

CS 294-13 Advanced Computer Graphics

Subdivision Basics

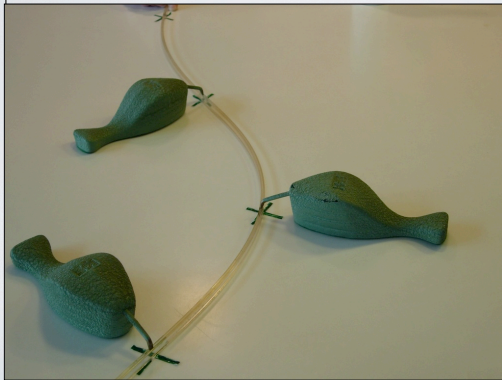
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Natural Splines

- Draw a "smooth" line through several points



A real draftsman's
spline.

Image from Carl de Boor's
webpage.

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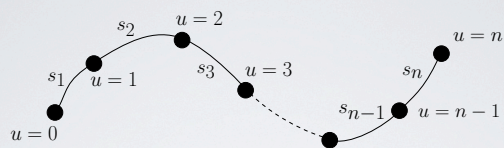
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Natural Cubic Splines

- Given $n + 1$ points
 - Generate a curve with n segments
 - Curves passes through points
 - Curve is C^2 continuous
- Use cubics because lower order is better..

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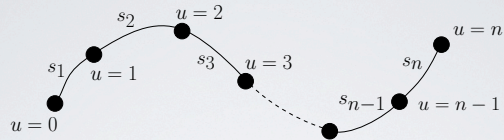
Natural Cubic Splines



$$\mathbf{x}(u) = \begin{cases} \mathbf{s}_1(u) & \text{if } 0 \leq u < 1 \\ \mathbf{s}_2(u-1) & \text{if } 1 \leq u < 2 \\ \mathbf{s}_3(u-2) & \text{if } 2 \leq u < 3 \\ \vdots & \\ \mathbf{s}_n(u-(n-1)) & \text{if } n-1 \leq u \leq n \end{cases}$$

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Natural Cubic Splines



$$s_i(0) = p_{i-1} \quad i = 1 \dots n$$

$$s_i(1) = p_i \quad i = 1 \dots n$$

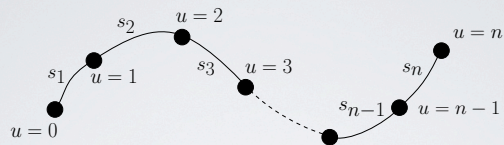
$$s'_i(1) = s'_{i+1}(0) \quad i = 1 \dots n-1$$

$$s''_i(1) = s''_{i+1}(0) \quad i = 1 \dots n-1$$

$$s''_1(0) = s''_n(1) = 0$$

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Natural Cubic Splines



$$s_i(0) = p_{i-1} \quad i = 1 \dots n \quad \leftarrow n \text{ constraints}$$

$$s_i(1) = p_i \quad i = 1 \dots n \quad \leftarrow n \text{ constraints}$$

$$s'_i(1) = s'_{i+1}(0) \quad i = 1 \dots n-1 \quad \leftarrow n-1 \text{ constraints}$$

$$s''_i(1) = s''_{i+1}(0) \quad i = 1 \dots n-1 \quad \leftarrow n-1 \text{ constraints}$$

$$s''_1(0) = s''_n(1) = 0 \quad \leftarrow 2 \text{ constraints}$$

Total 4n constraints

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Natural Cubic Splines

- Interpolate data points
- No convex hull property
- Non-local support
 - Consider matrix structure...
- C^2 using cubic polynomials

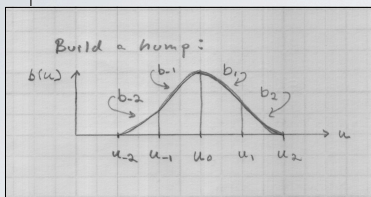
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B-Splines

- Goal: C^2 cubic curves with local support
 - Give up interpolation
 - Get convex hull property
- Build basis by designing “hump” functions

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B-Splines



$$\mathbf{b}(u) = \begin{cases} \mathbf{b}_{-2}(u) & \text{if } u_{-2} \leq u < u_{-1} \\ \mathbf{b}_{-1}(u) & \text{if } u_{-1} \leq u < u_0 \\ \mathbf{b}_{+1}(u) & \text{if } u_0 \leq u < u_{+1} \\ \mathbf{b}_{+2}(u) & \text{if } u_{+1} \leq u \leq u_{+2} \end{cases}$$

$$b''_{-2}(u_{-2}) = b'_{-2}(u_{-2}) = b_{-2}(u_{-2}) = 0$$

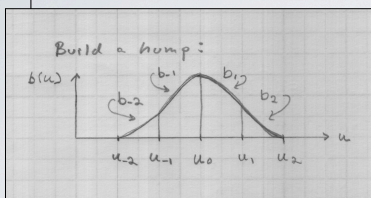
$$b'_{+2}(u_{+2}) = b'_{+2}(u_{+2}) = b_{+2}(u_{+2}) = 0$$

$$b_{-2}(u_{-1}) = b_{-1}(u_{-1})$$

$$b_{-1}(u_0) = b_{+1}(u_0)$$

$$b_{+1}(u_{+1}) = b_{+2}(u_{+1})$$

B-Splines



$$\mathbf{b}(u) = \begin{cases} \mathbf{b}_{-2}(u) & \text{if } u_{-2} \leq u < u_{-1} \\ \mathbf{b}_{-1}(u) & \text{if } u_{-1} \leq u < u_0 \\ \mathbf{b}_{+1}(u) & \text{if } u_0 \leq u < u_{+1} \\ \mathbf{b}_{+2}(u) & \text{if } u_{+1} \leq u \leq u_{+2} \end{cases}$$

$$b''_{-2}(u_{-2}) = b'_{-2}(u_{-2}) = b_{-2}(u_{-2}) = 0 \quad \leftarrow 3 \text{ constraints}$$

$$b'_{+2}(u_{+2}) = b'_{+2}(u_{+2}) = b_{+2}(u_{+2}) = 0 \quad \leftarrow 3 \text{ constraints}$$

$$b_{-2}(u_{-1}) = b_{-1}(u_{-1})$$

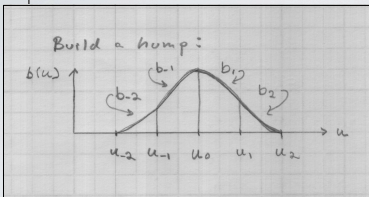
$$b_{-1}(u_0) = b_{+1}(u_0)$$

$$b_{+1}(u_{+1}) = b_{+2}(u_{+1})$$

← [Repeat for b' and b''
3×3=9 constraints

Total 15 constraints need one more

B-Splines



$$\mathbf{b}(u) = \begin{cases} \mathbf{b}_{-2}(u) & \text{if } u_{-2} \leq u < u_{-1} \\ \mathbf{b}_{-1}(u) & \text{if } u_{-1} \leq u < u_0 \\ \mathbf{b}_{+1}(u) & \text{if } u_0 \leq u < u_{+1} \\ \mathbf{b}_{+2}(u) & \text{if } u_{+1} \leq u \leq u_{+2} \end{cases}$$

$$b''_{-2}(u_{-2}) = b'_{-2}(u_{-2}) = b_{-2}(u_{-2}) = 0 \quad \leftarrow 3 \text{ constraints}$$

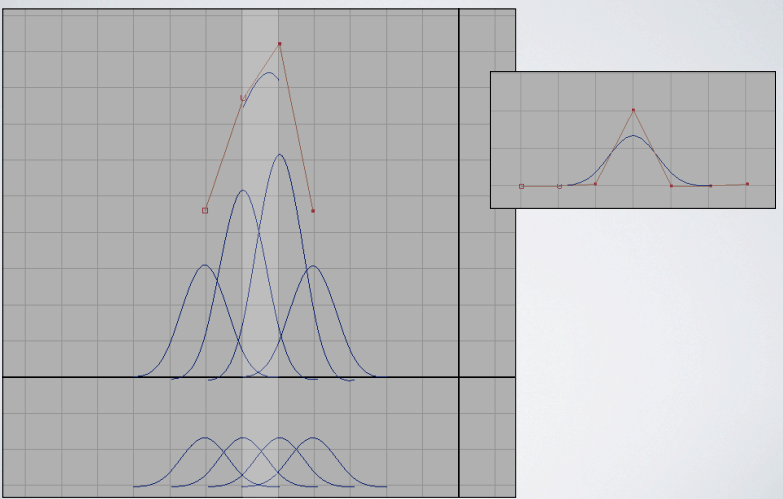
$$b'_{+2}(u_{+2}) = b'_{+2}(u_{+2}) = b_{+2}(u_{+2}) = 0 \quad \leftarrow 3 \text{ constraints}$$

$$\begin{matrix} b_{-2}(u_{-1}) = b_{-1}(u_{-1}) \\ b_{-1}(u_0) = b_{+1}(u_0) \\ b_{+1}(u_{+1}) = b_{+2}(u_{+1}) \end{matrix} \quad \leftarrow \begin{matrix} \text{Repeat for } b' \text{ and } b'' \\ 3 \times 3 = 9 \text{ constraints} \end{matrix}$$

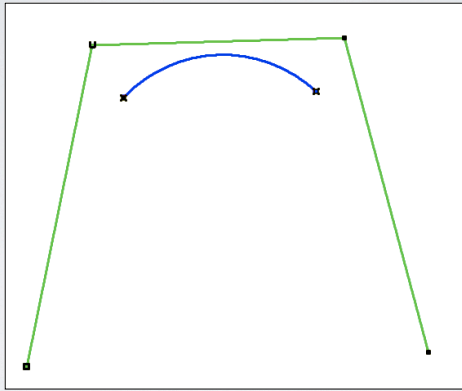
$$b_{-2}(u_{-2}) + b_{-1}(u_{-1}) + b_{+1}(u_0) + b_{+2}(u_{+1}) = 1 \quad \leftarrow 1 \text{ constraint (convex hull)}$$

Total 16 constraints

B-Splines

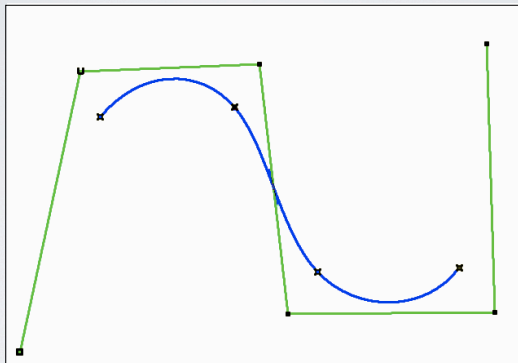


B-Splines



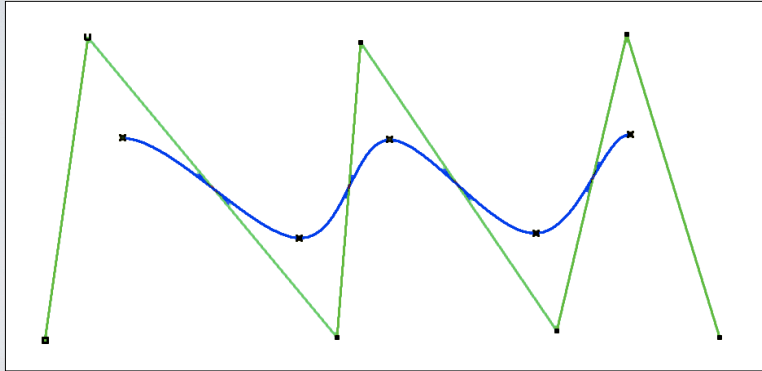
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B-Splines



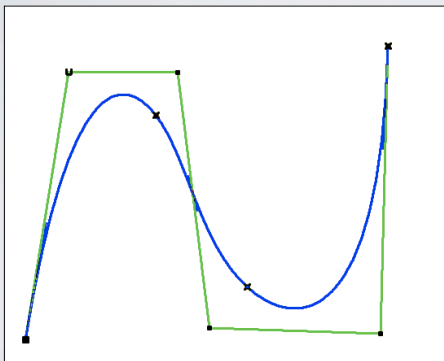
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B-Splines



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B-Splines



Example with end knots repeated

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B-Splines

- Build a curve w/ overlapping bumps
- Continuity
 - Inside bumps C^2
 - Bumps “fade out” with C^2 continuity
- Boundaries
 - Circular
 - Repeat end points
 - Extra end points

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B-Splines

- Notation
 - The basis functions are the $b_i(u)$
 - “Hump” functions are the concatenated function
 - Sometimes the humps are called basis... can be confusing
 - The u_i are the knot locations
 - The weights on the hump/basis functions are control points

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B-Splines

- Similar construction method can give higher continuity with higher degree polynomials
- Repeating knots drops continuity
 - Limit as knots approach each other
- Still cubics, so conversion to other cubic basis is just a matrix multiplication

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B-Splines

- Geometric construction
 - Due to Cox and de Boor
 - My own notation, beware if you compare w/ text

- Let hump centered on u_i be $N_{i,4}(u)$

Cubic is order 4

$N_{i,k}(u)$ is order k hump, centered at u_i

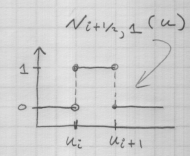
Note: i is integer if k is even
else $(i + 1/2)$ is integer

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B-Splines

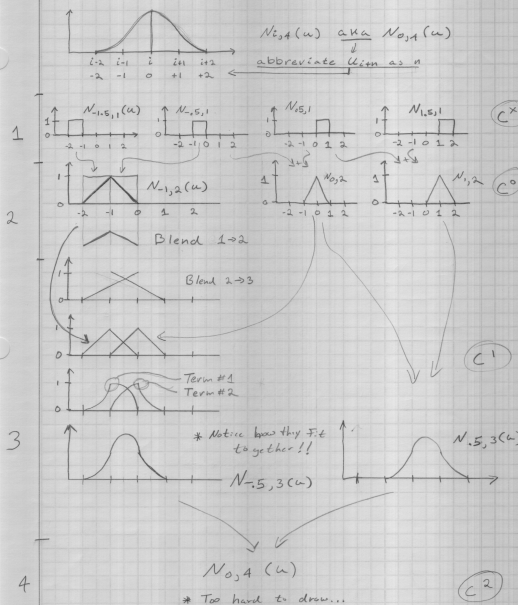
$$N_{i,1}(u) = \begin{cases} 1 & \text{if } u_{i-1/2} \leq u < u_{i+1/2} \\ 0 & \text{else} \end{cases}$$



$$N_{i,2}(u) = \frac{(u - u_{i-1/2}) N_{i-1/2,1}(u)}{u_{i+1/2-1} - u_{i-1/2}} + \frac{(u_{i+1/2} - u) N_{i+1/2,1}(u)}{u_{i+1/2} - u_{i+1/2+1}}$$

(Recursive defn.)

$N_{0,4}(u)$ aka $N_{0,4}(u)$
 abbreviate $u_{i+1/2}$ as n



1 $N_{-2,1}(u)$ $N_{-1,1}(u)$ $N_{0,1}(u)$ $N_{1,1}(u)$ $N_{2,1}(u)$ (C^x)
 2 $N_{-2,2}(u)$ $N_{-1,2}(u)$ $N_{0,2}(u)$ $N_{1,2}(u)$ $N_{2,2}(u)$ (C⁰)
 Blend 1 to 2
 Blend 2 to 3
 Term #1
 Term #2
 3 $N_{-5,3}(u)$ $N_{-5,4}(u)$
 * Metric how they fit together!!
 4 $N_{0,4}(u)$
 * Too hard to draw... (C²)

NURBS

- **Nonuniform Rational B-Splines**

- Basically B-Splines using homogeneous coordinates
- Transform under perspective projection
- A bit of extra control

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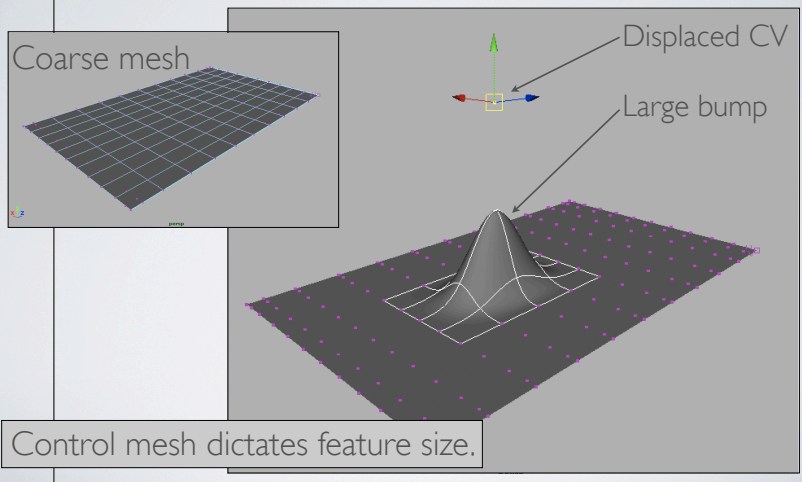
NURBS

$$\mathbf{p}_i = \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \\ p_{iw} \end{bmatrix} \quad \mathbf{x}(u) = \frac{\sum_i \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{bmatrix} N_i(u)}{\sum_i p_{iw} N_i(u)}$$

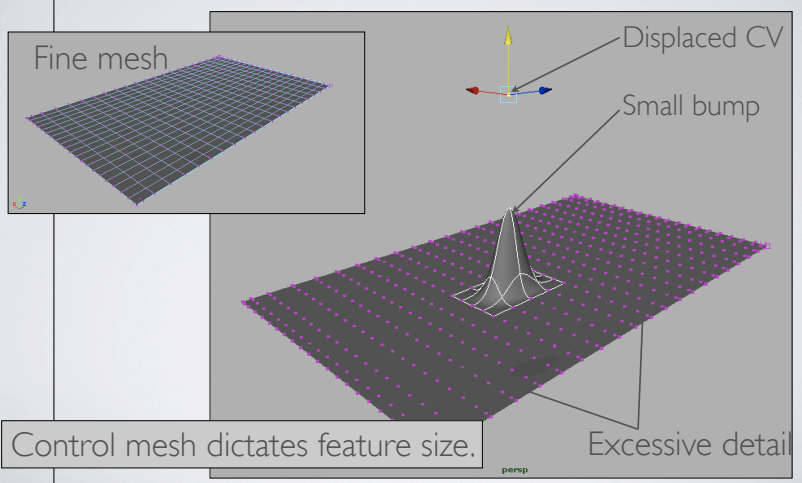
- Non-linear in the control points
- The p_{iw} are sometimes called “weights”

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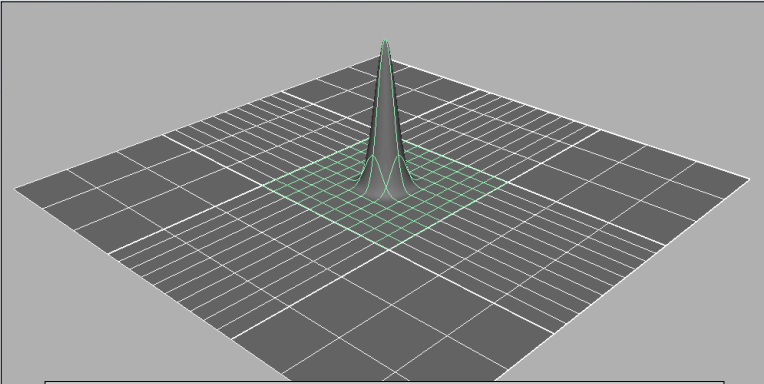
Consider NURBS Surface



Consider NURBS Surface



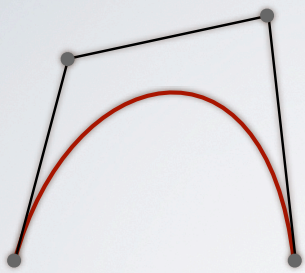
Tensor Product Surface Refinement



Refinement must be constant across u or v directions

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Bézier Subdivision



$u \in [0..1]$

$$\mathbf{x}(u) = \sum_i b_i(u) \mathbf{p}_i$$

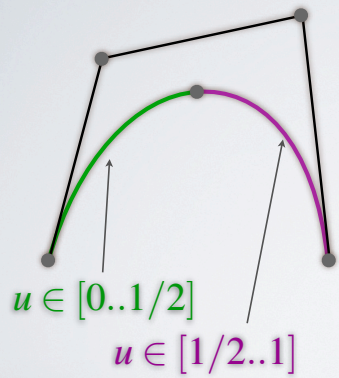
$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

Vector of control points

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

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Bézier Subdivision

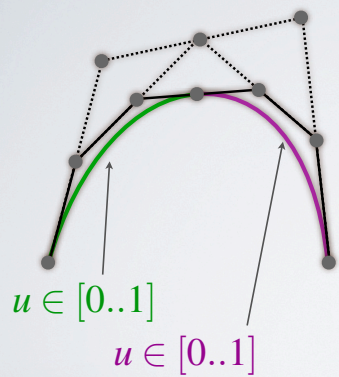


$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

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Bézier Subdivision



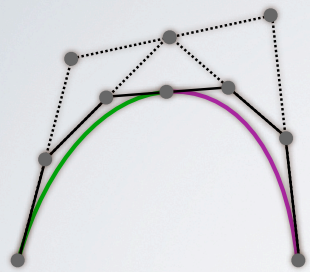
$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

Can't change these...

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

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Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \quad u \in [0, \frac{1}{2}]$$

$$\mathbf{x}(u) = [1, \frac{u}{2}, \frac{u^2}{4}, \frac{u^3}{8}] \beta_Z \mathbf{P} \quad u \in [0, 1]$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \mathbf{S}_1 \beta_Z \mathbf{P}$$

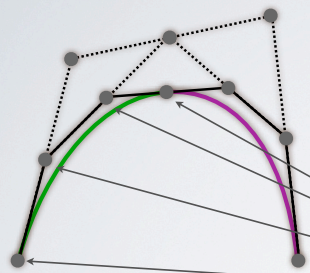
$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/8 \end{bmatrix}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \beta_Z^{-1} \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

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Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

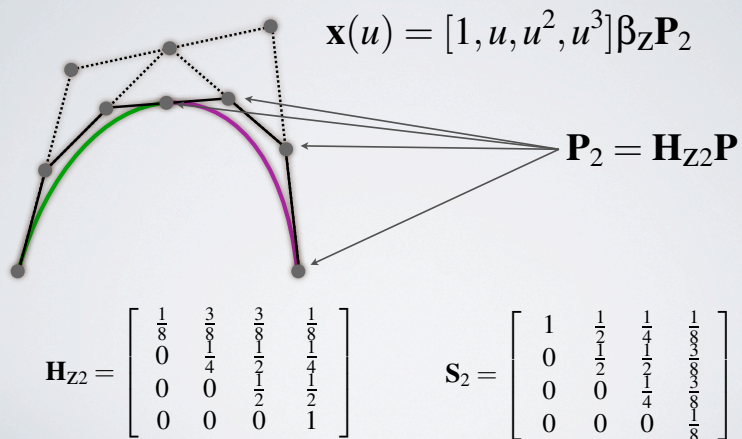
$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}_1$$

$$\mathbf{P}_1 = \mathbf{H}_{Z1} \mathbf{P}$$

$$\mathbf{H}_{Z1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 \\ 1/8 & 3/8 & 3/8 & 1/8 \end{bmatrix}$$

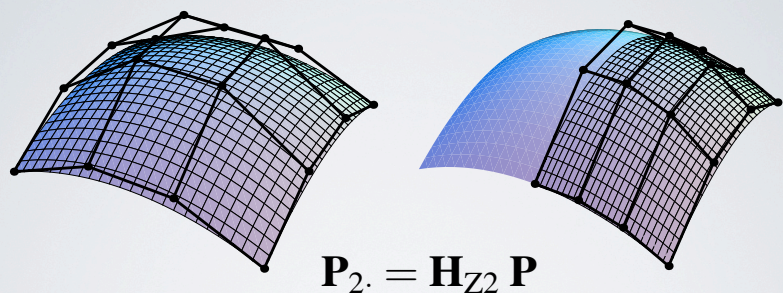
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Bézier Subdivision



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Bézier Subdivision

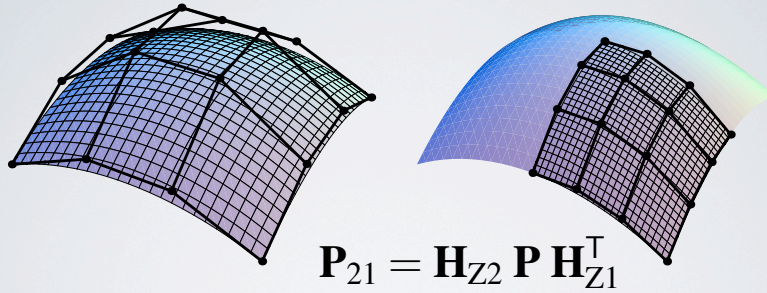


$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \beta_Z^T [1, v, v^2, v^3]^T$

4 x 4 matrix of control points \mathbf{P}

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Bézier Subdivision

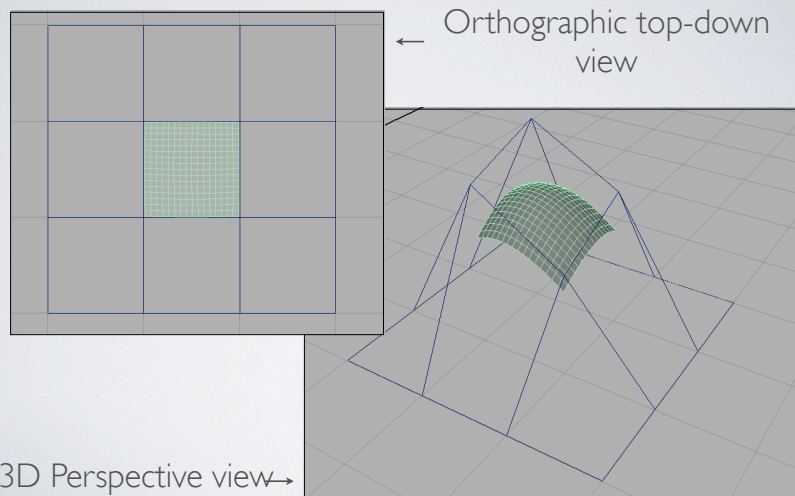


$$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \beta_Z^T [1, v, v^2, v^3]^T$$

4 × 4 matrix of control points

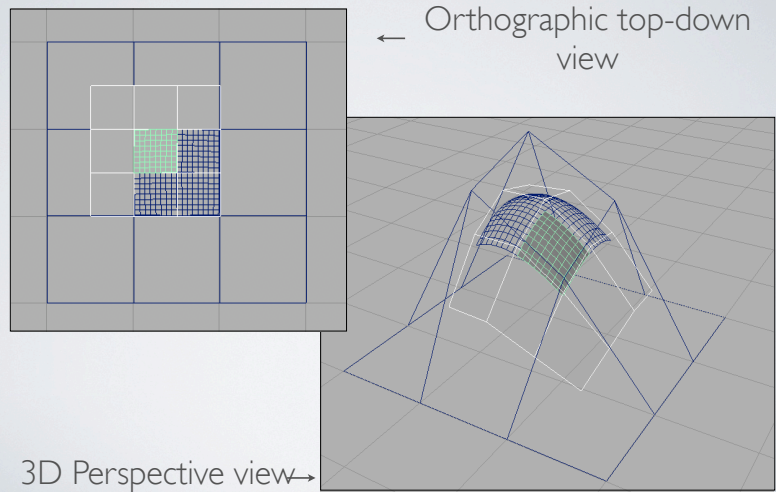
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Regular B-Spline Subdivision

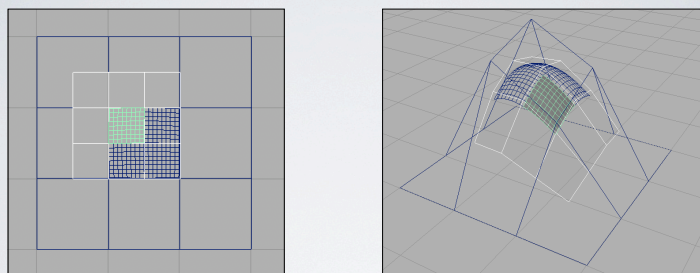


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Regular B-Spline Subdivision



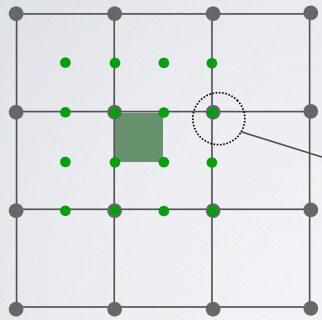
Regular B-Spline Subdivision



$$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_B \mathbf{P} \beta_B^T [1, v, v^2, v^3]^T$$

$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

Regular B-Spline Subdivision



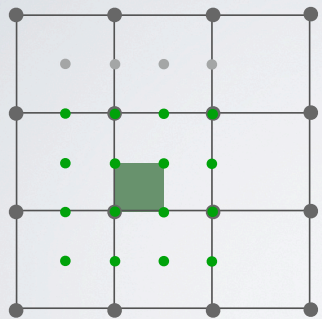
$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

In this parametric view these knot points are collocated.

The 3D control points are not.

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Regular B-Spline Subdivision

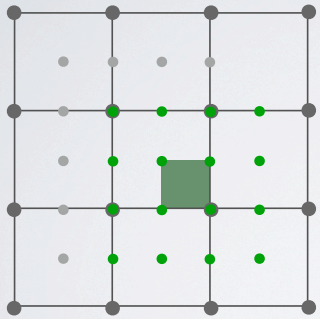


$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

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Regular B-Spline Subdivision



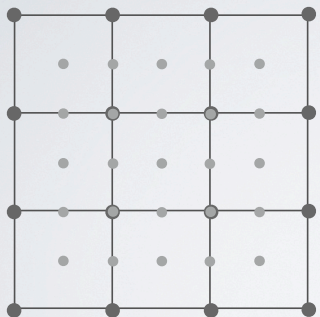
$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

$$\mathbf{P}_{22} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B2}^T$$

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Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

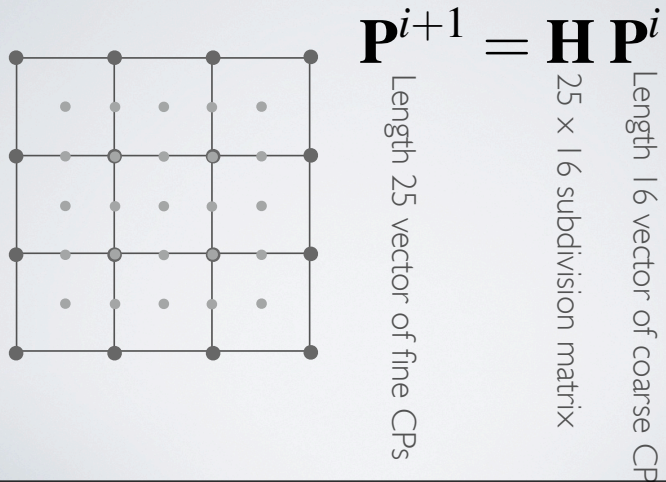
$$\mathbf{P}_{22} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B2}^T$$

$$\mathbf{P}_{21} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B1}^T$$

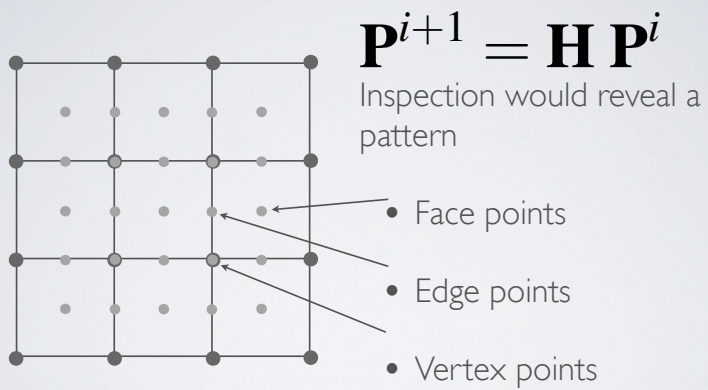
$$\mathbf{H}_{B1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \end{bmatrix} \quad \mathbf{H}_{B2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

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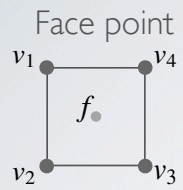
Regular B-Spline Subdivision



Regular B-Spline Subdivision

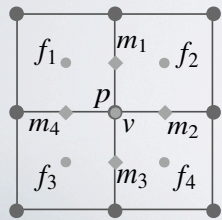
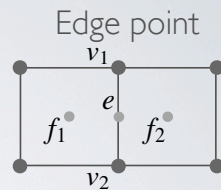


Regular B-Spline Subdivision



$$f = \frac{v_1 + v_2 + v_3 + v_4}{4}$$

$$e = \frac{v_1 + v_2 + f_1 + f_2}{4}$$



Vertex point

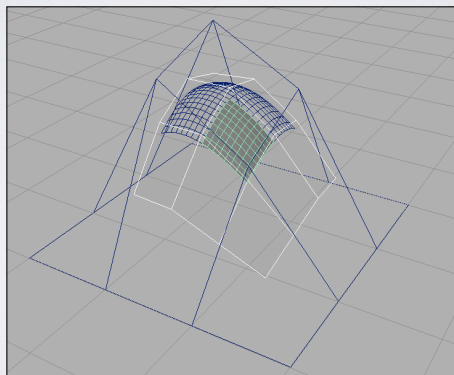
$$v = \frac{f_1 + f_2 + f_3 + f_4 + 2(m_1 + m_2 + m_3 + m_4) + 4p}{16}$$

m midpoint of edge, not "edge point"

p old "vertex point"

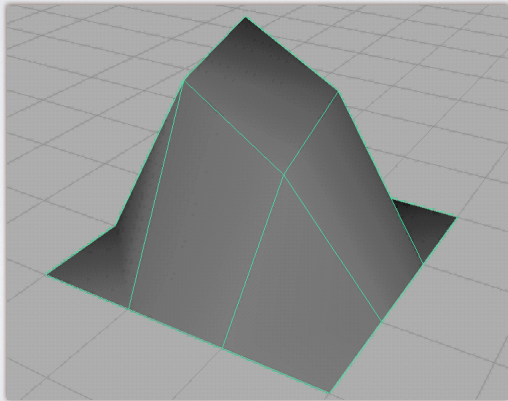
Regular B-Spline Subdivision

- Recall that control mesh approaches surface



Regular B-Spline Subdivision

- Limit of subdivision is the surface



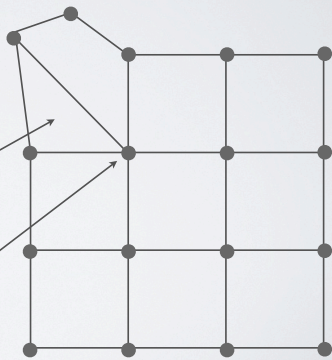
Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
 - Generalizes regular B-Spline subdivision

An irregular patch

Non-quad face

Extraordinary vertex



Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
 - Generalizes regular B-Spline subdivision
 - Rules reduce to regular for ordinary vertices/faces

f = average of surrounding vertices

$$e = \frac{f_1 + f_2 + v_1 + v_2}{4}$$

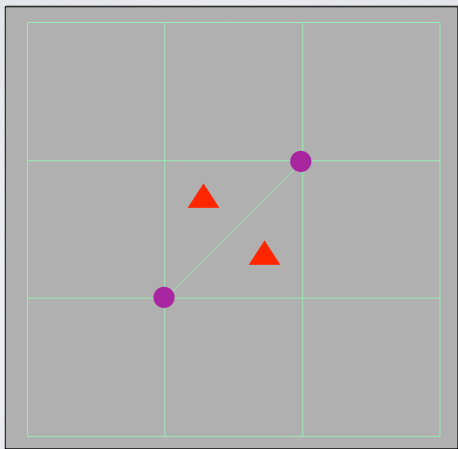
$$v = \frac{\bar{f}}{n} + \frac{2\bar{m}}{n} + \frac{p(n-3)}{n}$$

\bar{m} = average of adjacent midpoints

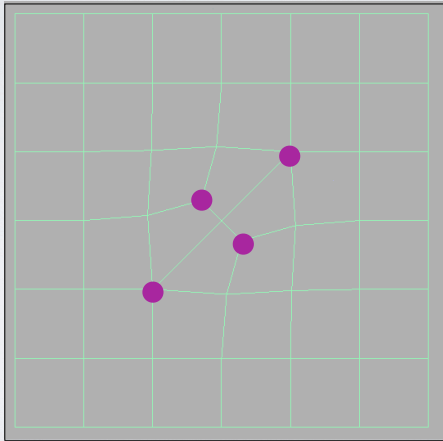
\bar{f} = average of adjacent face points

n = valence of vertex

Catmull-Clark Subdivision

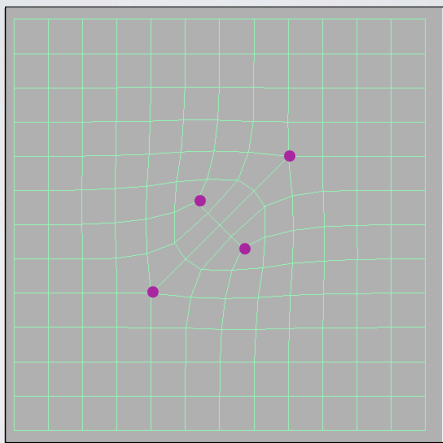


Catmull-Clark Subdivision



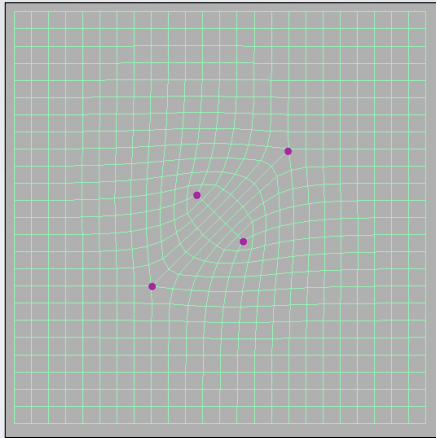
49

Catmull-Clark Subdivision



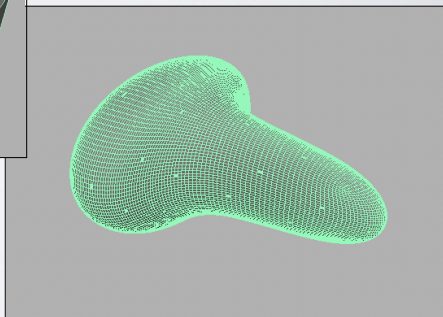
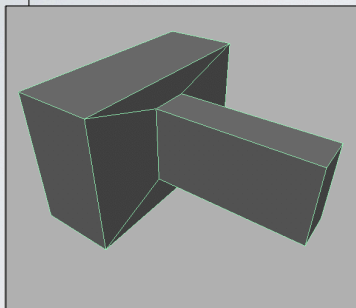
50

Catmull-Clark Subdivision



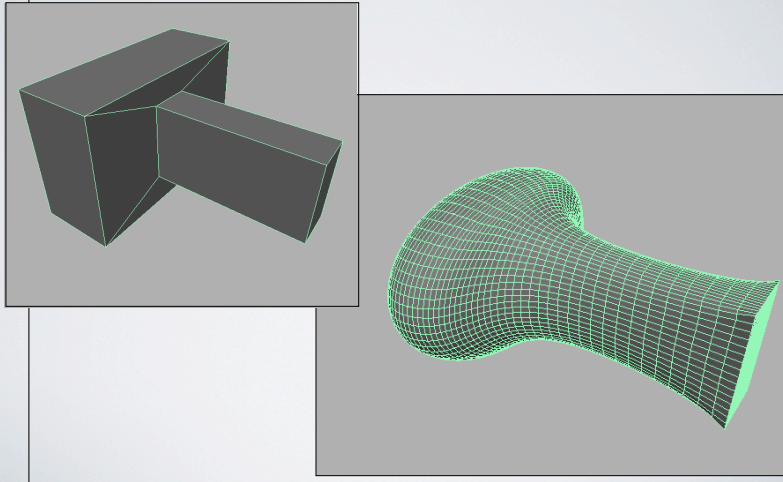
51

Catmull-Clark Subdivision

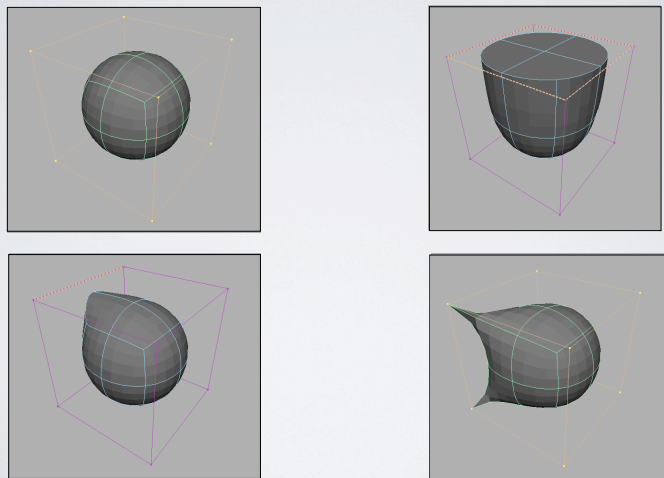


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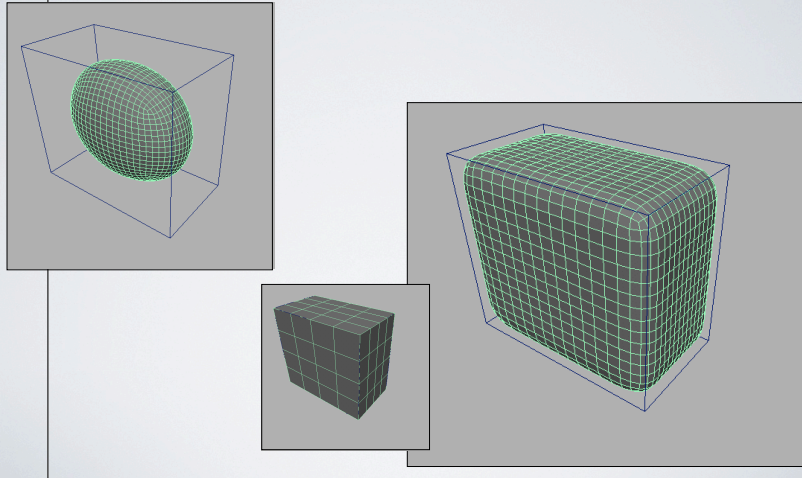
Catmull-Clark Subdivision



Catmull-Clark Subdivision



Catmull-Clark Subdivision

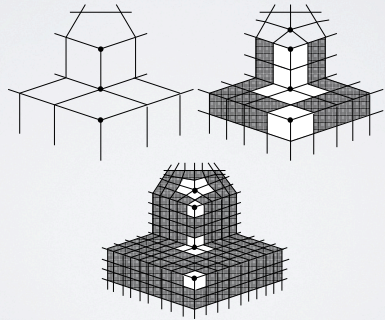


Practical Evaluation

- Uniform subdivision
 - Exponential growth of control mesh
 - Need several subdivisions before error is small
 - Ok if you are “drawing and forgetting”, otherwise impractical
- Evaluation at arbitrary points
- Tangent and other derivative evaluation needed
- Paper by Jos Stam SIGGRAPH 98 outlines efficient method

Isolated Extraordinary Points

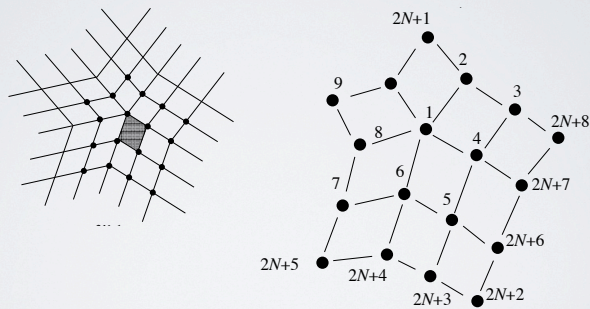
- Where surface is regular, treat as b-splines
- We can assume extraordinary points are isolated



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Isolated Extraordinary Points

- Neighborhood around an extraordinary point

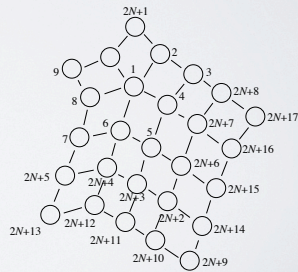
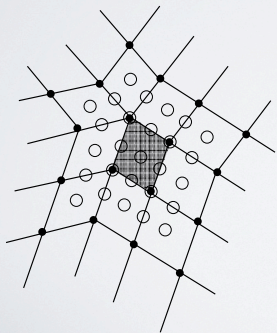


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Subdivision Matrix

$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0$$



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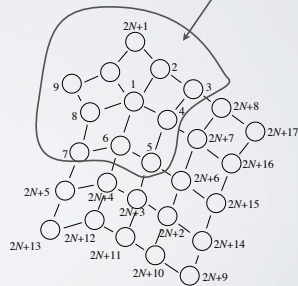
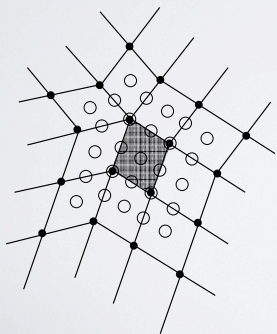
Subdivision Matrix

$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{S}_{11} & \mathbf{S}_{12} \end{pmatrix}$$

\mathbf{S} is $2n \times 2n$



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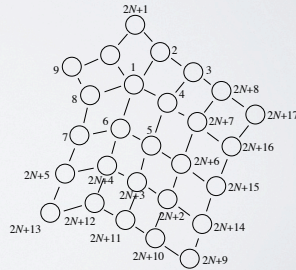
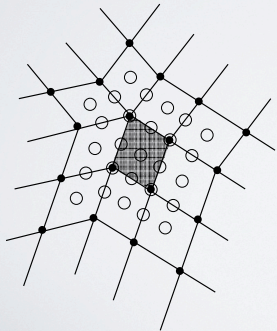
Subdivision Matrix

$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0$$

$$\bar{\mathbf{C}}_n = \bar{\mathbf{A}}\mathbf{C}_{n-1} = \bar{\mathbf{A}}\mathbf{A}^{n-1} \mathbf{C}_0, \quad n \geq 1$$

$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix}$$



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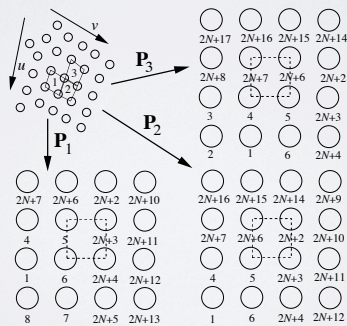
Subdivision Matrix

$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0$$

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$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix}$$



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Eigen Space

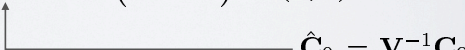
$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0 \quad \mathbf{A} = \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{S}_{11} & \mathbf{S}_{12} \end{pmatrix}$$

$$\mathbf{A}\mathbf{V} = \mathbf{V}\mathbf{\Lambda} \rightarrow \mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$$

$$\bar{\mathbf{C}}_n = \bar{\mathbf{A}}\mathbf{A}^{n-1}\mathbf{C}_0 = \bar{\mathbf{A}}\mathbf{V}\mathbf{\Lambda}^{n-1}\mathbf{V}^{-1}\mathbf{C}_0$$

$$\mathbf{s}_{k,n}(u, v) = \hat{\mathbf{C}}_0^T \mathbf{\Lambda}^{n-1} (\mathbf{P}_k \bar{\mathbf{A}}\mathbf{V})^T \mathbf{b}(u, v)$$


 $\hat{\mathbf{C}}_0 = \mathbf{V}^{-1}\mathbf{C}_0$

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Eigen Space

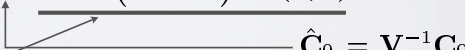
$$\mathbf{C}_1 = \mathbf{A}\mathbf{C}_0.$$

$$\mathbf{C}_n = \mathbf{A}\mathbf{C}_{n-1} = \mathbf{A}^n \mathbf{C}_0 \quad \mathbf{A} = \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{S}_{11} & \mathbf{S}_{12} \end{pmatrix}$$

$$\mathbf{A}\mathbf{V} = \mathbf{V}\mathbf{\Lambda} \rightarrow \mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$$

$$\bar{\mathbf{C}}_n = \bar{\mathbf{A}}\mathbf{A}^{n-1}\mathbf{C}_0 = \bar{\mathbf{A}}\mathbf{V}\mathbf{\Lambda}^{n-1}\mathbf{V}^{-1}\mathbf{C}_0$$

$$\mathbf{s}_{k,n}(u, v) = \hat{\mathbf{C}}_0^T \mathbf{\Lambda}^{n-1} (\mathbf{P}_k \bar{\mathbf{A}}\mathbf{V})^T \mathbf{b}(u, v)$$


 $\hat{\mathbf{C}}_0 = \mathbf{V}^{-1}\mathbf{C}_0$

Only depends on valence of extraordinary vertex.

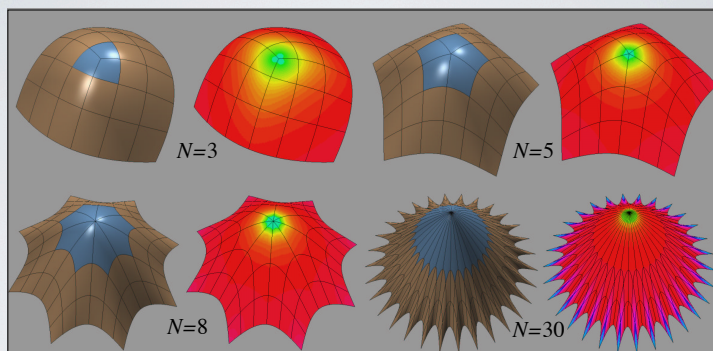
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Comments

- Computing Eigen Vectors is tricky
 - See Jos's paper for details
 - He includes the solutions up to valence 500
- All eigenvalues are (abs) less than one
 - except for lead value which is exactly one
 - well defined limit behaviour
- Exact evaluation allows "pushing to limit surface"

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Curvature Plots



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Thursday, November 12, 2009