## Advanced Computer Graphics (Fall 2009)

CS 294-13, Rendering Lecture 2: Lighting and Reflection

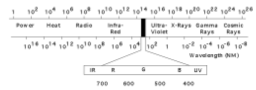
http://inst.eecs.berkeley.edu/~cs294-13/fa09

Many slides courtesy Pat Hanrahan

## Light

**Visible electromagnetic radiation**

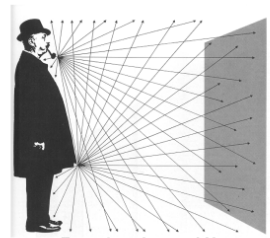
**Power spectrum**



**Polarization**

**Photon (quantum effects)**

**Wave (interference, diffraction)**



**From London and Upton**



## Radiometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - *Radiance, Irradiance*
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models
- Environment Maps

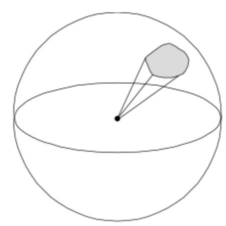
## Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

  $\Rightarrow$ **circle has 2$\pi$ radians**

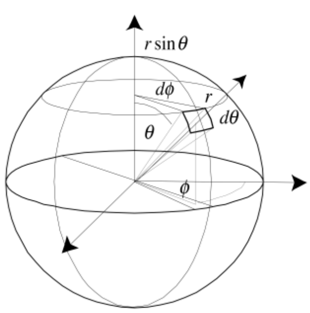
- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ **sphere has 4$\pi$ steradians**





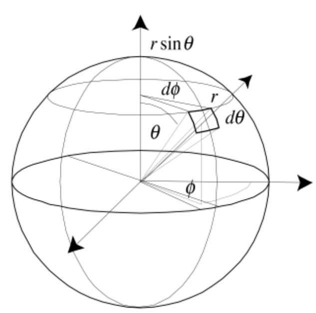
## Differential Solid Angles




$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$



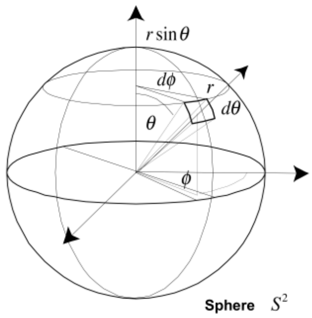
## Differential Solid Angles




$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

## Differential Solid Angles



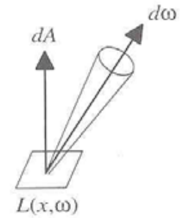

$$d\omega = \sin\theta \, d\theta \, d\phi$$

$$\Omega = \int_{S^2} d\omega = \int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\theta \, d\phi = \int_{-1}^{1}\int_0^{2\pi} d\cos\theta \; d\phi = 4\pi$$



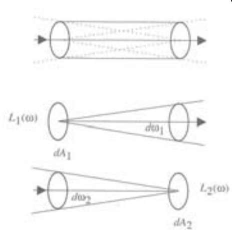
## Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ ($W/m^2$ sr)
- Flux given by
  $d\Phi = L(x,\omega) \cos\theta \, d\omega \, dA$




## Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.




$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

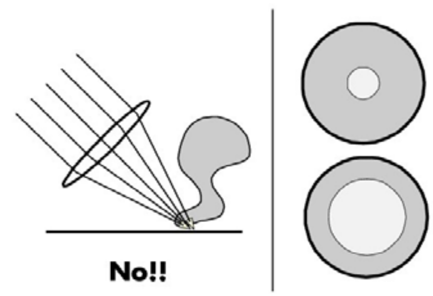
$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

## Quiz

**Does radiance increase under a magnifying glass?**

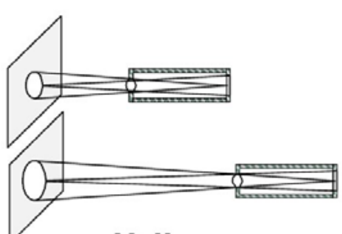


**No!!**



## Quiz

**Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?**



**No!!**



## Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances

Consequences
  - Radiance associated with rays in a ray tracer
  - Other radiometric quants derived from radiance

# Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle (cos θ dω)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: $W/m^2$

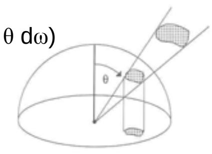
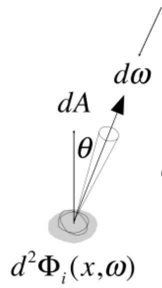



Figure 2.8: Projection of differential area.

- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

## Directional Power Arriving at a Surface




$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

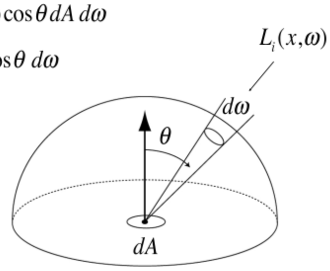


## Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta \, d\omega$$

Light meter




$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta \, d\omega$$

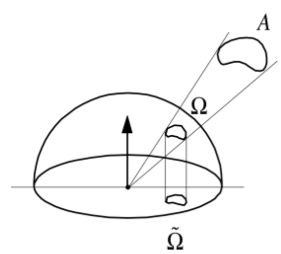


## Uniform Area Source

$$E(x) = \int_{H^2} L\cos\theta \, d\omega$$

$$= L\int_{\Omega} \cos\theta \, d\omega$$

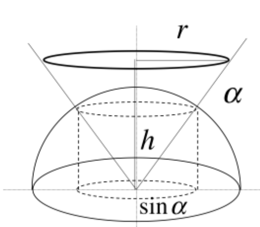
$$= L\tilde{\Omega}$$






## Uniform Disk Source

**Geometric Derivation**




$$\tilde{\Omega} = \pi \sin^2\alpha$$

**Algebraic Derivation**

$$\tilde{\Omega} = \int_1^{\cos\alpha}\int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

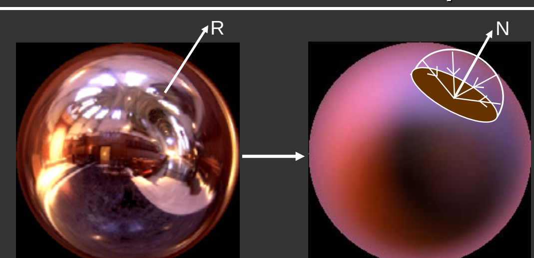
$$= 2\pi \left.\frac{\cos^2\theta}{2}\right|_1^{\cos\alpha}$$

$$= \pi \sin^2\alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$



## Irradiance Environment Maps




Incident Radiance (Illumination Environment Map)

Irradiance Environment Map

## Radiometry

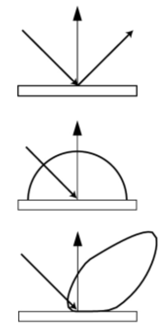
- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF*
  - *Reflection Equation*
  - Simple BRDF models
- Environment Maps

## Types of Reflection Functions

**Ideal Specular**
- **Reflection Law**
- **Mirror**

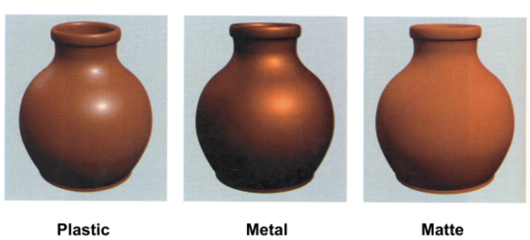
**Ideal Diffuse**
- **Lambert's Law**
- **Matte**

**Specular**
- **Glossy**
- **Directional diffuse**





## Materials




**From Apodaca and Gritz, *Advanced RenderMan***



## Spheres [Matusik et al.]

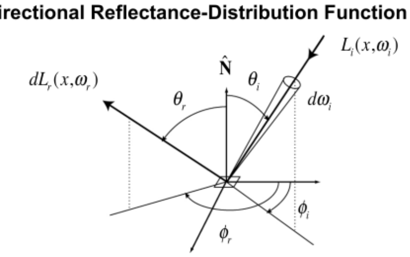


## Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

## The BRDF

**Bidirectional Reflectance-Distribution Function**




$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

## BRDF

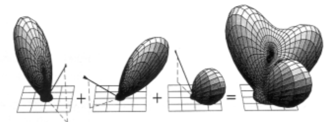
- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos\theta_i d\omega_i$$

## Properties of BRDF's

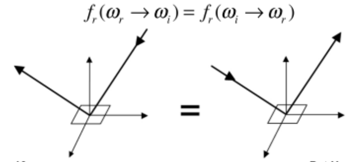
**1. Linearity**



**From Sillion, Arvo, Westin, Greenberg**

**2. Reciprocity principle**

$$f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$$



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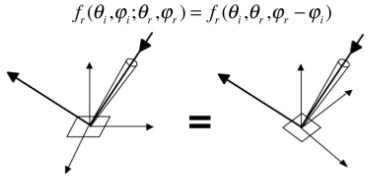
## Properties of BRDF's

**3. Isotropic vs. anisotropic**

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



**Reciprocity *and* isotropy**

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

**4. Energy conservation**

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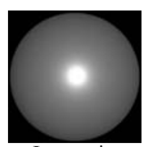
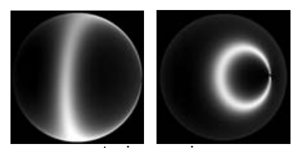
## Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic



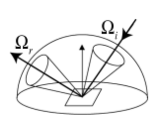
Anisotropic

## Energy Conservation

$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \to \omega_r) L_i(\omega_i)\cos\theta_i\, d\omega_i \cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$\leq 1$$

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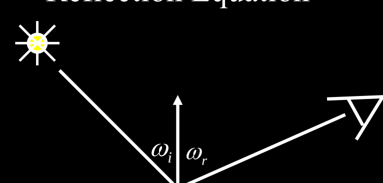
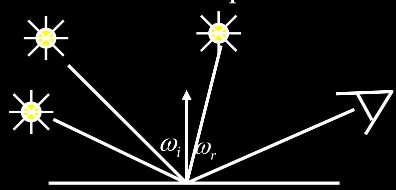
## Reflection Equation




$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) — Incident radiance (from light source) — BRDF — Cosine of Incident angle

## Reflection Equation

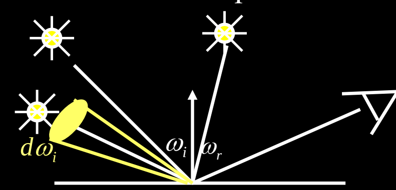



Sum over all light sources

$$L_r(\omega_r) = \sum_i L_i(\omega_i)\, f(\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

## Reflection Equation




Replace sum with integral

$$L_r(\omega_r) = \int_\Omega L_i(\omega_i)\, f(\omega_i,\omega_r)(\omega_i \bullet n)\, d\omega_i$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

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- Physical measurement of electromagnetic energy
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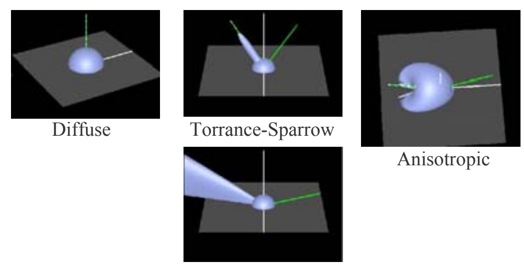
  - *Simple BRDF models*
- Environment Maps

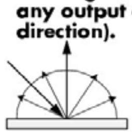
## Brdf Viewer plots




bv written by Szymon Rusinkiewicz

## Ideal Diffuse Reflection

**Assume light is equally likely to be reflected in any output direction (independent of input direction).**



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i\, d\omega_i$$
$$= f_{r,d}\int L_i(\omega_i)\cos\theta_i\, d\omega_i$$
$$= f_{r,d}E$$

$$M = \int L_r(\omega_r)\cos\theta_r\, d\omega_r = L_r\int\cos\theta_r\, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d}E}{E} = \pi f_{r,d} \quad\Rightarrow\quad f_{r,d} = \frac{\rho_d}{\pi}$$

**Lambert's Cosine Law** $M = \rho_d E = \rho_d E_s \cos\theta_s$

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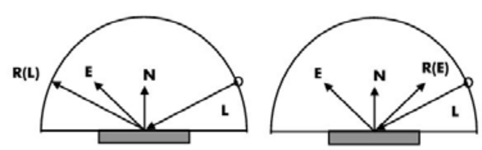
## Phong Model




$\left(\hat{\mathbf{E}}\bullet\mathbf{R}(\hat{\mathbf{L}})\right)^s$ $\left(\hat{\mathbf{L}}\bullet\mathbf{R}(\hat{\mathbf{E}})\right)^s$

**Reciprocity:** $\left(\hat{\mathbf{E}}\bullet\mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}}\bullet\mathbf{R}(\hat{\mathbf{E}})\right)^s$

**Distributed light source!**

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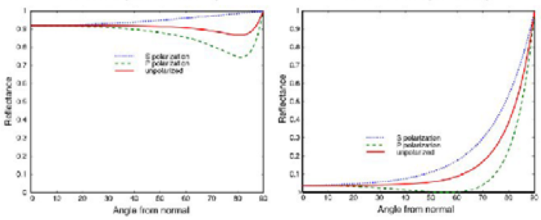
## Fresnel Reflectance




Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

**Schlick Approximation** $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

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## Experiment

**Reflections from a shiny floor**

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

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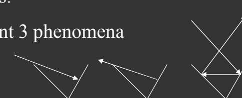
## Analytical BRDF: TS example

- One famous analytically derived BRDF is the Torrance-Sparrow model.
- T-S is used to model specular surface, like the Phong model.
  - more accurate than Phong
  - has more parameters that can be set to match different materials
  - derived based on assumptions of underlying geometry. (instead of 'because it works well')

## Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level.

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena




## Torrance-Sparrow Result

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

- Fresnel term: allows for wavelength dependency
- Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.
- Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.
- How much of the macroscopic surface is visible to the light source
- How much of the macroscopic surface is visible to the viewer

## Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

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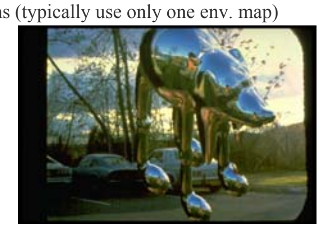
- *Environment Maps*

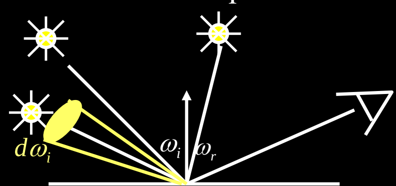
## Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)



Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

## Reflection Equation




Replace sum with integral

$$L_r(\omega_r) = \int_\Omega L_i(\omega_i)\, f(\omega_i, \omega_r)(\omega_i \bullet n)\, d\omega_i$$

Reflected Radiance (Output Image) — ***Environment Map (continuous)*** — BRDF — Cosine of Incident angle

## Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

## Demo