

CS61A Notes – Week 14: Logic programming

Paradigm Shift Again (Why Not?)

With this many paradigm shifts in a single semester, we expect you to at least be able to pronounce the word “paradigm” correctly after this class.

To reiterate, we are now in the realm of logic or declarative programming, at least for a week. Here in the magical world of the non-imperative, we can say exactly what we want – and have the computer figure out how to get it for us. Instead of saying how to get the solution, we describe – declare – what the solution looks like.

The mind-boggling part of all of this is that it all just works through pattern matching. That is, there are no “procedures” in the way you’re used to; when you write out a parenthesized statement, it’s not really a procedure call, and you don’t really get a return value. Instead, either you get entries from some database, or nothing at all.

Be Assertive And Ask Questions (Fitter, Happier, More Productive)

There are two things you can type into our query system: an **assertion** and a **query**.

A **query** asks whether a given expression matches some fact that’s already in the database. If the query matches, then the system prints out all matches in the database. If the query doesn’t match to anything, you get no results.

An **assertion** states something true; it adds an entry to the database. You can either assert a simple fact, or a class of facts (specified by a “rule”).

So here’s an assertion that you’ve seen: `(assert! (justin likes chocolate))`

You can also assert rules. In general, a rule looks like: `(rule <conclusion> <subconditions>)`

And it’s read: “conclusion is true if and only if all the subconditions are true”. Note that you don’t have to have subconditions! Here’s a very simple rule:

```
(rule (same ?x ?x))
```

The above says two things satisfy the “same” relation if they can be bound to the same variable. It’s deceptively simple – no subconditions are provided to check the “sameness” of the two arguments. Instead, either the query system can bind the two arguments to the same variable `?x` – and it can only do so if the two are equivalent – or, the query system can’t.

And, of course, the rule of love:

```
(assert! (rule (?person1 loves ?person2)
               (and (?person1 likes ?something)
                    (?person2 likes ?something)
                    (not (same ?person1 ?person2))))))
```

The above rule means that `?person1 loves ?person2` if the three conditions following can all be satisfied – that is, `?person1` likes `?something` that `?person2` also likes, and that `?person1` is not the same person as `?person2`.

Note the “and” special form enclosing the three conditions – an entry in the database must satisfy ALL three conditions to be a match for the query. If you would like a rule to be satisfied by this *or* that condition, you can either use the `or` special form in the obvious way, or you can make two separate rules. For example, if one loves another if they like the same things *or* if `?person1` is a parent of `?person2`, we would add the following to the database:

```
(assert! (rule (?person1 loves ?person2) (parent ?person1 ?person2)))
```

Note the new rule does NOT “overwrite” the previous rule; this is not the same thing as redefining a procedure in Scheme. Instead, the new rule complements the previous rule.

To add to confusion, you can also use the `and` special form for queries. For example,

```
(and (justin loves ?someone) (?someone likes chocolate))
```

is a query that finds a person Justin loves because that person likes chocolate.

There's another special form called `lisp-value`: `(lisp-value <pred?> <arg1, arg2, ...>)`

The `lisp-value` condition is satisfied if the given `pred?` applied to the list of `args` return `#t`. For example, `(lisp-value even? 4)` is satisfied, but `(lisp-value < 3 4 1)` is not. `lisp-value` is useful mostly for numerical comparisons (things that the logic system isn't so great at).

A note on writing rules: it's often tempting to think in terms of procedures – “this rule takes in so and so, and returns such and such”. This is not the right way to approach these problems – remember, nothing is returned; an expression or query either has a match, or it doesn't. So often, you need to have both the “arguments” and the “return value” in the expression of the rule, and the rule is satisfied if “return value” is what would be returned if the rule were a normal Scheme procedure given the “arguments”. Always keep in mind that everything is a Yes or No question, and your rule can only say if something is a correct answer or not. So when you write a rule, instead of trying to “build” to a solution like you've been doing in Scheme, think of it as trying to check if a given solution is correct or not.

In fact, **this is so important I'll say it again:** when you define rules, don't think of it as defining procedures in the traditional sense. Instead, think of it as, *given arguments and a proposed answer, check if the answer is correct*. The proposed answer can either be derived from the arguments, or it can't.

A different approach for writing declarative rules is to try to convert a Scheme program to a rule. For example, let's take a crack at the popular `append`:

```
(define (append ls1 ls2)
  (cond ((null? ls1) ls2)
        (else (cons (car ls1) (append (cdr ls1) ls2)))))
```

The `cond` specifies an “either-or” relationship; either `ls1` is null or it is not. This implies that, for logic programming, we'd need two separate rules, each corresponding to each `cond` clause. The first one is straightforward:

```
(rule (append () ?ls2 ?ls2))
```

The second `cond` clause breaks `ls1` into two parts – its `car` and its `cdr` – and basically says that appending `ls1` and `ls2` is the same as consing the first element of `ls1` to the list obtained by appending the `cdr` of `ls1` to `ls2`. Translated to logic programming, it means the `cdr` of the result is equivalent to appending the `cdr` of `ls1` to `ls2`, and that the `car` of the result is just the `car` of `ls1`. This implies:

```
(rule (append (?car . ?cdr) ?ls2 (?car . ?r-cdr))
      (append ?cdr ?ls2 ?r-cdr))
```

We will try the above techniques on some of the harder problems below.

The Search For Truth and Honor (defined as such in our database)

Don't think declarative programming is useless – you might see it more than you think. The most common use of it is in database queries, and if you're worked with databases before, you know that SQL is a declarative language:

```
SELECT * FROM people WHERE age=13 ORDER BY first_name;
```

The above is a valid query into the table (or “relation”) called “people” to select all columns of entries with column “age” equal to 13, and we want the result to be ordered by the column “first_name”. Note how we did *not* instruct to the database on how to give us the result; we simply trust the database management system to figure out the most efficient way to satisfy our desires. This is one place where declarative programming is absolutely natural (and think of the nightmare you'd have if you have to specify your queries imperatively!)

Of course, database queries is about the easiest use of logic programming, and you've had plenty of practice with `microshaft` (a rather obscure name if you ask me) in lab and homework. So let's move on to the interesting stuff.

Lists Again (and again, and again, and again, and again...)

Since lists are just patterns of symbols, logic programming is especially good at dealing with them.

```
(my-list (1 2 3 4))
```

Let's look at the following queries. Note the explanations – it's easy to see intuitively, sometimes, why something is bound to something, but you should get used to the way the query processor thinks!

```
(my-list ?x) => ?x is bound to (1 2 3 4) because there's an entry in the database starting with "my-list" and followed by one more thing.
```

```
(my-list (1 ?x 3 4)) => ?x is bound to 2 because there's an entry in the database starting with "my-list" and followed by a list of four elements, the first, third and fourth of which are 1, 3 and 4.
```

```
(my-list (1 ?x)) => nothing. There is no entry in the database that starts with "my-list" followed by a two-element list – and, note carefully, (1 ?x) is a list of two elements!
```

```
(my-list (1 . ?x)) => ?x is bound to (2 3 4) because there's an entry in the database starting with "my-list" followed by a list whose first element is 1. ?x is simply bound to the rest. Note that this makes sense, since (1 . (2 3 4)) is equivalent to (1 2 3 4).
```

Now, let's play a few.

REVIEW

```
car      (assert! (rule (car (?car . ?cdr) ?car)))
```

```
cdr      (assert! (rule (cdr (?car . ?cdr) ?cdr)))
```

member

```
(assert! (rule (member ?item (?item . ?cdr))))  
(assert! (rule (member ?item (?car . ?cdr))  
             (member ?item ?cdr)))
```

```
deep-member  ??
```

QUESTIONS

1. Using the above, write a query that would bind `?x` to the `car` of `my-list`. Write another query that would bind `?y` to the `cdr` of `my-list`.

2. Define another old friend, `reverse`, so that `(reverse (1 2 3) (3 2 1))` would be satisfied.

```
(assert!
```

3. (HARD!) Define its cousin, `deep-reverse`, so that `(deep-reverse (1 2 (3 4) 5) (5 (4 3) 2 1))` would be satisfied.

4. Write the rule `remove` so that `(remove 3 (1 2 3 4 3 2) ?what)` binds `?what` to `(1 2 4 2)` – the list with 3 removed.

5. Write the rule `interleave` so that `(interleave (1 2 3) (a b c d) ?what)` would bind `?what` to `(1 a 2 b 3 c d)`.

6. Consider this not very interesting rule: `(rule (listify ?x (?x)))`. So if we do `(listify 3 ?what)`, `?what` would be bound to `(3)`.

Define a rule `map` with syntax `(map procedure list result)`, so that `(map listify (1 2 3) ((1) (2) (3)))` would be satisfied, as would `(map reverse ((1 2) (3 4 5)) ((2 1) (5 4 3)))`. In fact, we should be able to do something cool like `(map ?what (1 2 3) ((1) (2) (3)))` and have `?what` bound to the word “listify”. Assume the “procedures” we pass into `map` are of the form `(procedure-name argument result)`.

7. We can let predicates have the form `(predicate-name argument)`. Define a rule `even` so that `(even 3)` is not satisfied, and `(even 4)` is satisfied.

8. The above is a way to make predicates. And once we have predicates, we can – and will, of course – write a `filter` rule with the syntax `(filter predicate list result)` so that `(filter even (1 2 3 4 5 6) (2 4 6))` returns Yes, and querying `(filter ?what (10 11 12 13) (10 12))` would bind `?what` to the word “even”.

EXTRA PRACTICE

Number Theory (The Bizarre Way)

Logic programming is bad at working with numbers, since arithmetic isn't really pattern matching. Or isn't it? Suppose we use a new numbering system:

```
0           => zero
(s 0)      => one
(s (s 0))  => two
```

And so on. Think of s as a "successor function". But, there's no need for such a function (the fact that there are no functions, period, in logic-programming should've tipped you off!) That is, $(s\ 0)$ doesn't "return" one; it IS one! What we've done is that we've made up a completely new way to represent numbers – a system that's different from the decimal representation that you're used to. Each number is represented by *the number of nestings*, or *the number of s* it has. For example, 2 has two nestings of s until you get to 0, and 100 would have 100 nestings of s . The "successor function" is purely conceptual, not functional. And, more importantly, it's purely *syntactical*, so the nestings of s are just patterns that we can match to!

Try to wrap your head around that, and try the rule `sum`:

```
(rule (sum 0 ?x ?x)) ;; the sum of 0 and a number is itself
(rule (sum (s ?x) ?y ?z)
      (sum ?x (s ?y) ?z))
```

The first rule is obvious, but the second is a bit puzzling. Note that the first rule is the "base case", and as in everywhere else in this course, we're always trying to reduce a problem down to a base case. So how do you reduce an addition of two numbers into an addition of 0 and a number?

Consider this sequence of equivalences:

$$2+3=5 \iff 1+4=5 \iff 0+5=5$$

Ah, see? We went from a general case $(2+3)$ to a base case $(0+5)$. And we did this by subtracting one from the first argument to `sum` and adding one to the second argument to `sum`. But our "successor function" merely adds; how do we subtract? Well, if we match one s , we would be left with a number with one less s , which is basically subtracting one. Or, note that if we try to match $(s\ (s\ (s\ 0)))$ to $(s\ ?x)$, $?x$ would be bound to $(s\ (s\ 0))$! This should explain the second rule. The magic is, again, in pattern matching!

QUESTIONS

1. Write the rule `subtract` using the same syntax as `sum`. Assume that the first argument will always be greater than the second (since we don't support negative numbers with our system!)
2. Write the rule `product`.

3. Define the rule `exp` (for exponent, of course), with the first argument the base and second the power, so that `(exp (s (s 0)) (s (s 0)) ?what)` would bind `?what` to `(s (s (s (s 0))))`.

4. Write the rule `factorial`, so that `(factorial (s (s (s 0))) ?what)` would bind `?what` to `(s (s (s (s (s (s 0))))))`.

5. Write the rule `max` with syntax `(max number1 number2 the-bigger-number)` so that `(max (s 0) (s (s 0)) ?what)` would bind `?what` to `(s (s 0))`.

6. Write the rule `appearances` that counts how many times something appears in a list. For example, `(appearances 3 (1 2 3 3 2 3 3) ?what)` would bind `?what` to `(s (s (s (s 0))))`.

7. Note that we can represent negative numbers by putting `s` to the right. For example, negative two would be `((0 s) s)`. Write `*-1` so that `(*-1 ((0 s) s) ?what)` binds `?what` to `(s (s 0))` and `(*-1 (s (s 0)) ?what)` binds `?what` to `((0 s) s)`.

8. (HARD!) Write rules `sum2` and `subtract2` that can take in negative numbers.