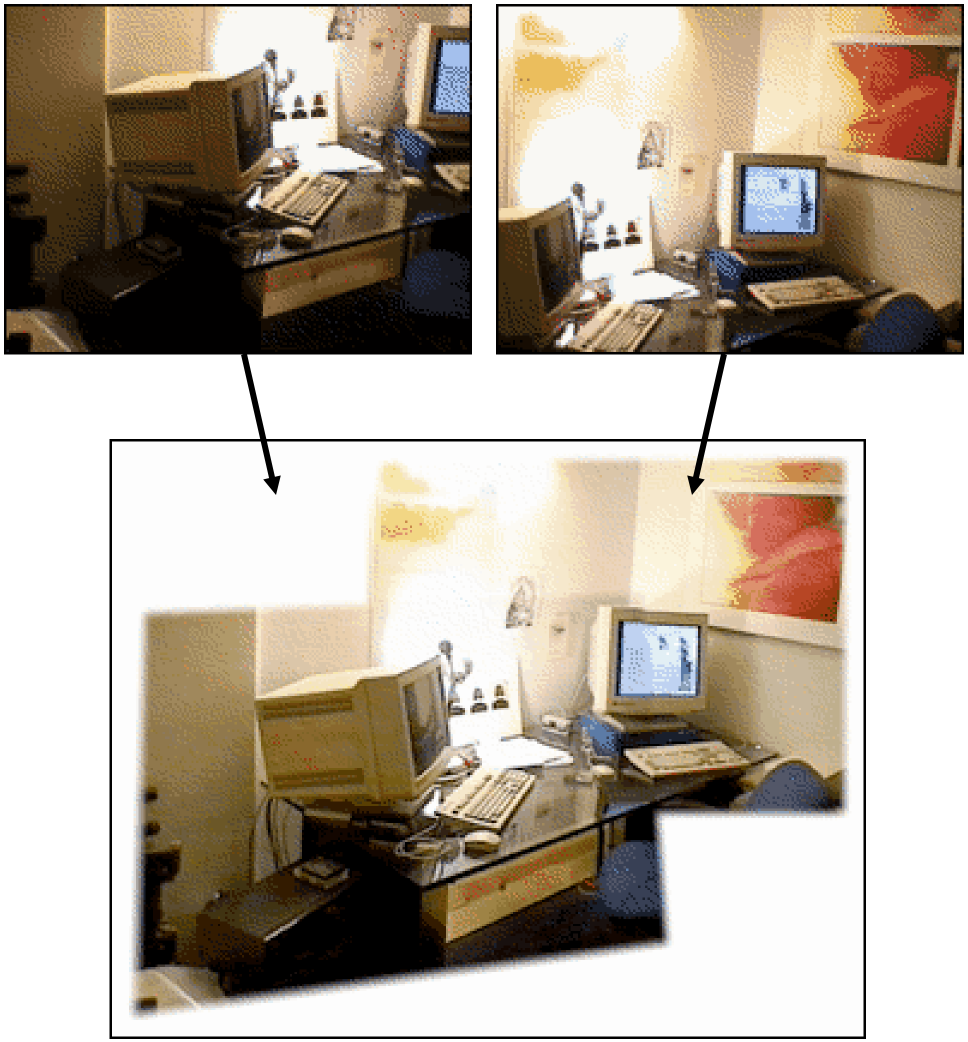
## What can we do?

- Use pyramid search to limit starting/stopping/step values
- For special cases (rotational panoramas), can reduce search slightly to $O(N^4)$:
  - $H = K_1 R_1 R_2^{-1} K_2^{-1}$ (4 DOF: f and rotation)

## Alternative: gradient decent on the error function

- i.e. how do I tweak my current estimate to make the SSD error go down?
- Can do sub-pixel accuracy
- BIG assumption?
  - Images are already almost aligned (<2 pixels difference!)
  - Can improve with pyramid
- Same tool as in **motion estimation**

# Image alignment



# Feature-based alignment

1. Find a few important features (aka Interest Points)
2. Match them across two images
3. Compute image transformation as per Project #5 Part I

How do we <u>choose</u> good features?

- They must be prominant in both images
- Easy to localize
- Think how you did that by hand in Project #5 Part I
- Corners!

# Feature Detection

# Feature Matching

How do we match the features between the images?

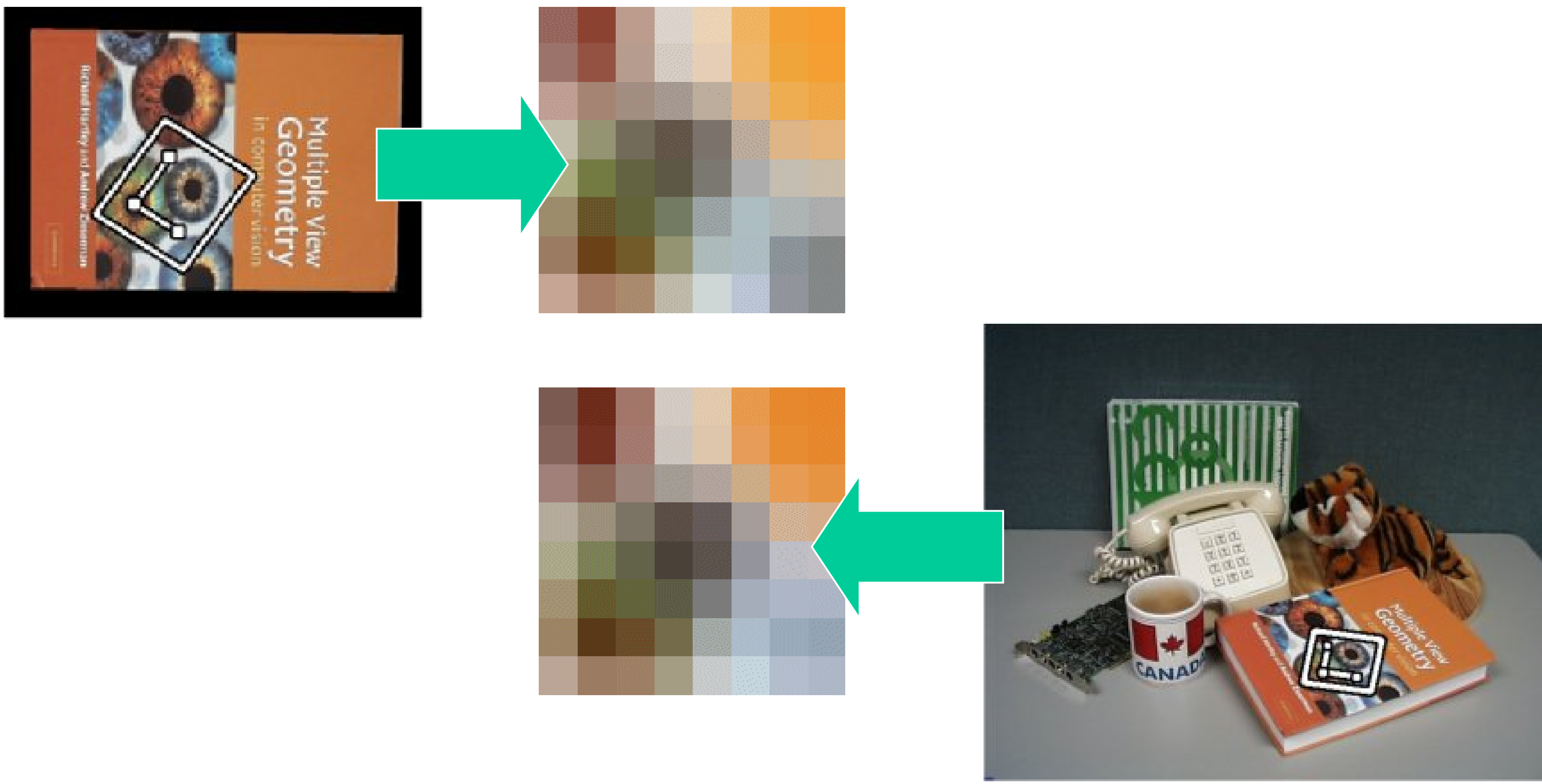
- Need a way to describe a region around each feature
  - e.g. image patch around each feature
- Use successful matches to estimate homography
  - Need to do something to get rid of outliers

Issues:

- What if the image patches for several interest points look similar?
  - Make patch size bigger
- What if the image patches for the same feature look different due to scale, rotation, etc.
  - Need an invariant descriptor

# Invariant Feature Descriptors

Schmid & Mohr 1997, Lowe 1999, Baumberg 2000, Tuytelaars & Van Gool 2000, Mikolajczyk & Schmid 2001, Brown & Lowe 2002, Matas et. al. 2002, Schaffalitzky & Zisserman 2002



# Today's lecture

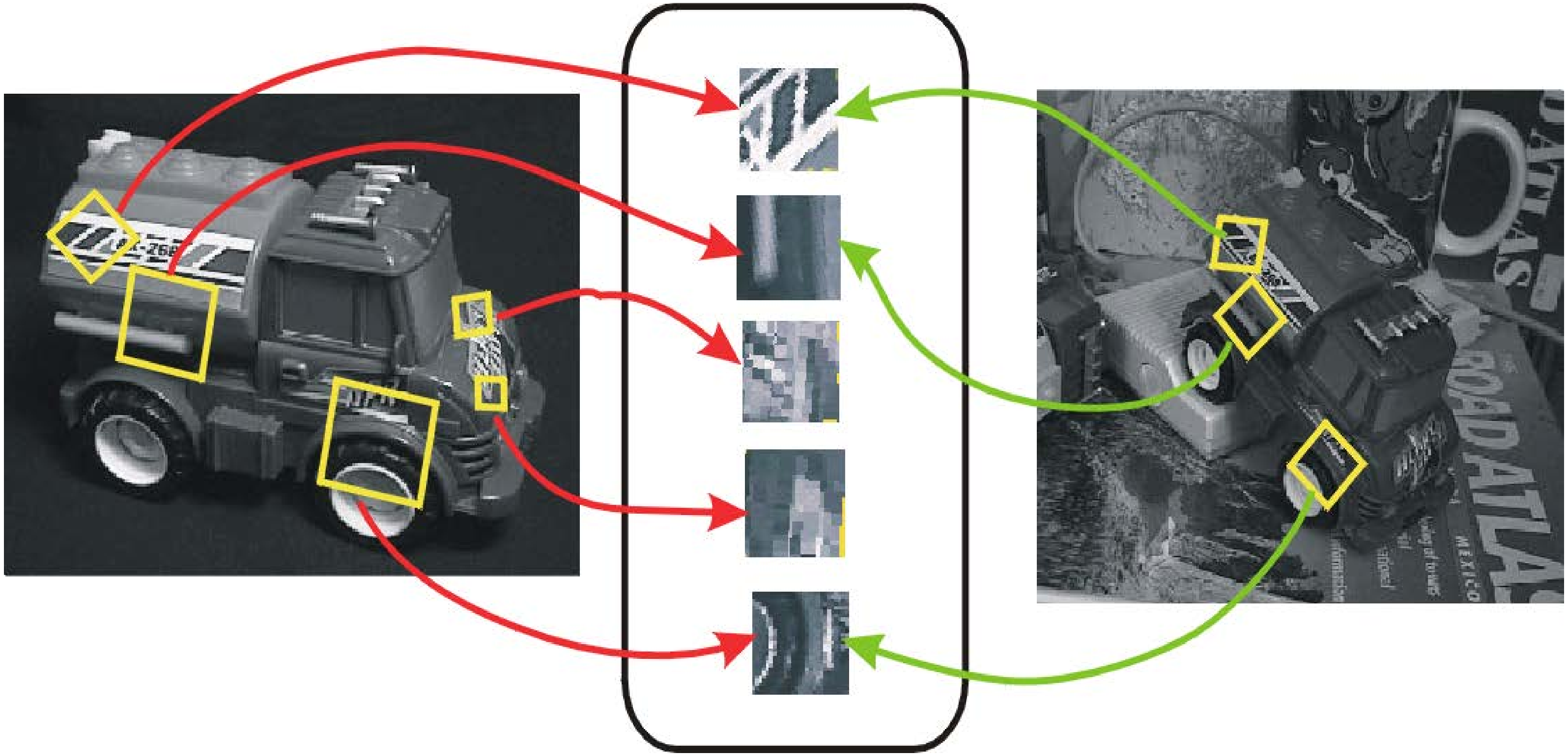
- 1 Feature <u>detector</u>
  - scale invariant Harris corners
- 1 Feature <u>descriptor</u>
  - patches, oriented patches

Reading:

**Multi-image Matching using Multi-scale image patches, CVPR 2005**

# Invariant Local Features

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



**Features Descriptors**

# Applications

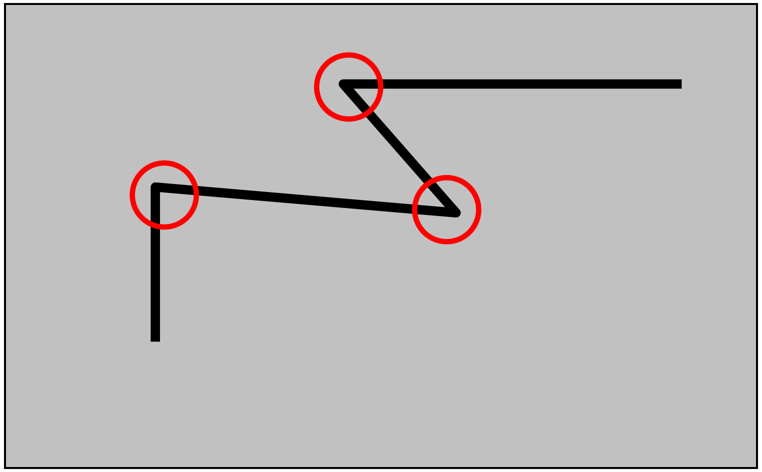
Feature points are used for:

- Image alignment (homography, fundamental matrix)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval
- Robot navigation
- … other

# Feature Detector – Harris Corner

# Harris corner detector

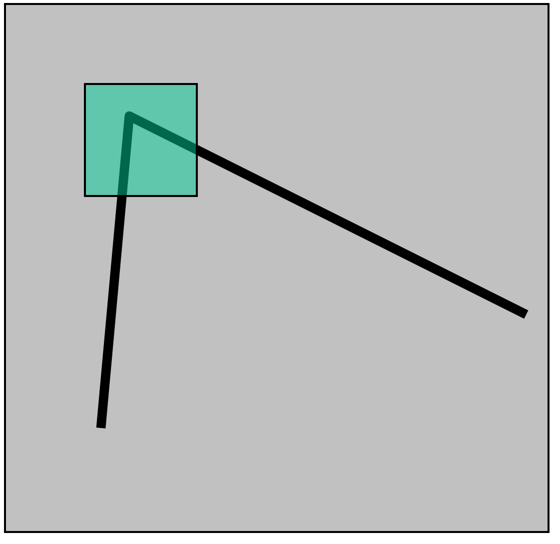
C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988



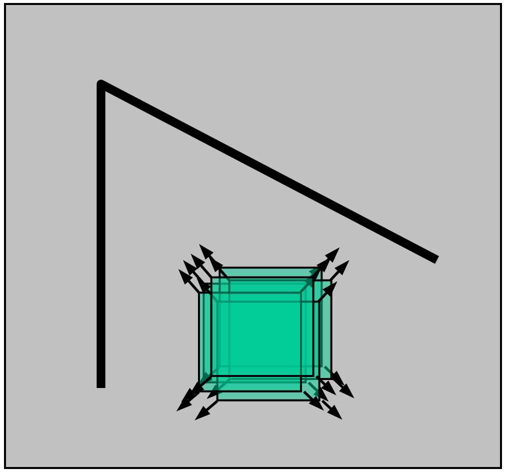
# The Basic Idea

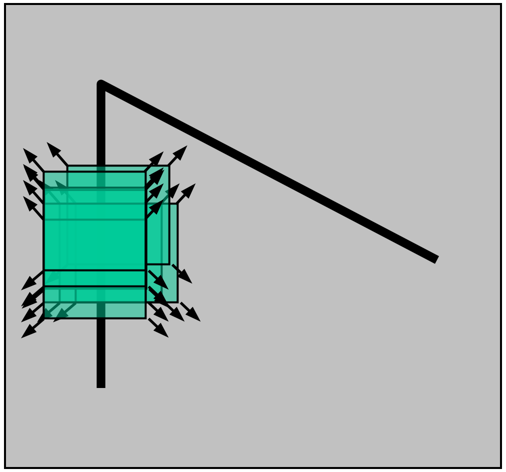
We should easily recognize the point by looking through a small window

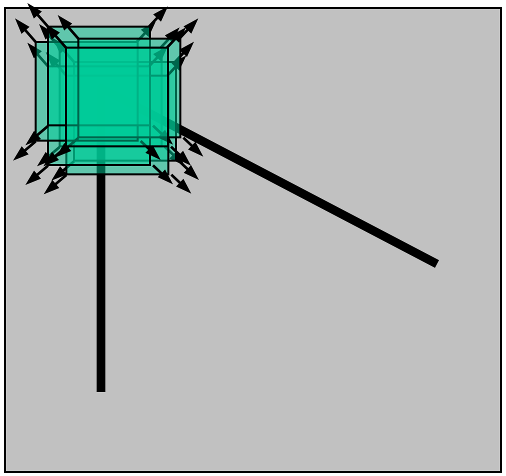
Shifting a window in *any direction* should give *a large change* in intensity



# Harris Detector: Basic Idea







"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

# Harris Detector: Mathematics

Change of intensity for the shift [$u$,$v$]:

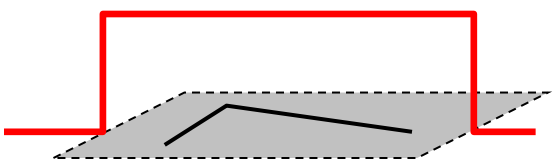
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

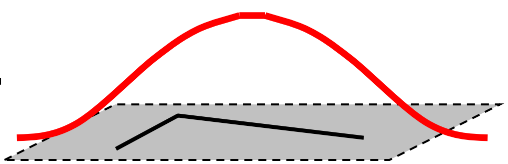
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

# Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

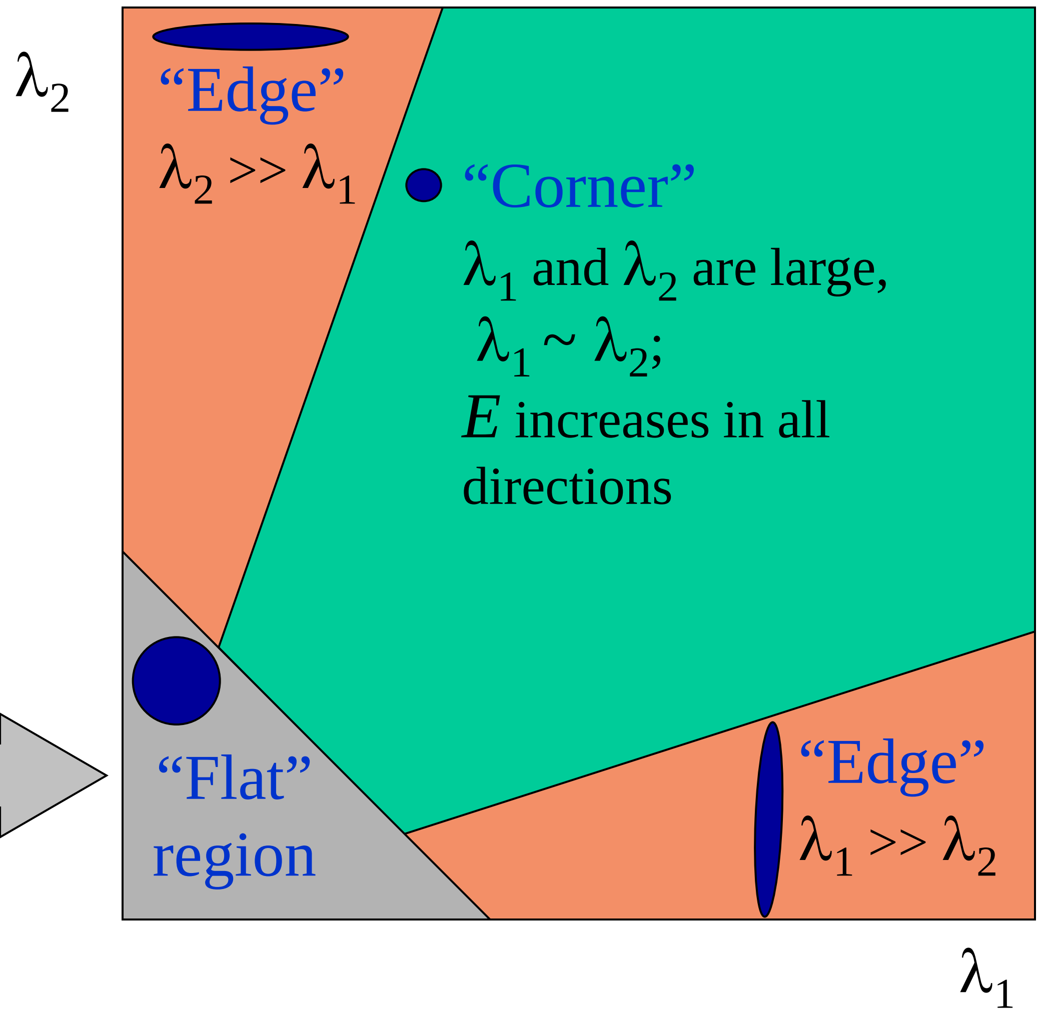
where $M$ is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

# Harris Detector: Mathematics

Classification of image points using eigenvalues of $M$:




$\lambda_1$ and $\lambda_2$ are small;
$E$ is almost constant in all directions

# Harris Detector: Mathematics

Measure of corner response:

$$R = \frac{\det M}{\operatorname{Trace} M}$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

# Harris Detector

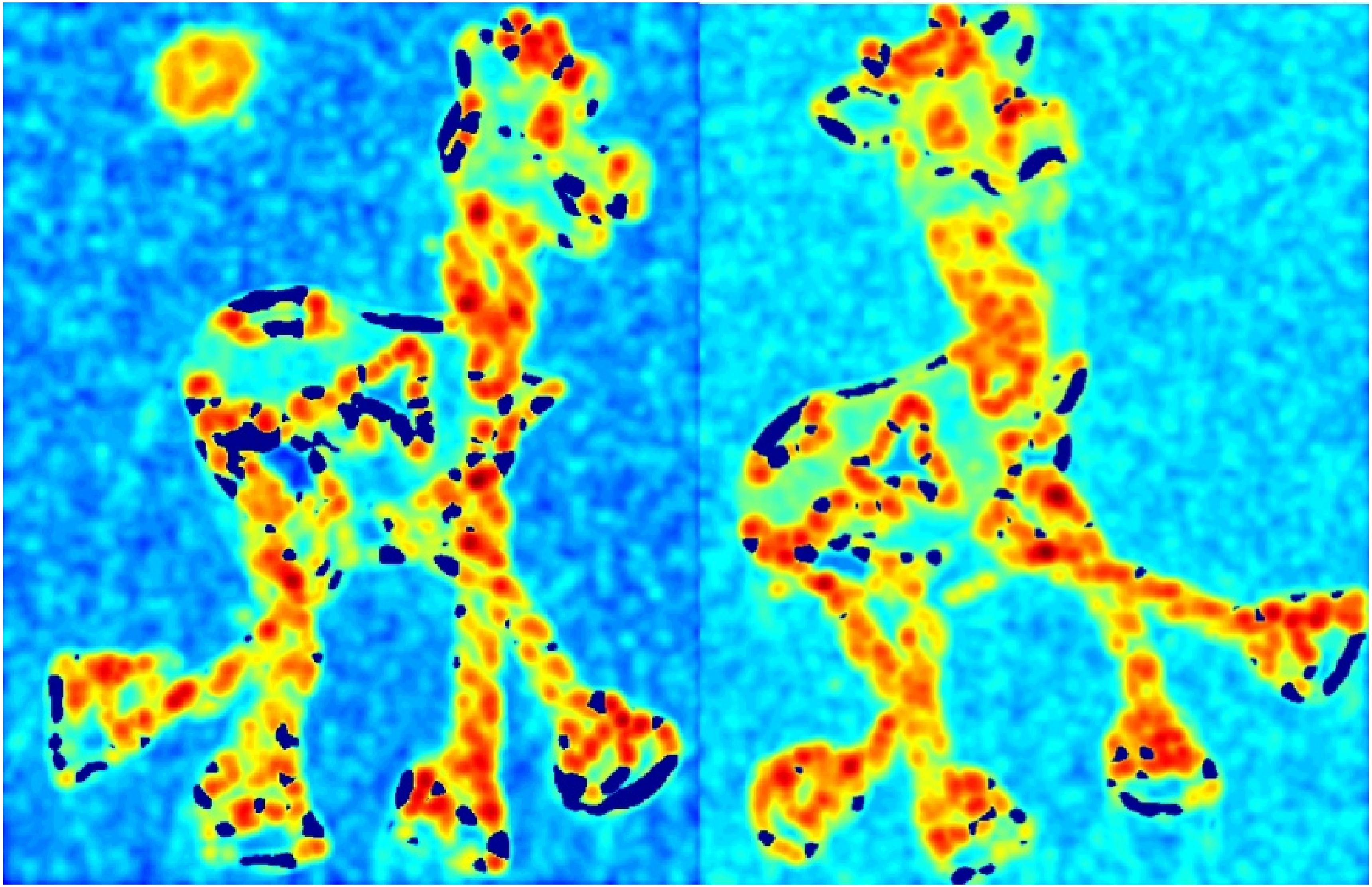
The Algorithm:

- Find points with large corner response function $R$ ($R$ > threshold)
- Take the points of local maxima of $R$

# Harris Detector: Workflow

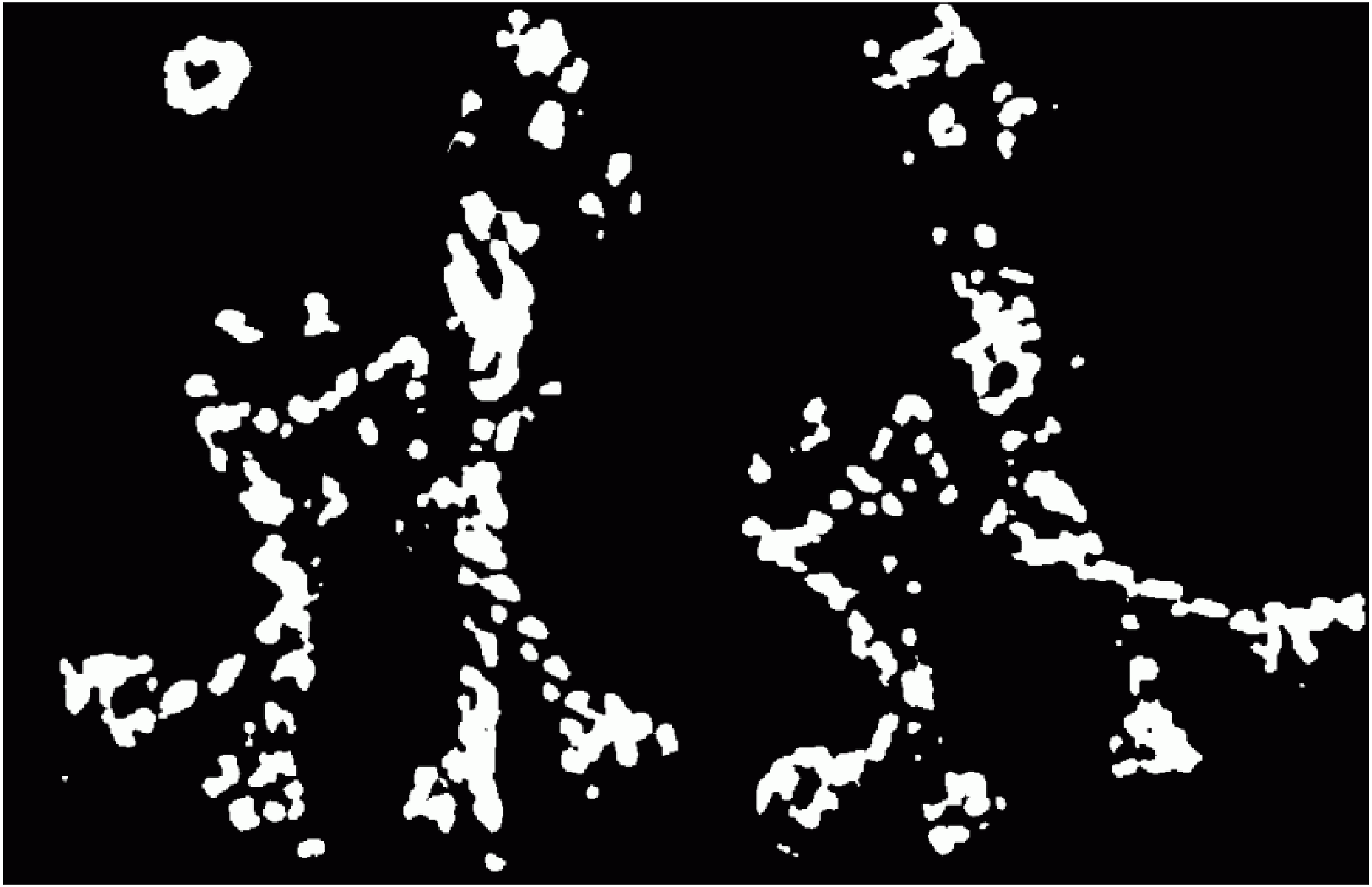
# Harris Detector: Workflow

Compute corner response $R$



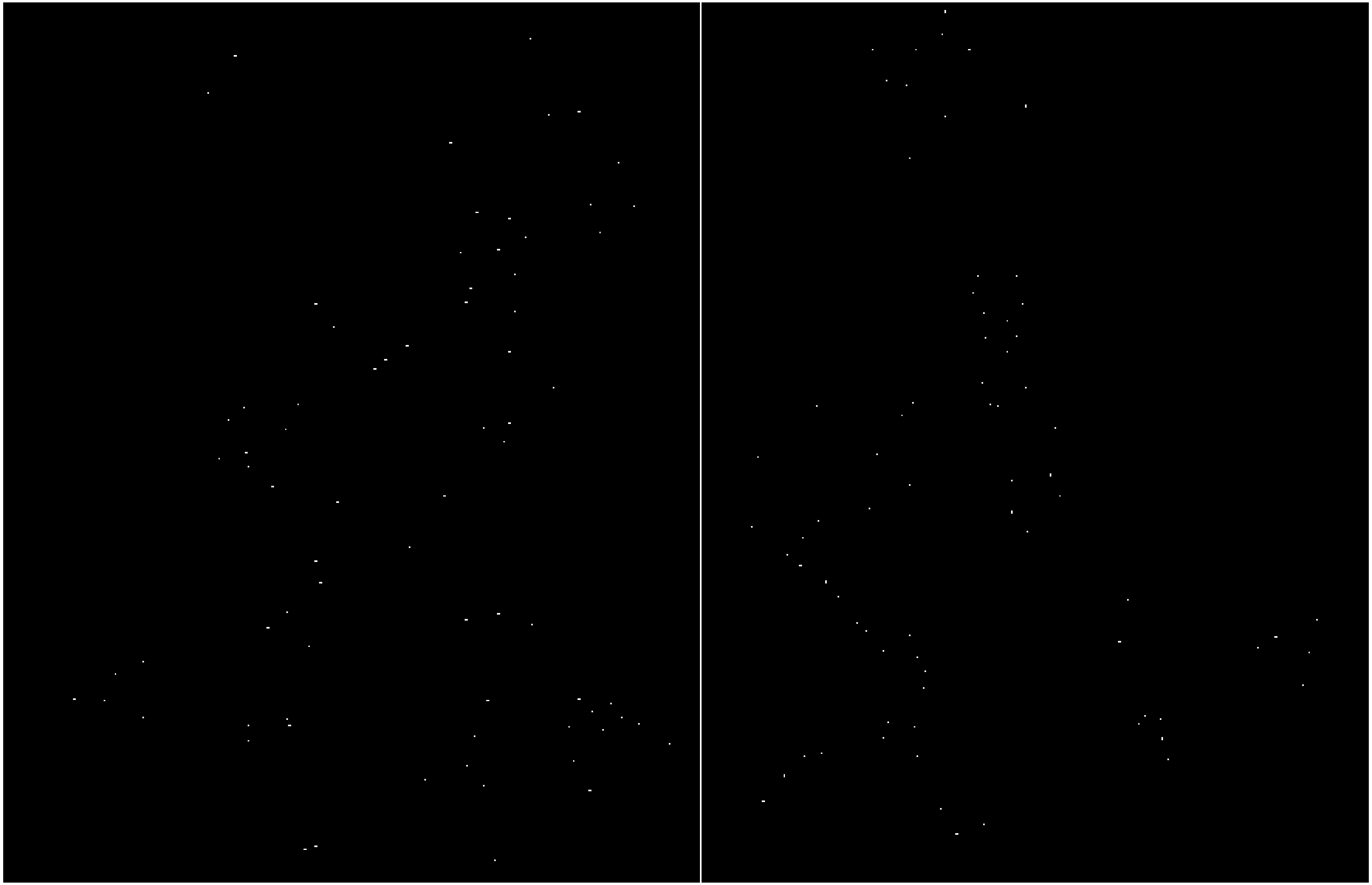
# Harris Detector: Workflow

Find points with large corner response: $R$>threshold



# Harris Detector: Workflow

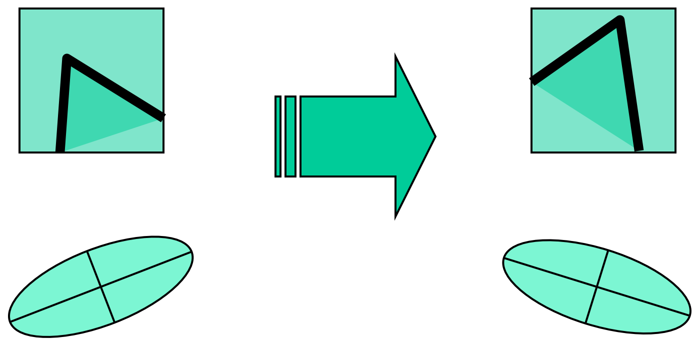
Take only the points of local maxima of $R$



# Harris Detector: Workflow

# Harris Detector: Some Properties

Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

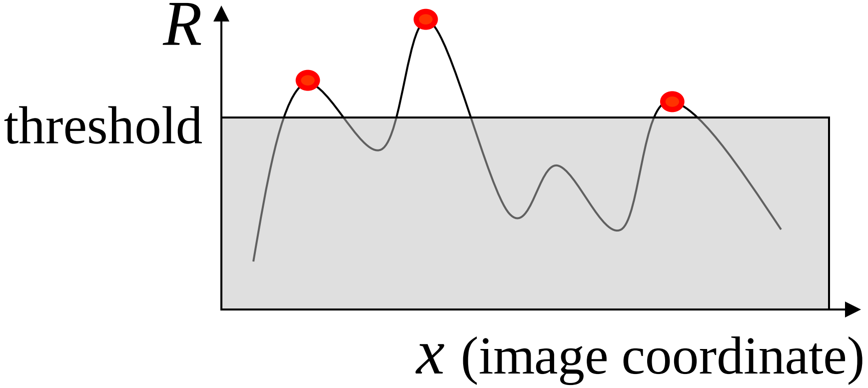
*Corner response* $R$ is invariant to image rotation

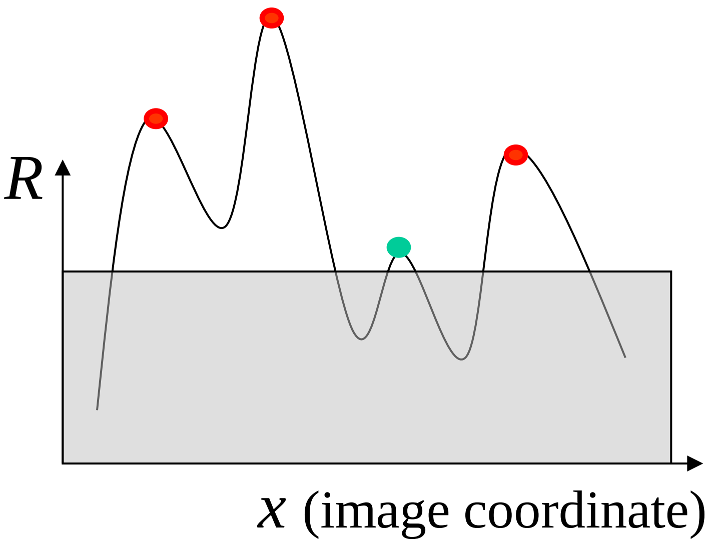
# Harris Detector: Some Properties

Partial invariance to *affine intensity* change

✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

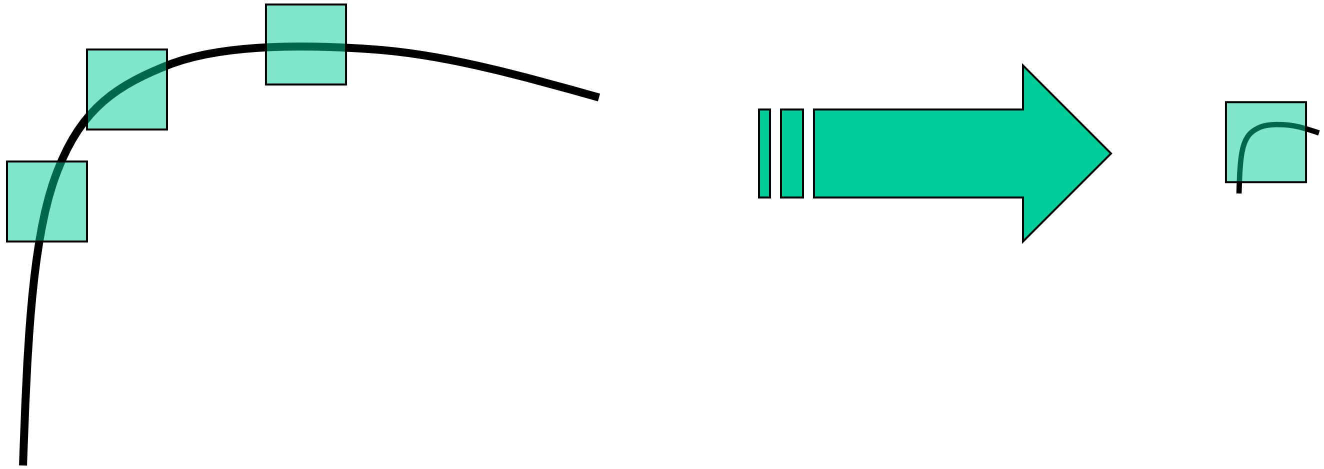
✓ Intensity scale: $I \rightarrow a\, I$



# Harris Detector: Some Properties

But: non-invariant to *image scale*!

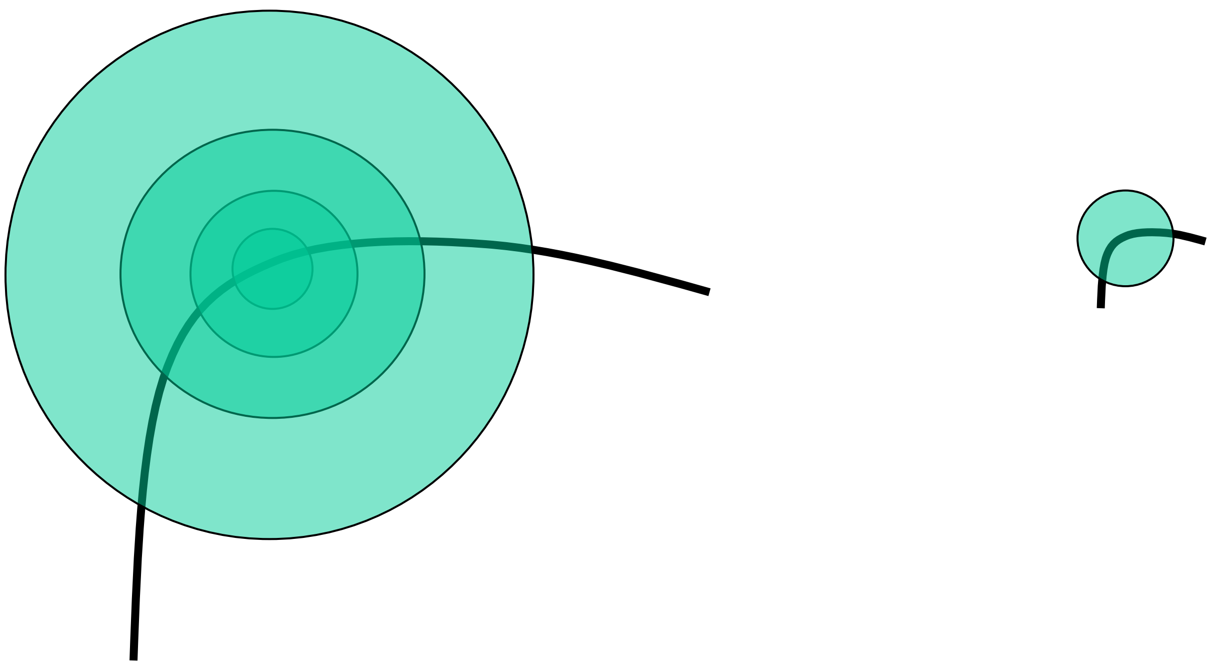


All points will be classified as edges

Corner !

# Scale Invariant Detection

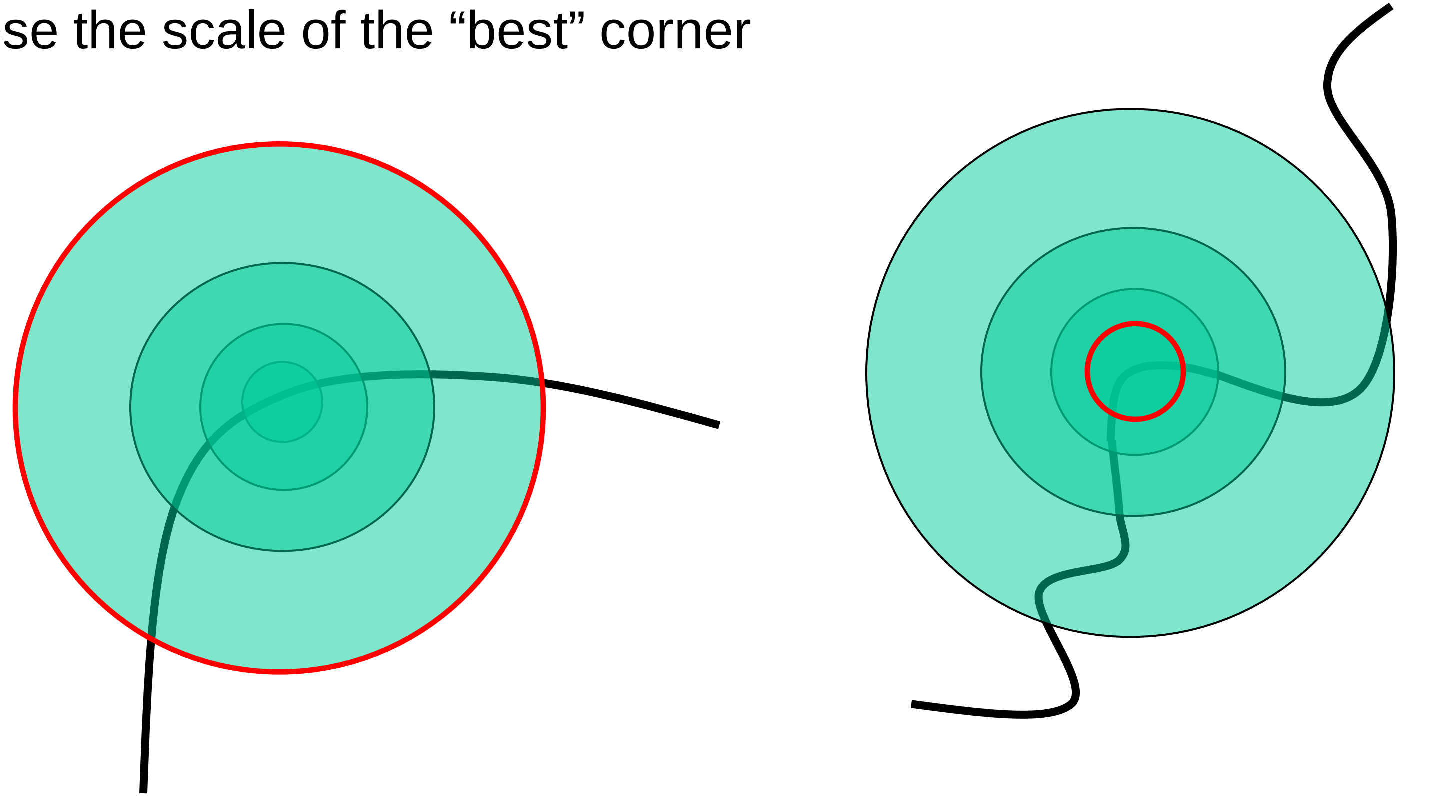
Consider regions (e.g. circles) of different sizes around a point
Regions of corresponding sizes will look the same in both images



# Scale Invariant Detection

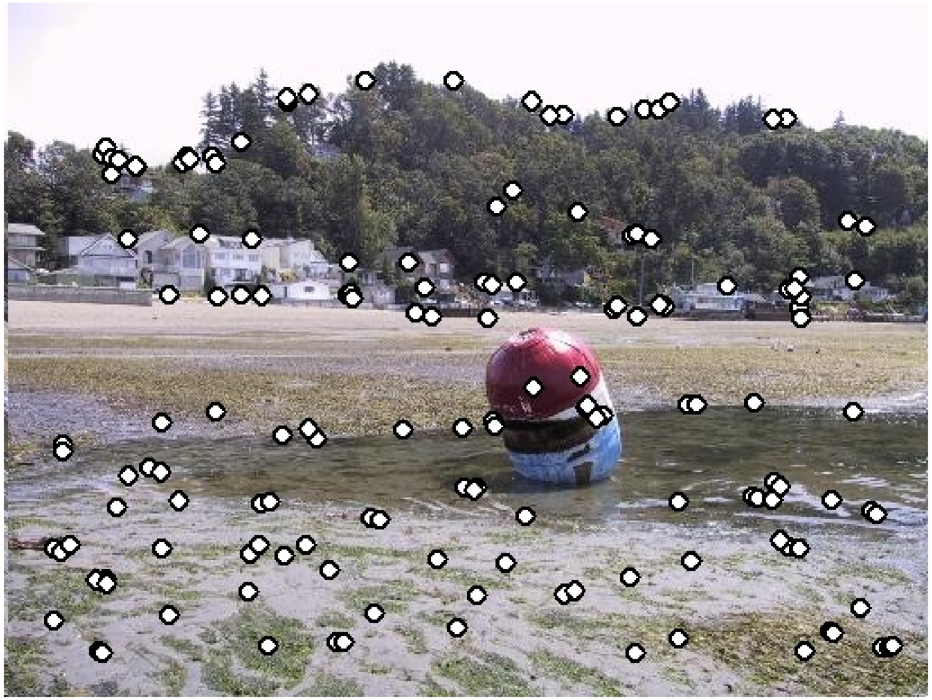
The problem: how do we choose corresponding circles *independently* in each image?

Choose the scale of the "best" corner



# Feature selection

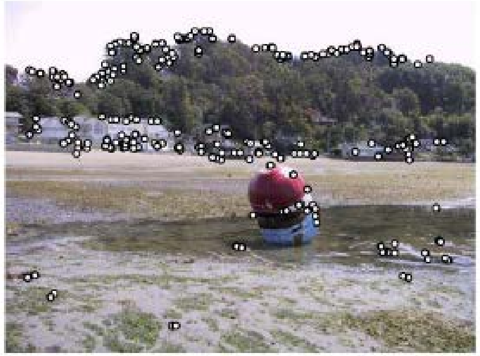
Distribute points evenly over the image



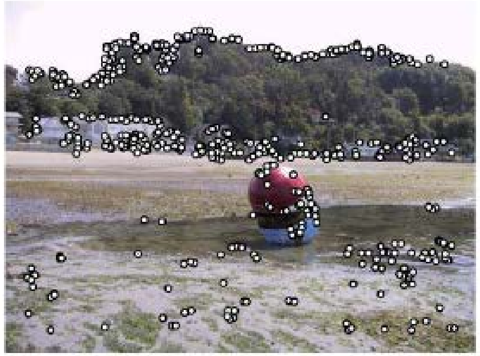
# Adaptive Non-maximal Suppression

Desired: Fixed # of features per image

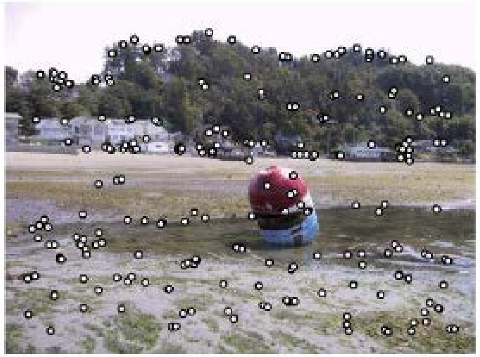
- Want evenly distributed spatially…
- Sort points by non-maximal suppression radius [Brown, Szeliski, Winder, CVPR'05]



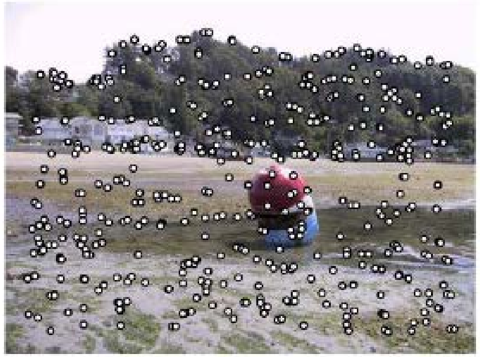
(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, $r = 24$

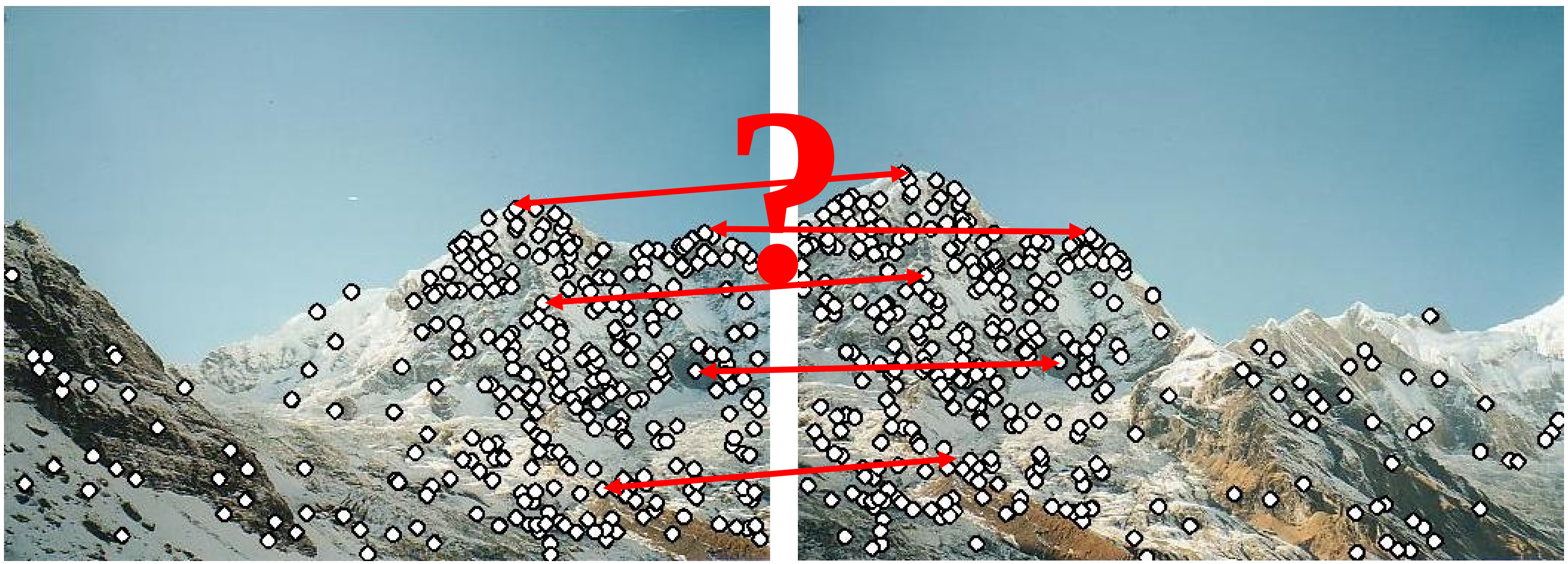


(d) ANMS 500, $r = 16$

# Feature descriptors

We know how to detect points

Next question: **How to match them?**




Point descriptor should be:

1. Invariant
2. Distinctive

# Feature Descriptor – MOPS

# Descriptors Invariant to Rotation

Find local orientation

Dominant direction of gradient

- Extract image patches relative to this orientation

# Multi-Scale Oriented Patches

Interest points

- Multi-scale Harris corners
- Orientation from blurred gradient
- Geometrically invariant to rotation

Descriptor vector

- Bias/gain normalized sampling of local patch (8x8)
- Photometrically invariant to affine changes in intensity

[Brown, Szeliski, Winder, CVPR’2005]

# Descriptor Vector

Orientation = blurred gradient

Rotation Invariant Frame

- Scale-space position (x, y, s) + orientation ($\theta$)

# Detections at multiple scales

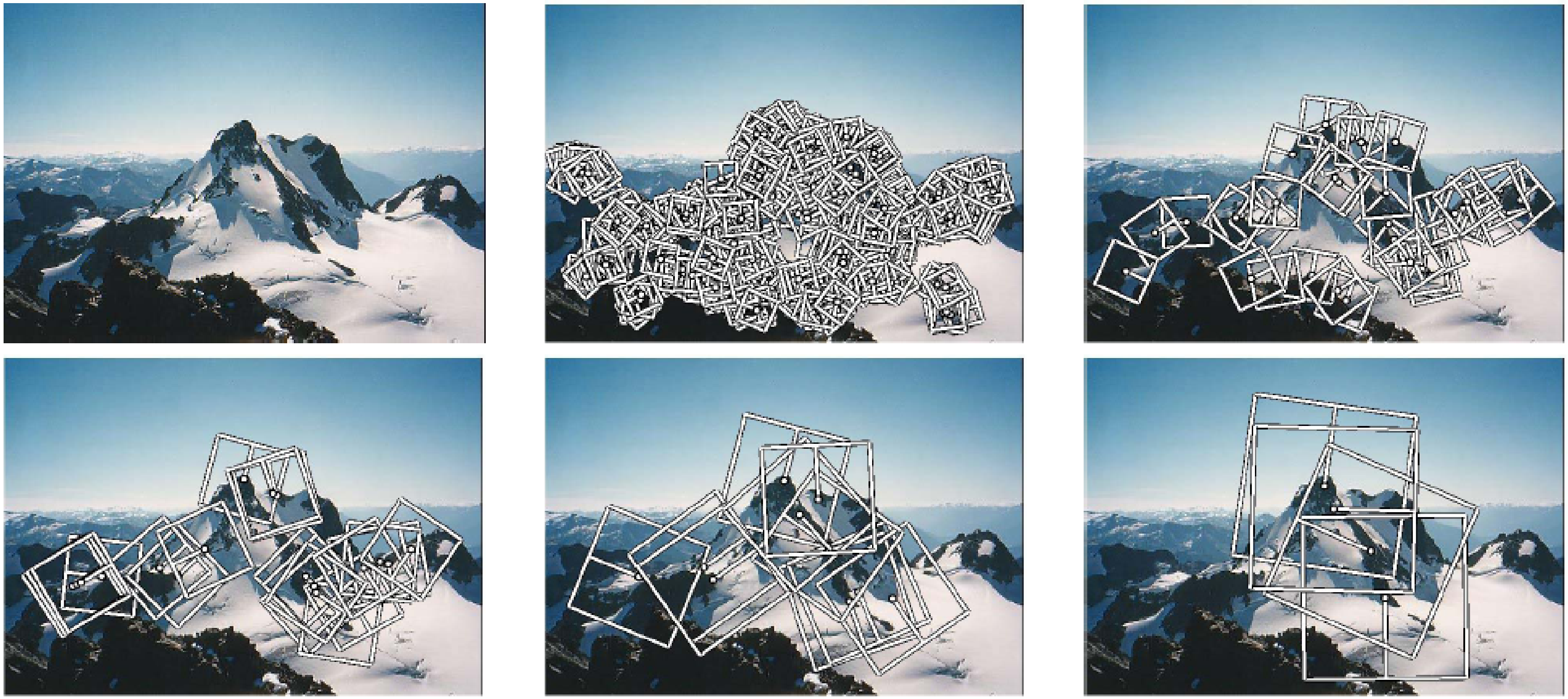


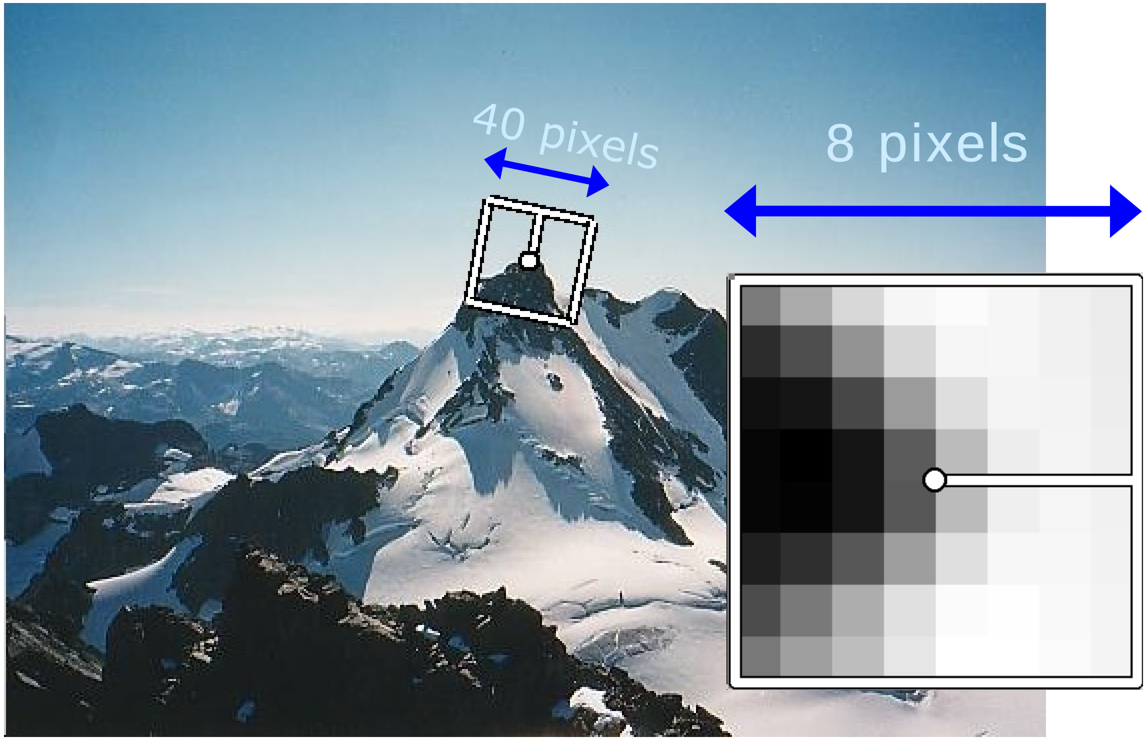
*Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.*

# MOPS descriptor vector

8x8 oriented patch

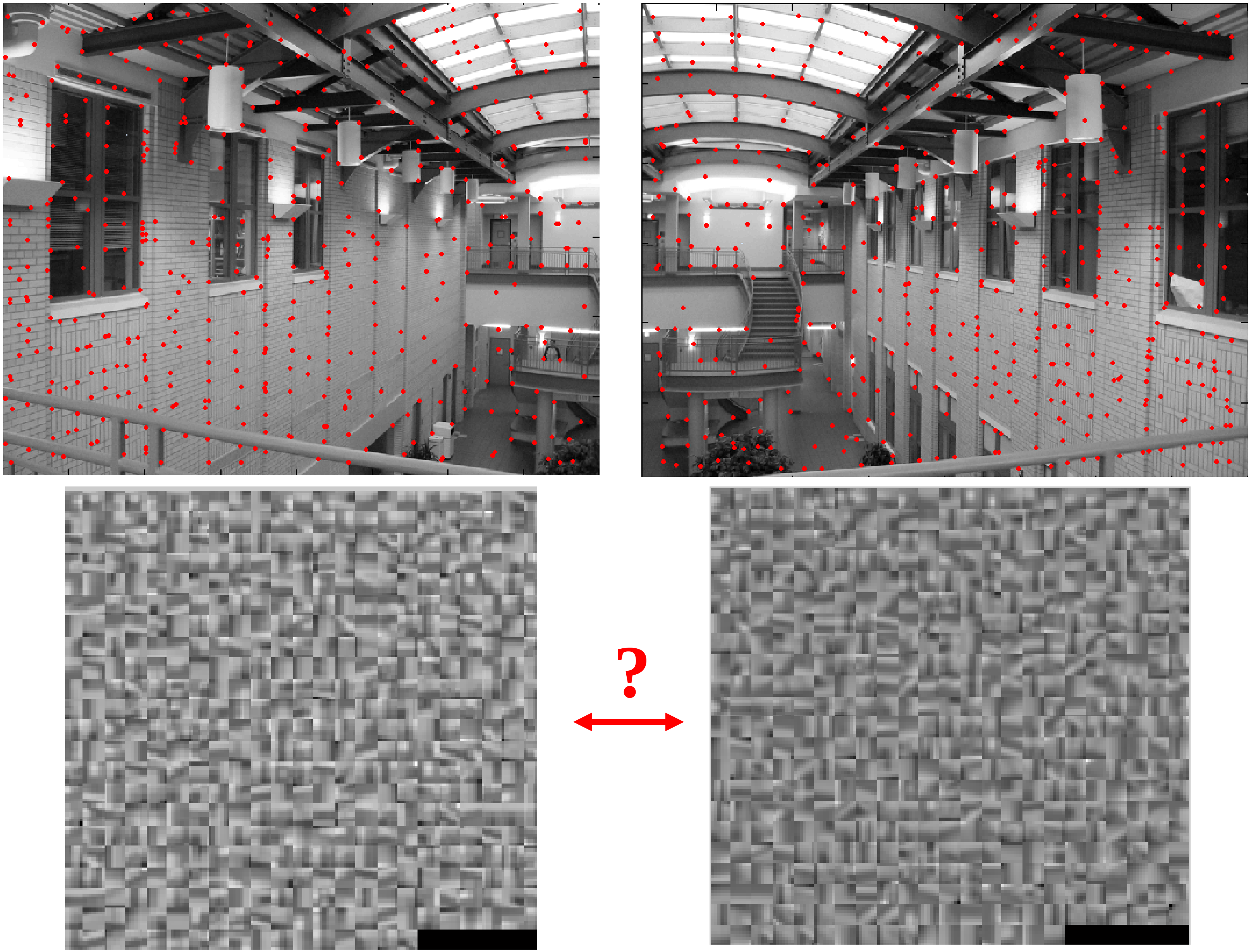
- Sampled at 5 x scale

Bias/gain normalisation: $I' = (I - \mu)/\sigma$



# Automatic Feature Matching

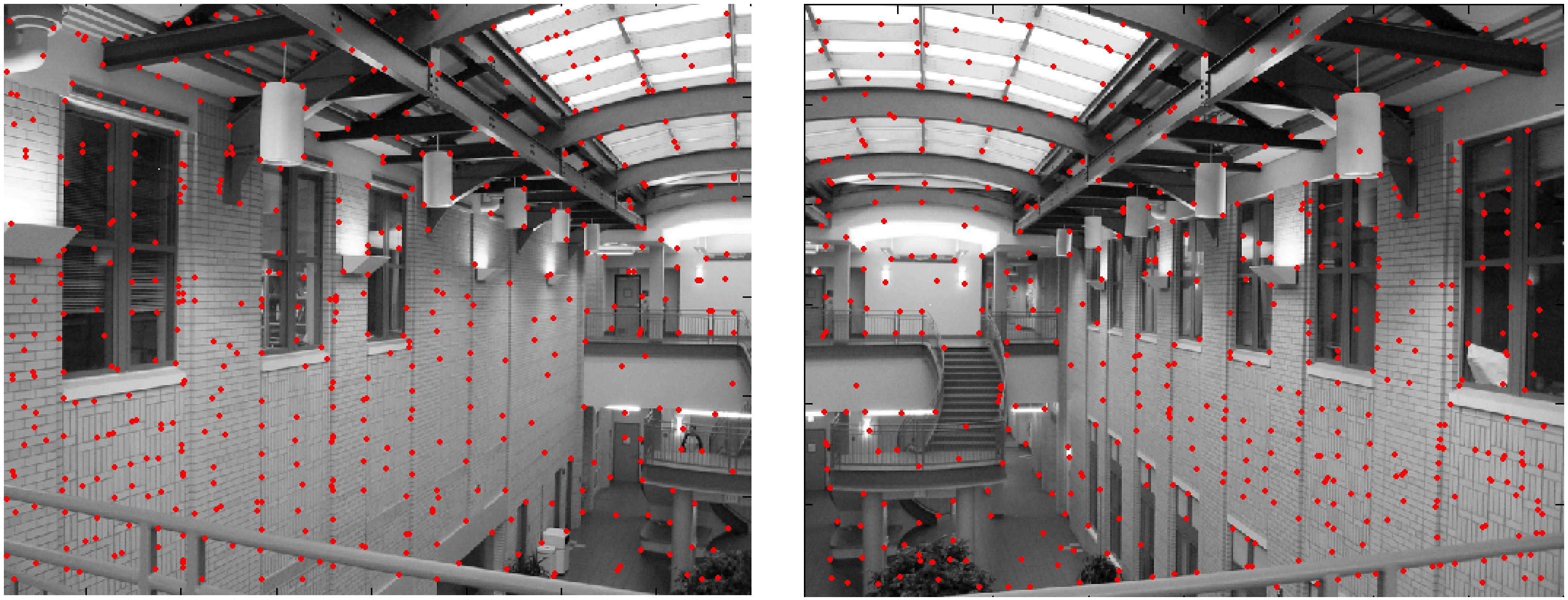
# Feature matching

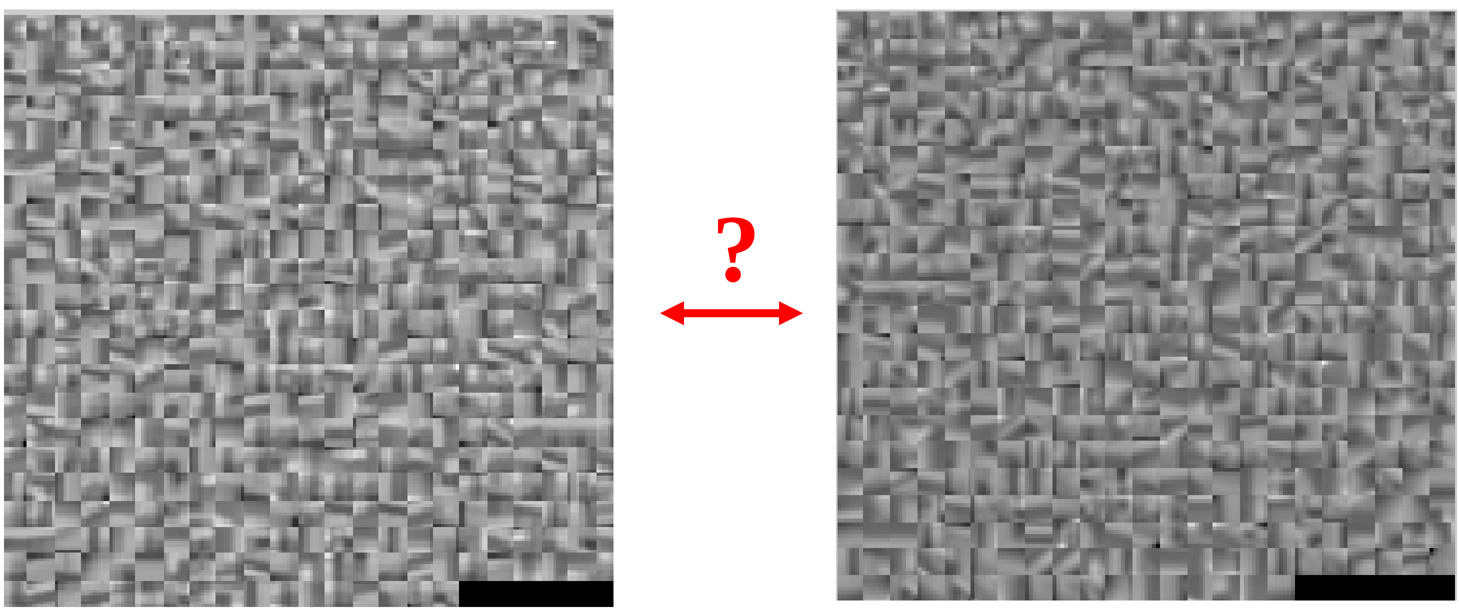


# Feature matching

- Pick best match!
  - For every patch in image 1, find the most similar patch (e.g. by SSD).
  - Called “nearest neighbor” in machine learning
- Can do various speed ups:
  - Hashing
    - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
  - Fast Nearest neighbor techniques
    - *kd*-trees and their variants
  - Clustering / Vector quantization
    - So called “visual words”

# What about outliers?



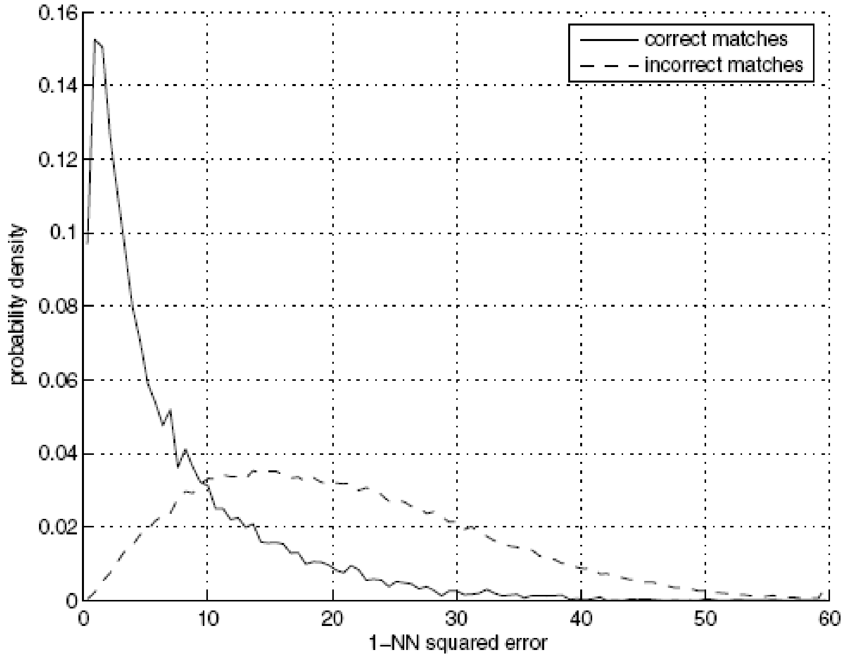


# Feature-space outlier rejection

Let's not match all features, but only these that have "similar enough" matches?

How can we do it?

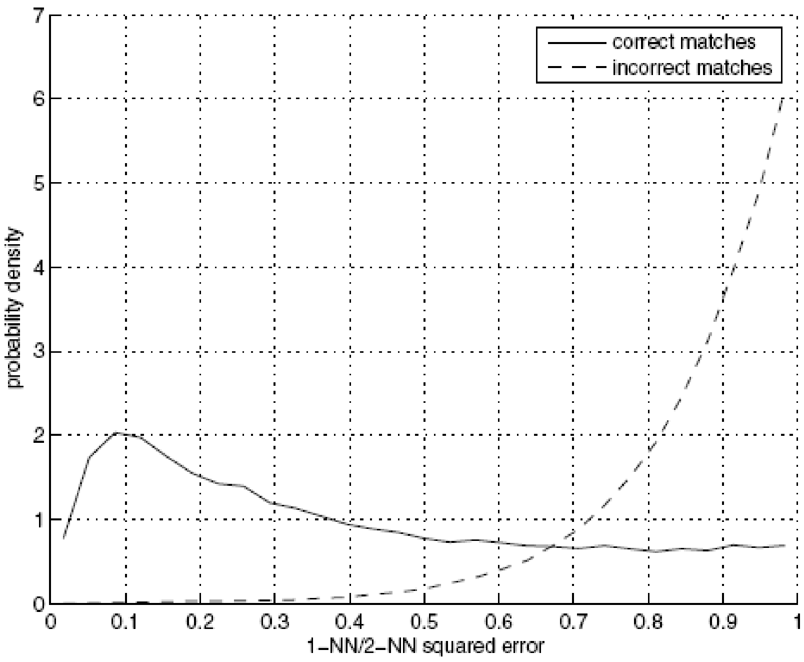
- SSD(patch1,patch2) < threshold
- How to set threshold?



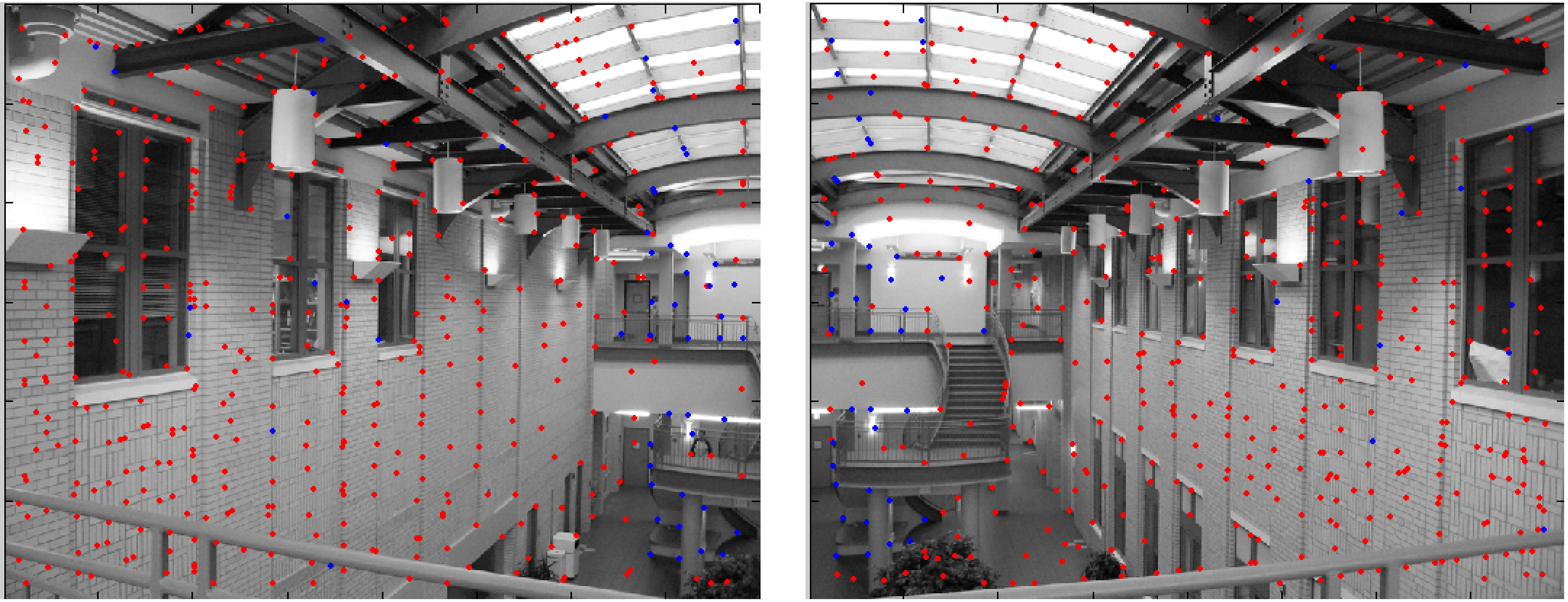
# Feature-space outlier rejection

A better way [Lowe, 1999]:

- 1-NN: SSD of the closest match
- 2-NN: SSD of the second-closest match
- Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
- That is, is our best match so much better than the rest?



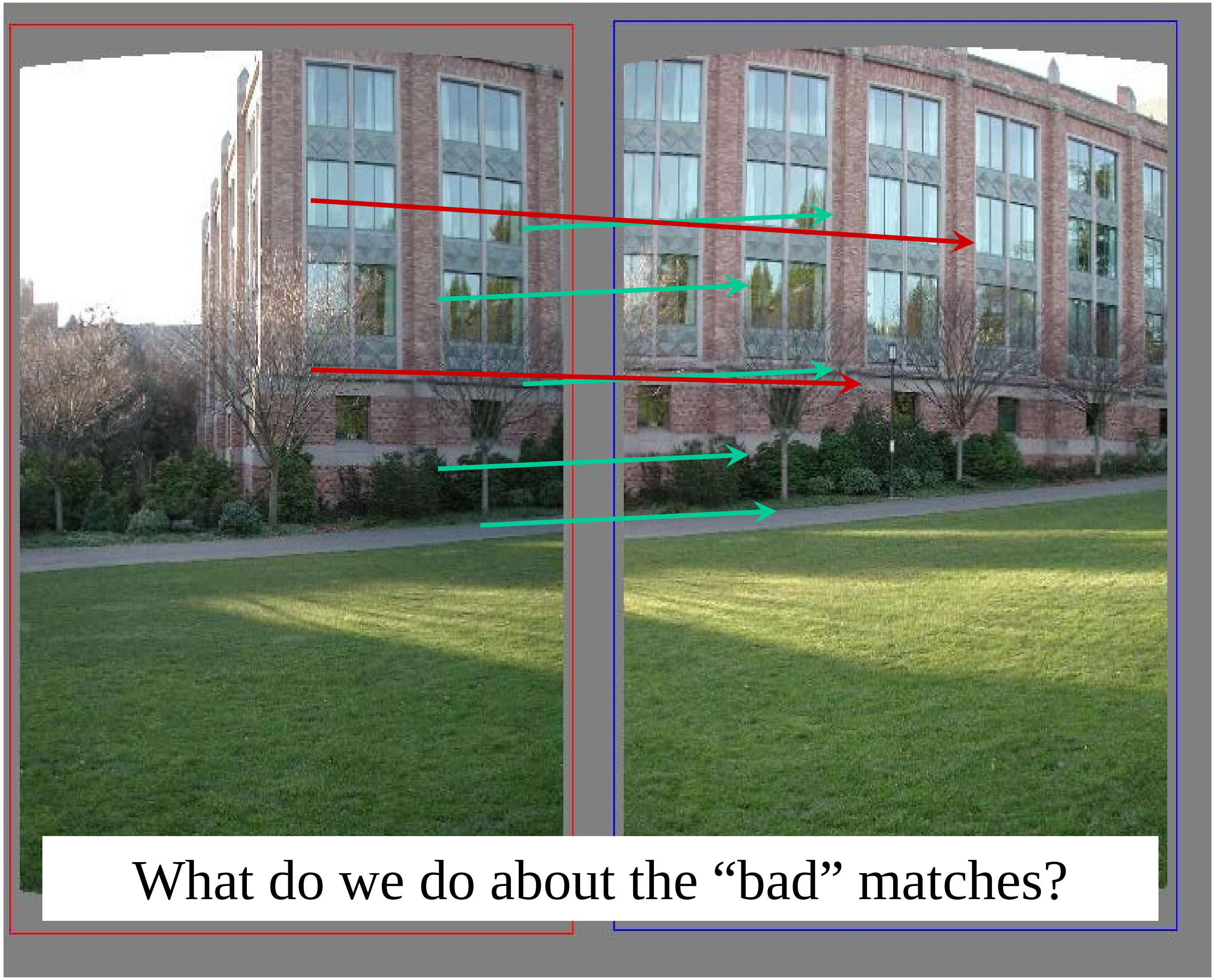
# Feature-space outliner rejection



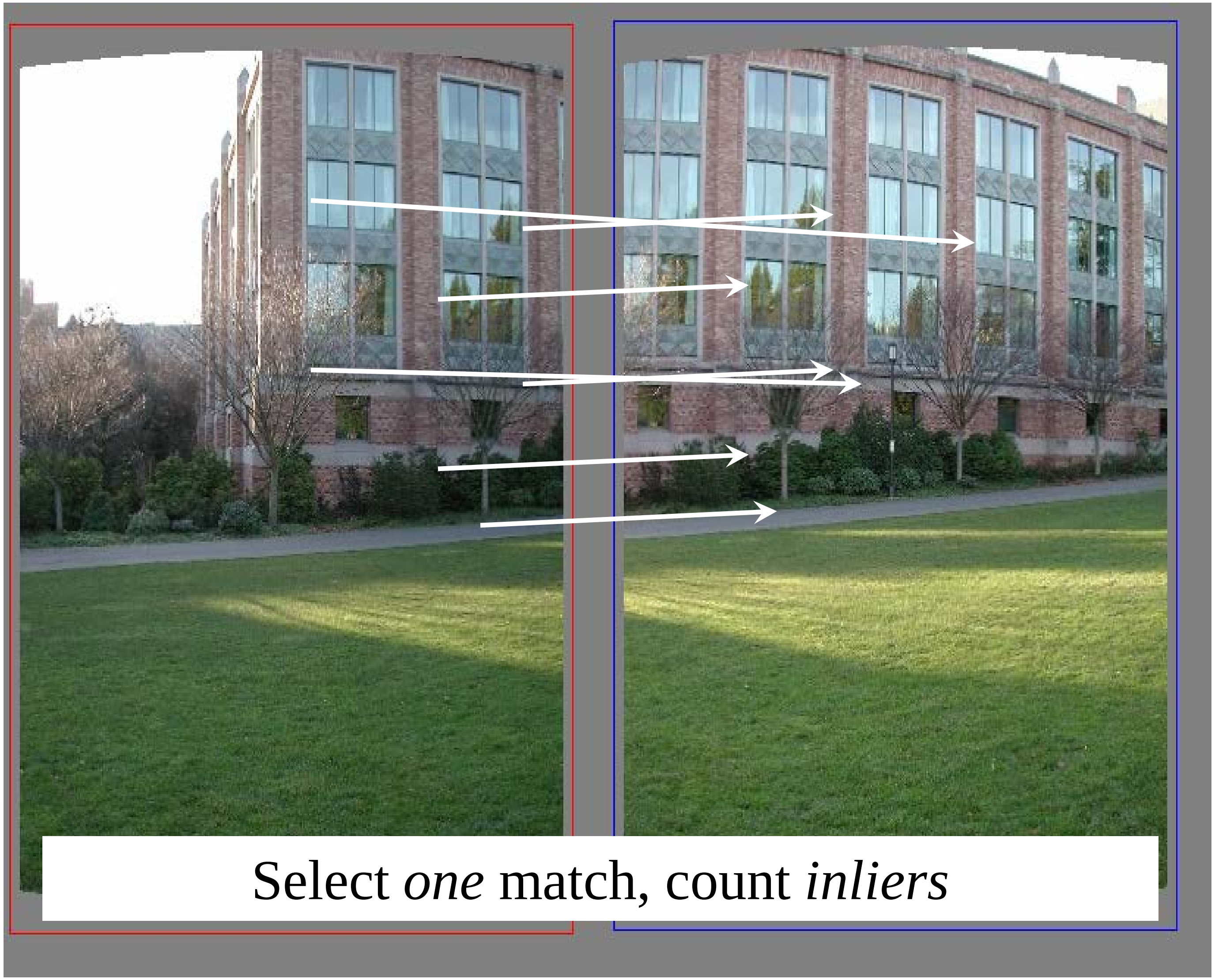
Can we now compute H from the blue points?

- No!  Still too many outliers…
- What can we do?

# Matching features

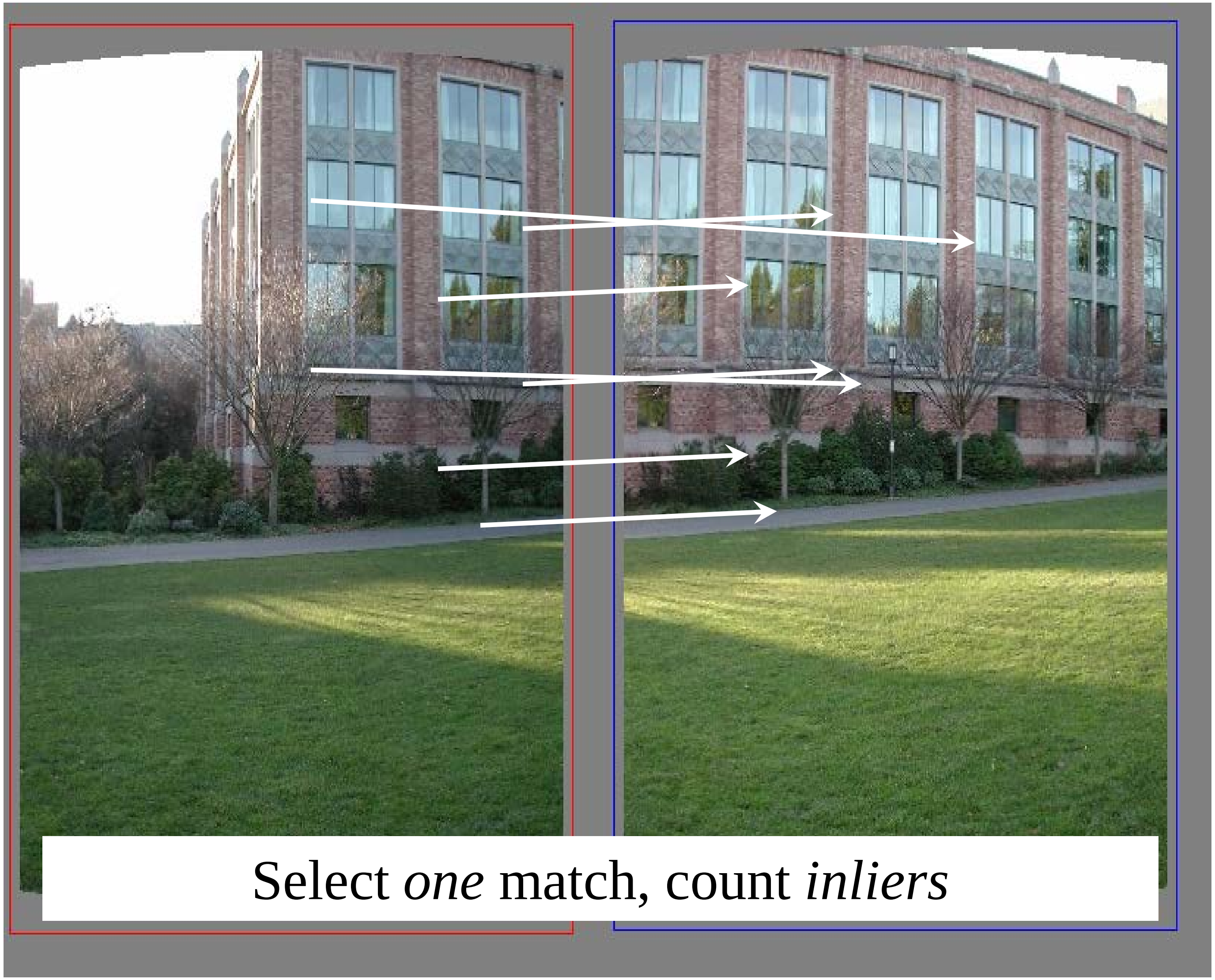


# RAndom SAmple Consensus



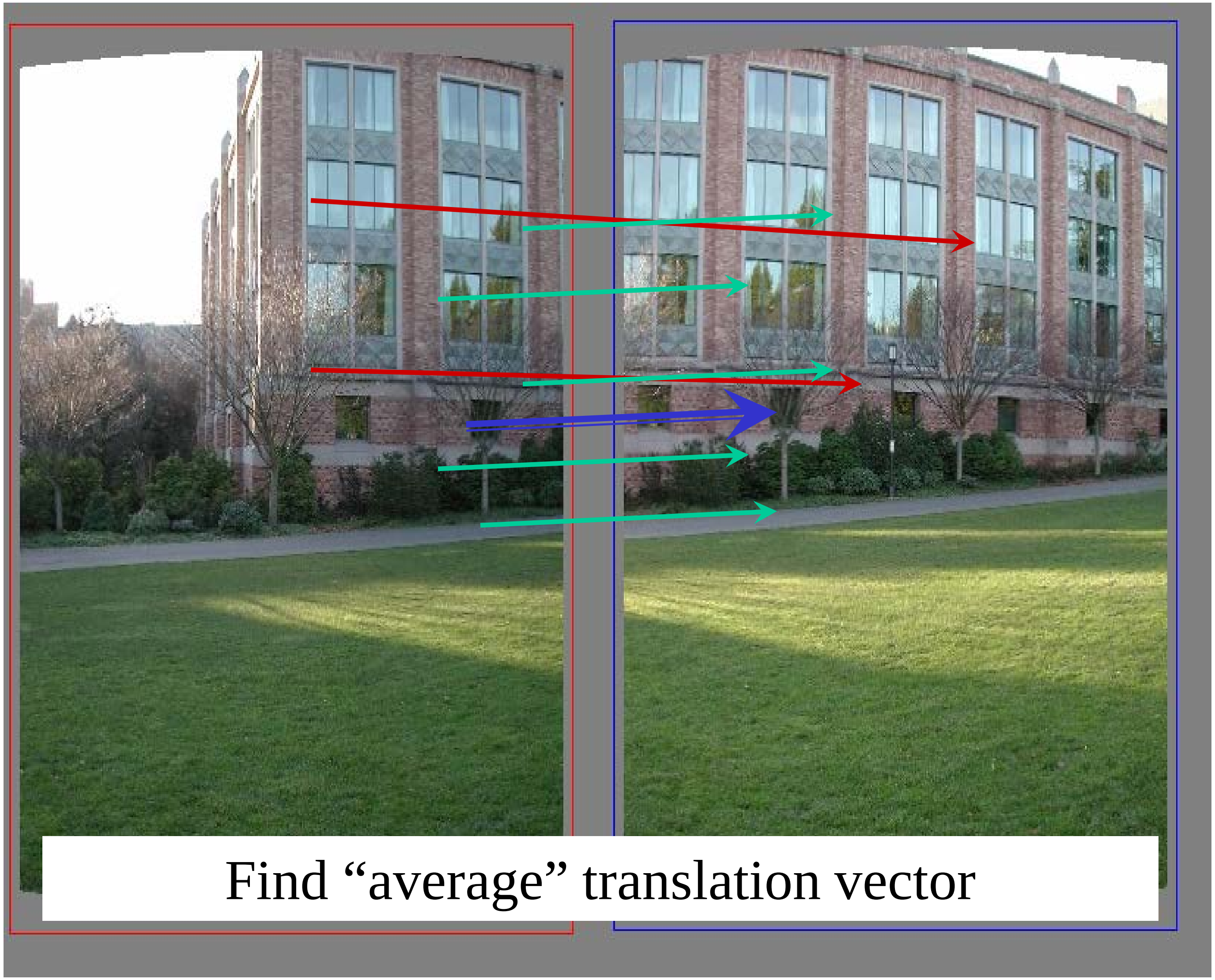
Select *one* match, count *inliers*

# RAndom SAmple Consensus



Select *one* match, count *inliers*

# Least squares fit



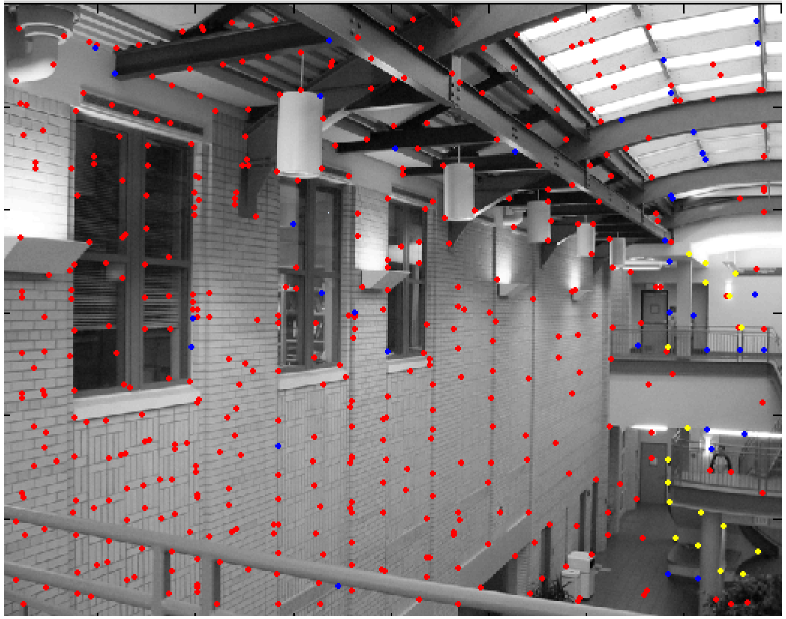
Find "average" translation vector

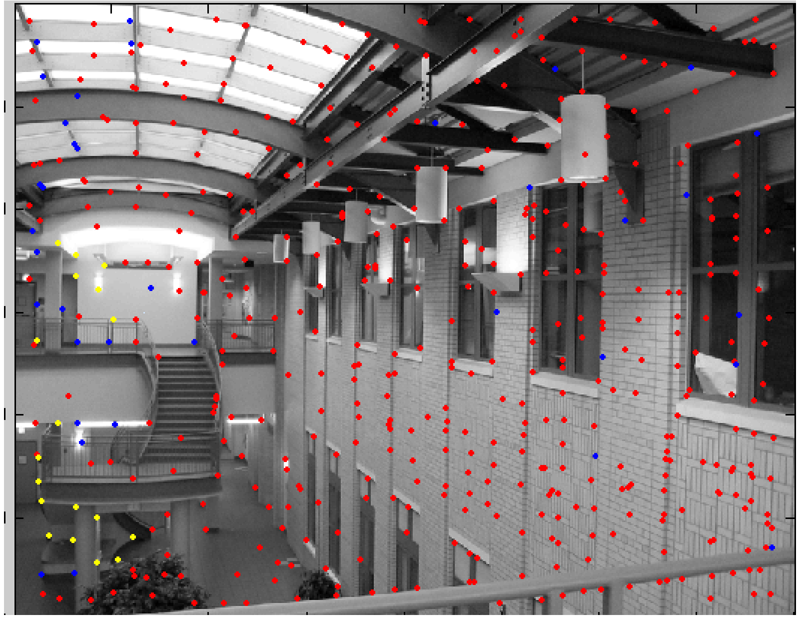
# RANSAC for estimating homography

RANSAC loop:

1. Select four feature pairs (at random)
2. Compute homography H (exact)
3. Compute *inliers* where $SSD(p_i', \mathbf{H}\, p_i) < \varepsilon$
4. Keep largest set of inliers
5. Re-compute least-squares H estimate on all of the inliers

# RANSAC





# Example: Recognising Panoramas

M. Brown and D. Lowe,
University of British Columbia

# Why “Recognising Panoramas”?

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1D Rotations ($\theta$)

- Ordering $\Rightarrow$ matching images

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1D Rotations ($\theta$)

- Ordering $\Rightarrow$ matching images

- 2D Rotations ($\theta$, $\phi$)
  - Ordering $\not\Rightarrow$ matching images

# Why “Recognising Panoramas”?

1D Rotations ($\theta$)

- Ordering $\Rightarrow$ matching images

- 2D Rotations ($\theta$, $\phi$)
  - Ordering $\nRightarrow$ matching images

# Why "Recognising Panoramas"?

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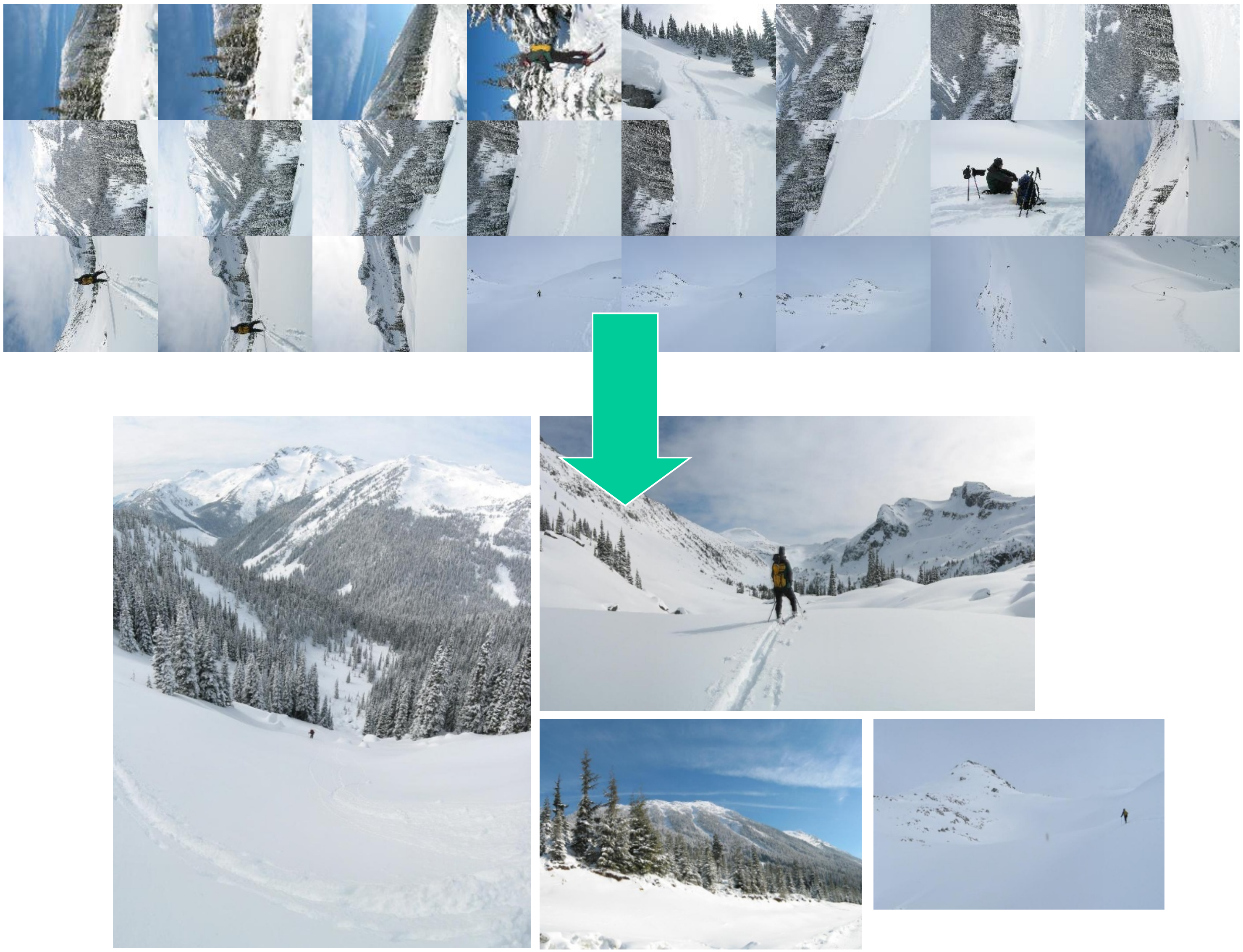
- 2D Rotations ($\theta$, $\phi$)
  - Ordering $\nRightarrow$ matching images

# Why "Recognising Panoramas"?



# Overview

Feature Matching

Image Matching

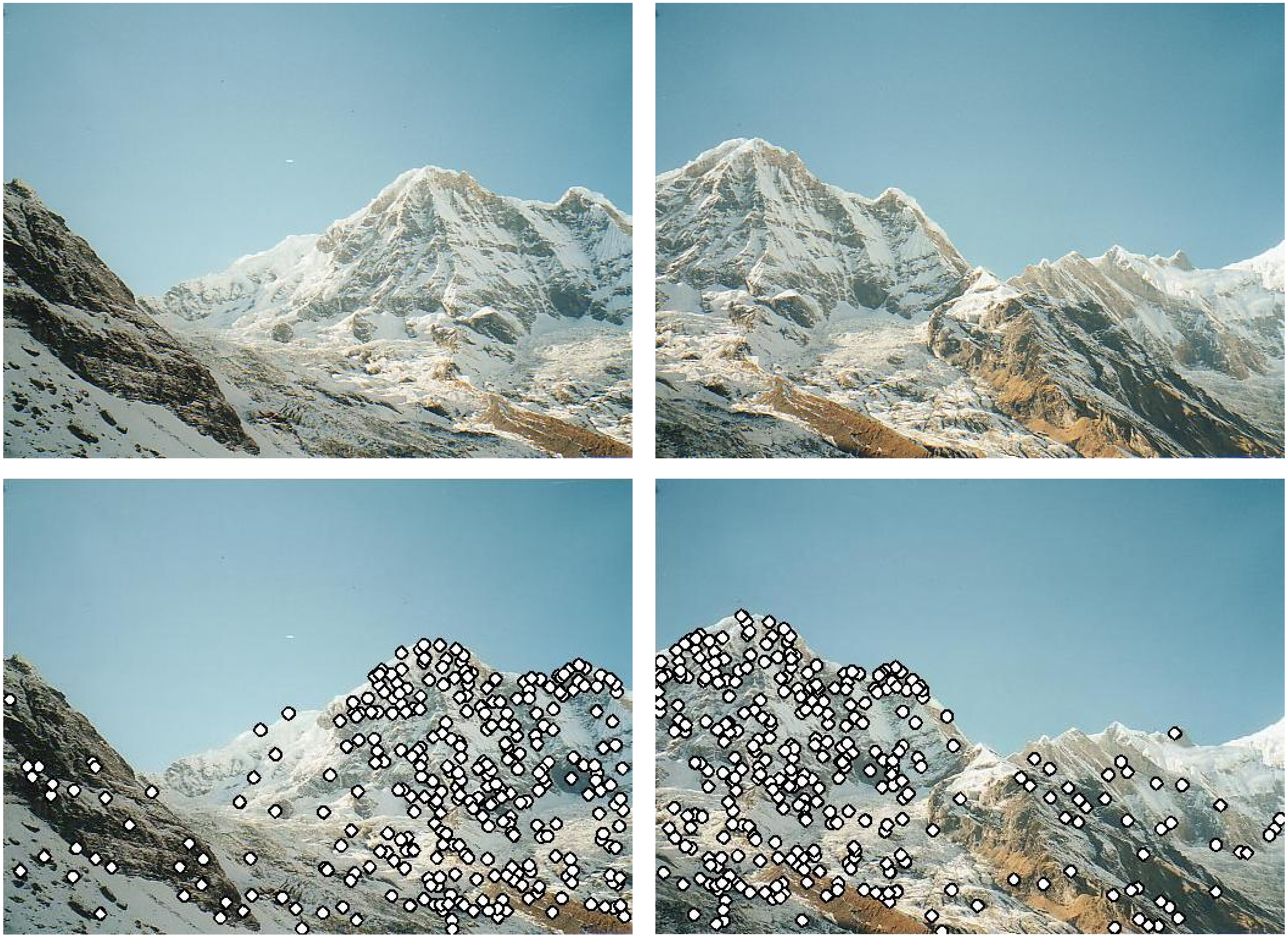
Bundle Adjustment

Multi-band Blending

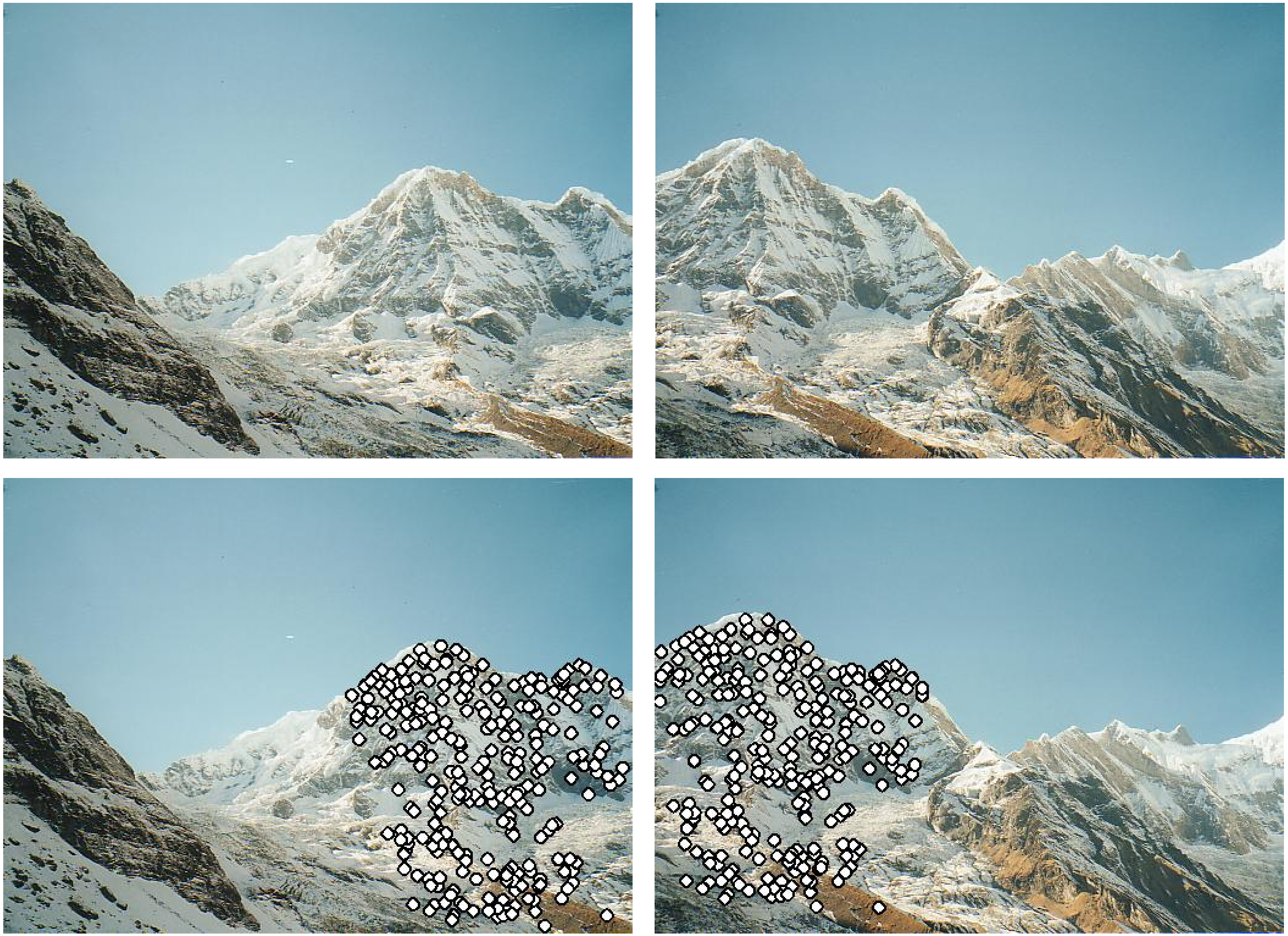
Results

Conclusions

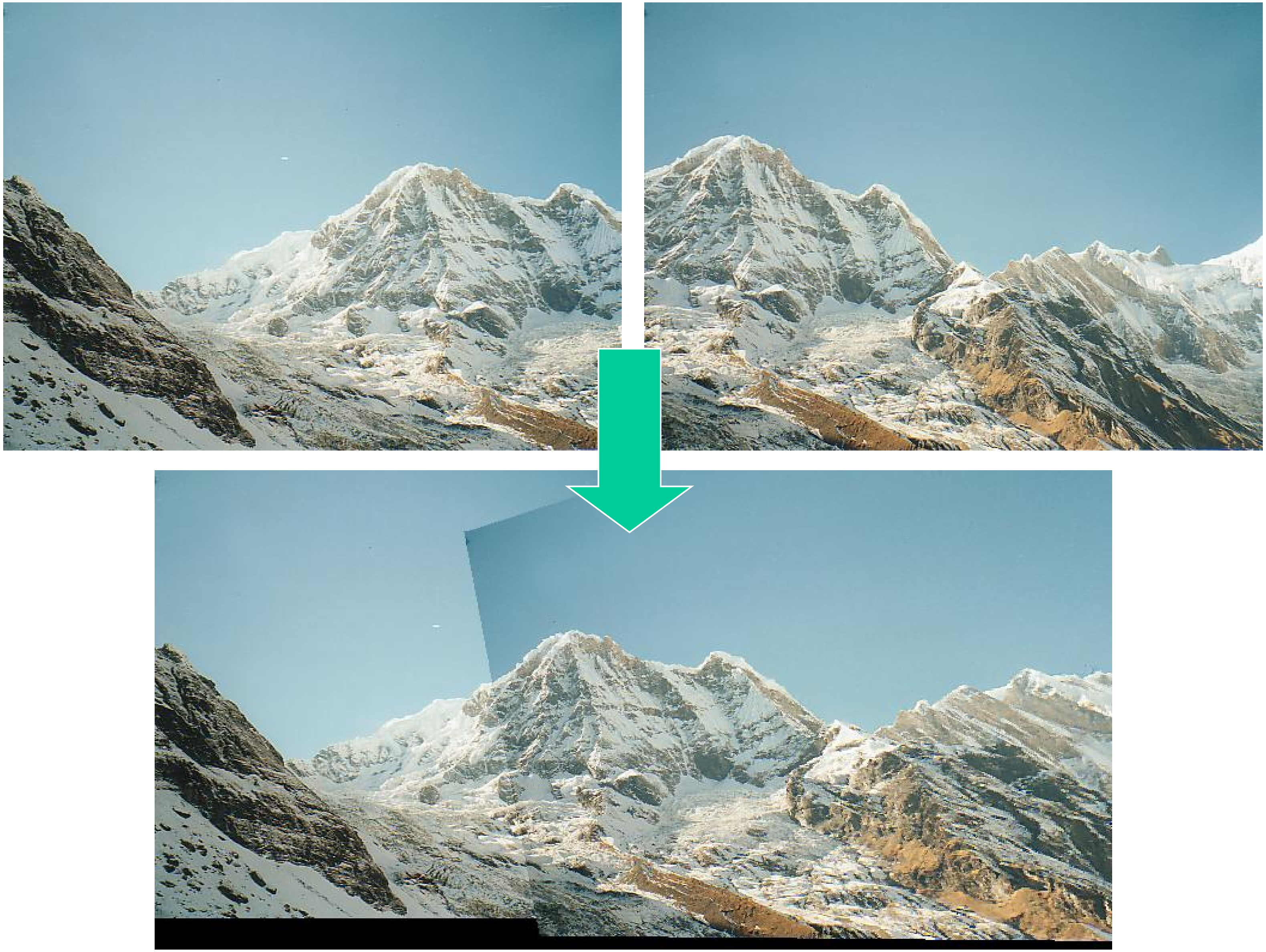
# RANSAC for Homography



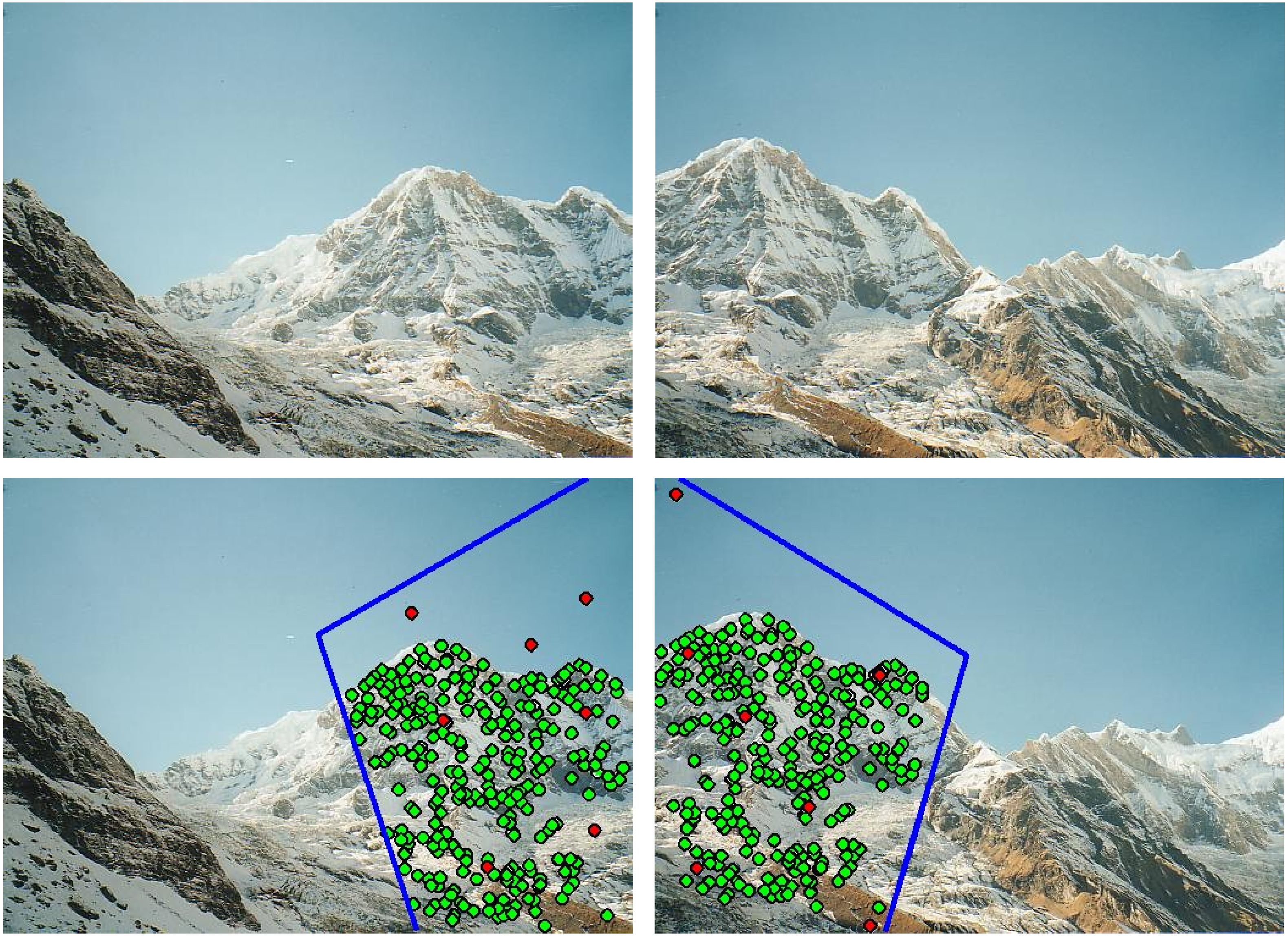
# RANSAC for Homography



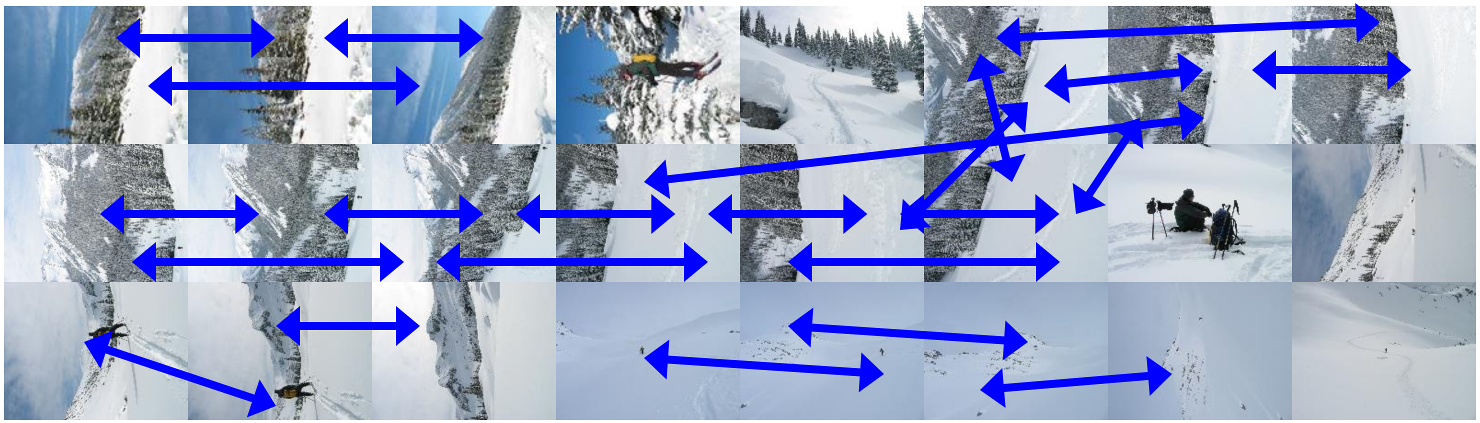
# RANSAC for Homography



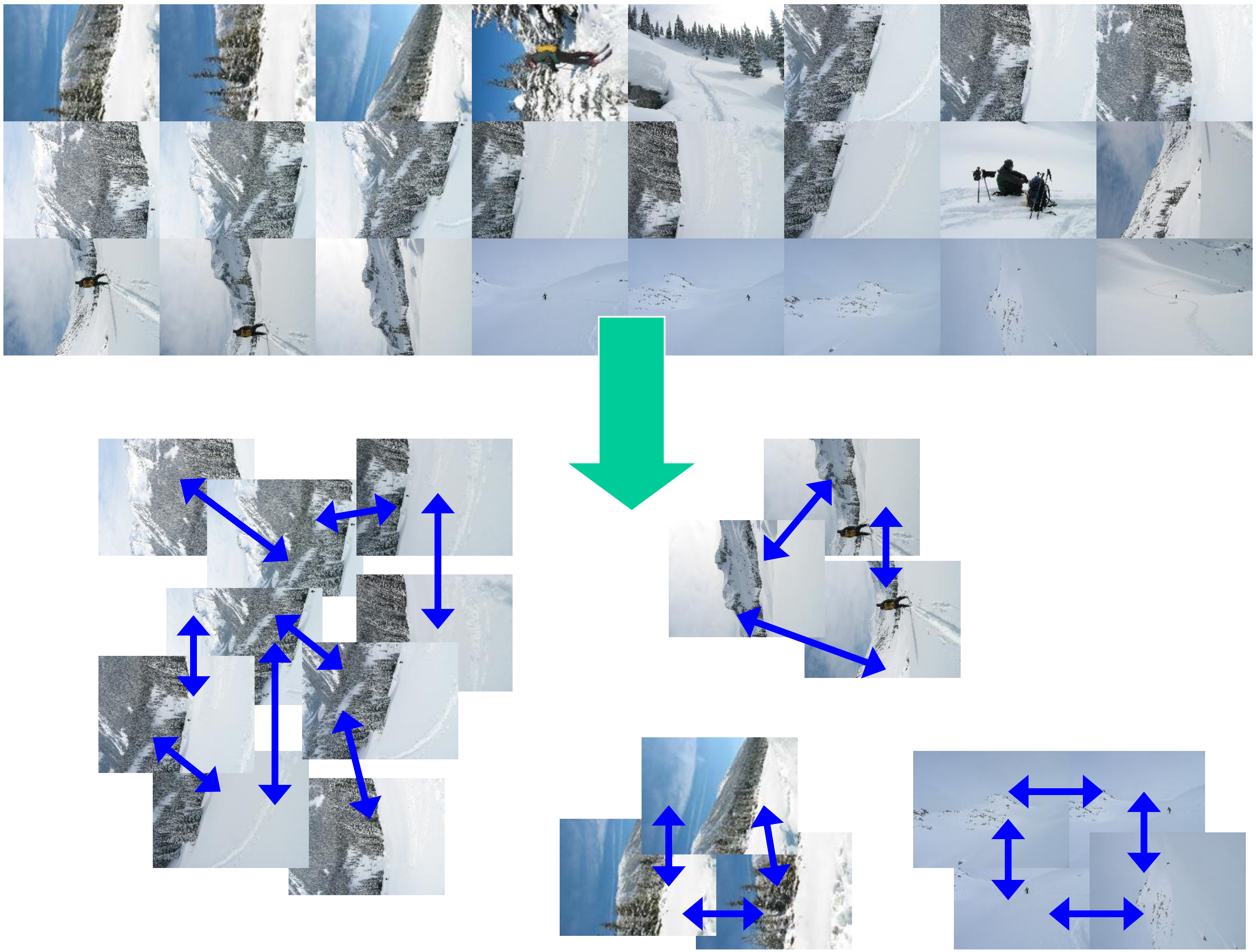
# Probabilistic model for verification



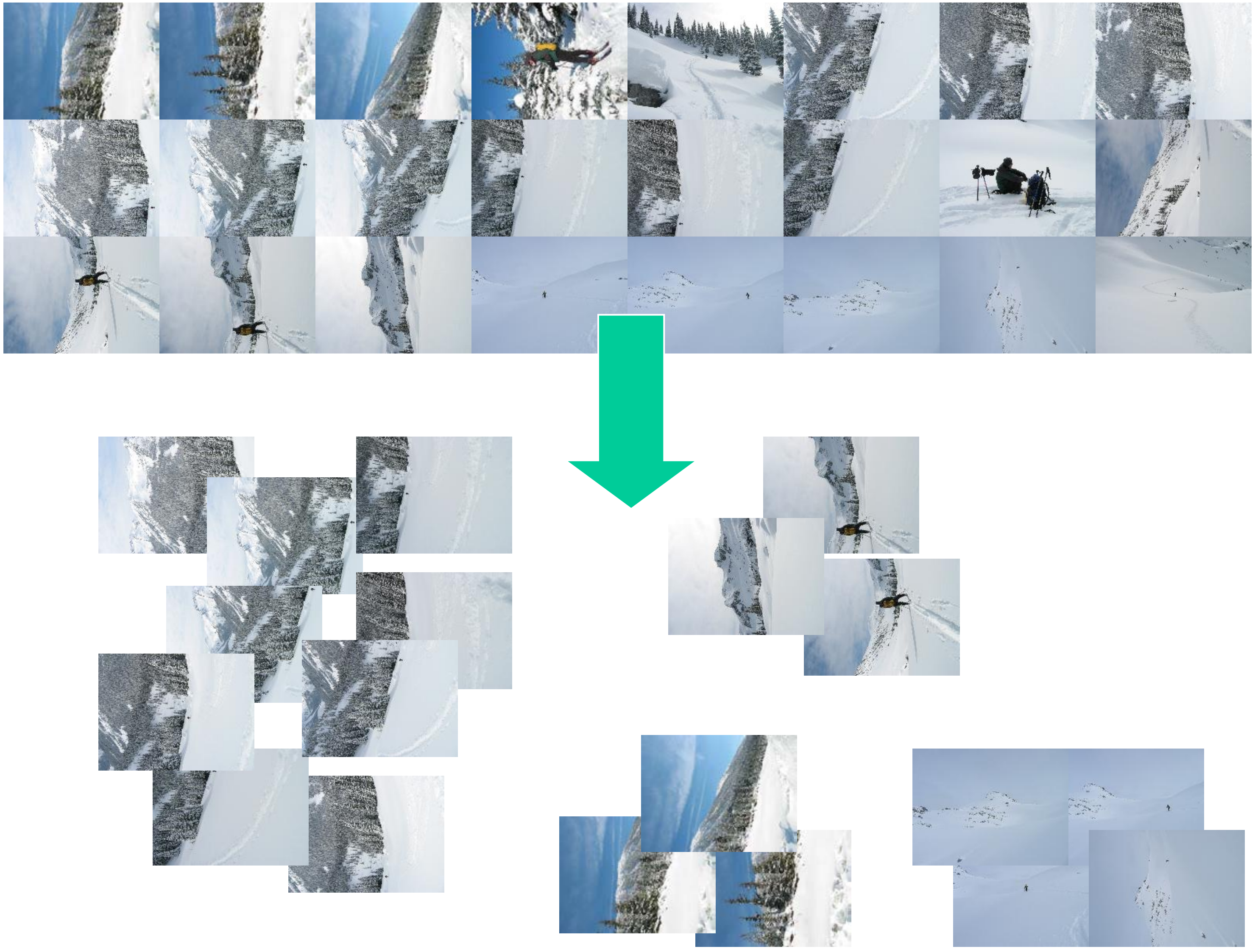
# Finding the panoramas



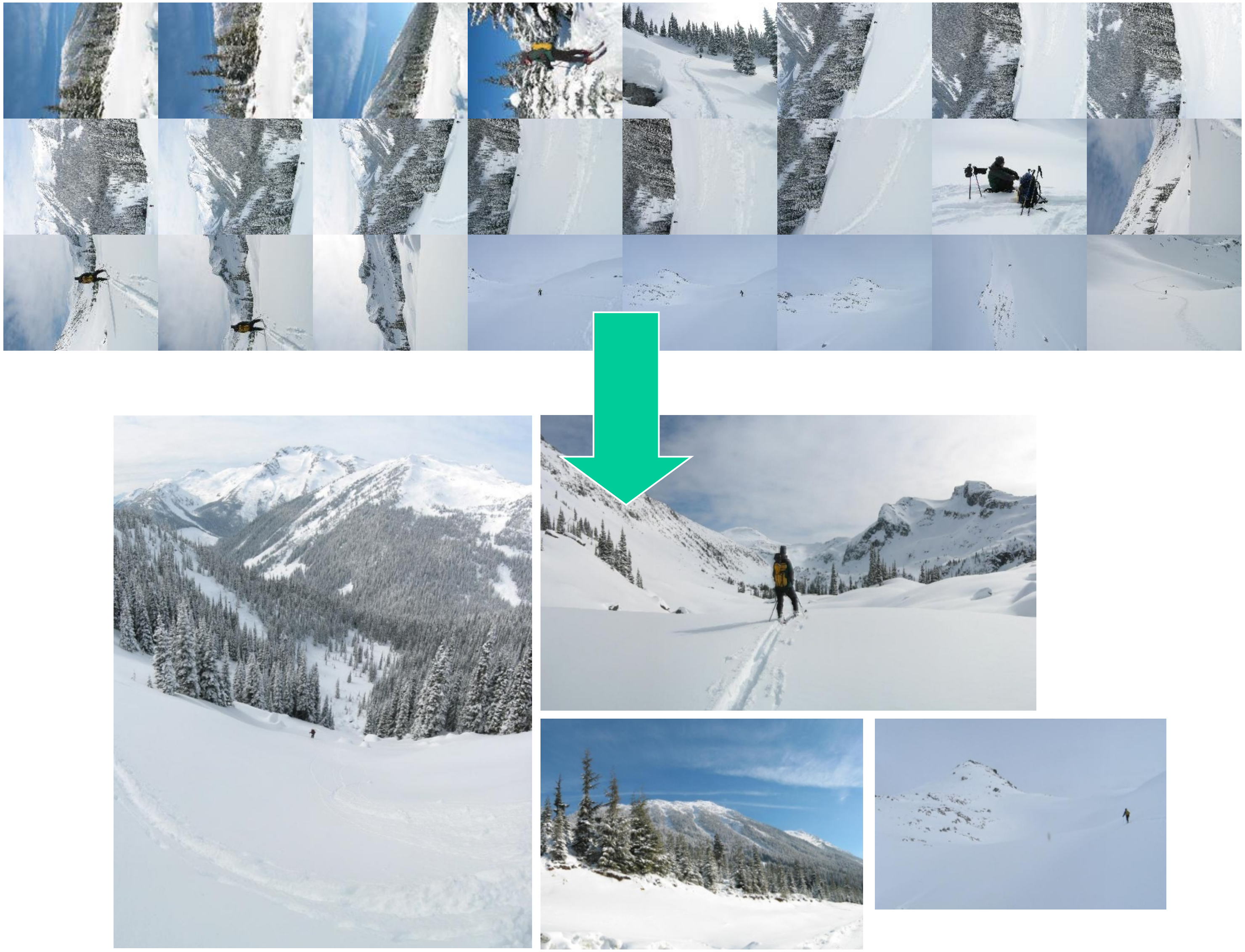
# Finding the panoramas



# Finding the panoramas



# Finding the panoramas



# Homography for Rotation

Parameterise each camera by rotation and focal length

$$\mathbf{R}_i = e^{[\boldsymbol{\theta}_i]_\times}, \;\; [\boldsymbol{\theta}_i]_\times = \begin{bmatrix} 0 & -\theta_{i3} & \theta_{i2} \\ \theta_{i3} & 0 & -\theta_{i1} \\ -\theta_{i2} & \theta_{i1} & 0 \end{bmatrix}$$

$$\mathbf{K}_i = \begin{bmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This gives pairwise homographies

$$\tilde{\mathbf{u}}_i = \mathbf{H}_{ij}\tilde{\mathbf{u}}_j, \;\; \mathbf{H}_{ij} = \mathbf{K}_i\mathbf{R}_i\mathbf{R}_j^T\mathbf{K}_j^{-1}$$

# Bundle Adjustment

New images initialised with rotation, focal length of best matching image

# Bundle Adjustment

New images initialised with rotation, focal length of best matching image

# Multi-band Blending

Burt & Adelson 1983

- Blend frequency bands over range $\propto \lambda$

# Results