Abstraction, List, & Cons

CS61A Lecture 7

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Very sad code

```
(define (total hand)
    (if (empty? hand)
        0
        (+ (butlast (last hand))
           (total (butlast hand)))))
STk> (total '(3h 10c 4d))
17
STk> (total '(3h ks 4d))
;;;EEEK!
```

Happier Code

```
(define (total hand)
    (if (empty? hand)
        0
        (+ (rank (one-card hand))
           (total (remaining-cards hand)
                                   ))))
```

```
(define (rank card) (butlast card))
(define (suit card) (last card))
(define suit last)
(define (one-card hand) (last hand))
(define (remaining-cards hand) (bl hand))
```

Selectors

Goals

- To talk about things using meaning not how it is represented in the computer
- To be able to change how it is represented in the computer without people who use our program caring
- Invented by: Turing Award Winner:
  Barbara Liskov

Constructors

- **GOAL:** To talk about things using meaning not how it is represented in the computer

```
STk> (total '(3h 10c 4d))
STk> (total
        (make-hand
           (make-card 3 'heart)
           (make-card 10 'club)
           (make-card 4 'diamond)))
```

You still have to teach people to use your program

Constructors

```
STk> (total
        (make-hand
           (make-card 3 'heart)
           (make-card 10 'club)
           (make-card 4 'diamond)))
```

```
(define (make-card rank suit)
    (word rank (first suit)))
(define make-hand se)
```

Constructors

Data Abstraction ☺

```
(define (total hand)
    (if (empty? hand)
        0
        (+ (rank (one-card hand))
           (total (remaining-cards hand)
                                      ))))
```

```
(define (rank card)
     (butlast card))
(define (suit card)
     (last card))
(define (one-card hand)
     (last hand))
(define (remaining-cards hand)
     (bl hand))
```

```
(define (make-card rank suit)
     (word rank (first suit)))
(define make-hand se)
```

Try It!

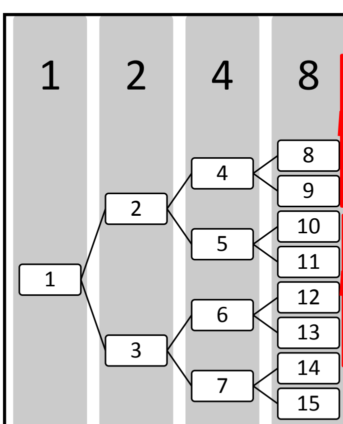
Rewrite what you need to:

- Cards are represented as numbers 1-52
  - 1-13 is A-K of Hearts
  - 14-26 is A-K of Spades
  - 27-39 is A-K of Diamonds
  - 40-52 is A-K of Clubs

Runtimes Continued

Exponential Runtime

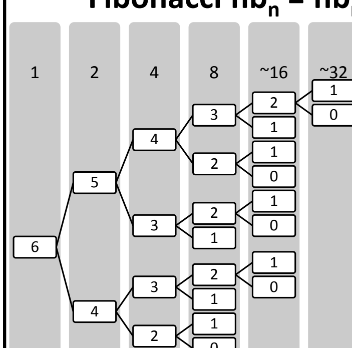
2^n



0 branches = 1 call
1 branches = 3
2 branches = 7
3 branches = 15

b branches = 2^{b+1} - 1 calls

Fibonacci $fib_n = fib_{n-1} + fib_{n-2}$



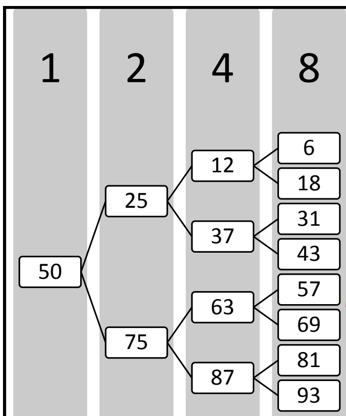
n branches = 2^{n+1} - 1 calls

Logarithmic Runtime

$Log_2(N)$

Number Guessing Game

- I'm thinking of a number between 1 and 100
- How many possible guesses could it take you? (WORST CASE)
- Between 1 and 10000000?
- How many possible guesses could it take you? (WORST CASE)



b branches = 2^{b+1} - 1 calls

n possible numbers & h calls $n=2^{h+1}-1$

Divide and Conquer

- If we can divide the problem up in half each time
  - like the number guessing game
- How many recursive calls will it take?

n is the original problem size if h calls then: $n=2^{h+1}-1$

$$n = 2^{h+1} - 1$$

// Take the log of both sides

$$log_2(n) = log_2(2^{h+1} - 1)$$

// Remember:

$$\log_b m^n = n * \log_b m$$

$$\log_2(n) \approx (h+1)(\log_2 2)$$

$$log_2 n \approx h$$

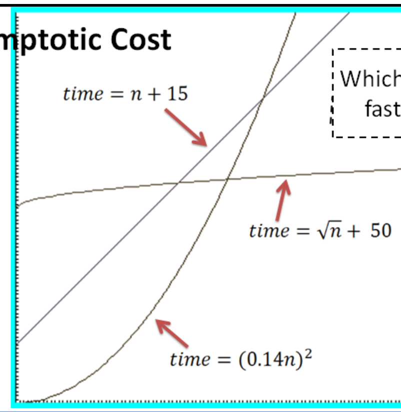
$Log_2(N)$

- When we're able to keep dividing the problem in half (or thirds etc.)
- Looking through a phone book

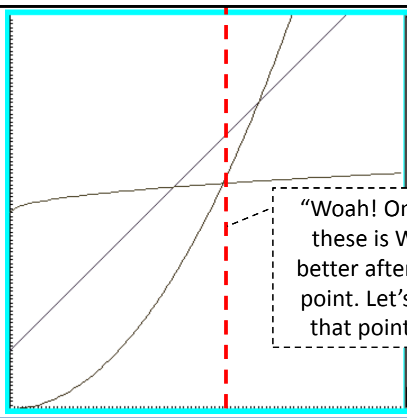
Asymptotic Cost

- We want to express the speed of an algorithm *independently* of a *specific implementation* on a *specific machine.*
- We examine the cost of the algorithms for large input sets i.e. *the asymptotic cost*.
- In later classes (CS70/CS170) you'll do this in more detail

Asymptotic Cost



Which one is fastest?



"Woah! One of these is WAY better after this point. Let's call that point N"

Important Big-Oh Sets

Subset of ↓

Function	Common Name
$O(1)$	Constant
$O(\log n)$	Logarithmic
$O(\log^2 n)$	Log-squared
$O(\sqrt{n})$	Root-n
$O(n)$	Linear
$O(n \log n)$	n log n
$O(n^2)$	Quadratic
$O(n^3)$	Cubic
$O(n^4)$	Quartic
$O(2^n)$	Exponential
$O(e^n)$	Bigger exponential

Which is fastest after some value N?

$$time = \sqrt{n} - 1000$$

$$time = 0.75\sqrt{n} + 50$$

$$time = \sqrt{n}$$

WAIT – who cares? These are all proportional!

Sometimes we do care, but for simplicity we ignore constants

Formal definition

$$T(n) \in O(f(n))$$

if and only if

$$T(n) \le c * f(n)$$

for all n > N

Simplifying stuff is important

$$f(n) \in O(5n^3 + 10n^2 + 1000n)$$

$$T(n) \in O(n^3)$$

Cons and Lists

DEMO

cons

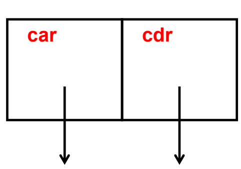
```
STk> (cons 1 2)
(1 . 2)
STk> (define a (cons 1 2))
a
STk> (define b (cons 'hi 'bye))
b
STk> b
(hi . bye)
```

car / cdr

```
STk> (car a)
1
STk> (cdr a)
2
STk> (car b)
hi
STk> (cdr b)
bye
```

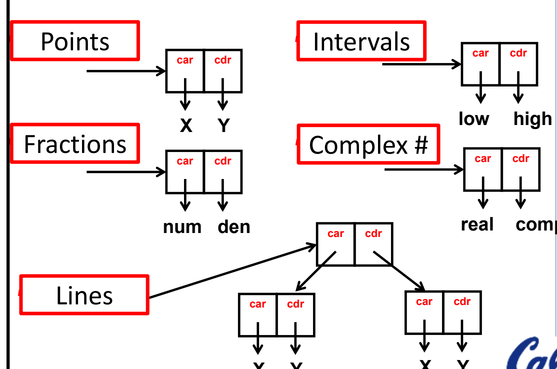
Data Abstraction ☺

Pairs



Selectors	Constructors
car cdr	cons

Uses of Pair from textbook



Lists

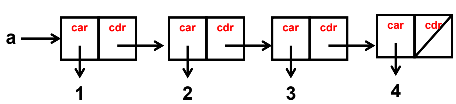
Demo

list

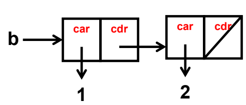
```
STk> (list 1 2 3 4 5)
(1 2 3 4 5)
STk> (list 1)
(1)
STk> (list)
()
STk> (list +)
(#[closure arglist=args 7ff53de8])
```

Lists are made with pairs!

```
STk> (define a (list 1 2 3 4))
```

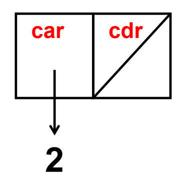



```
STk> (define b (list 1 2))
```



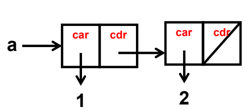
The Empty List

```
STk> (cons 2 '())
(2)
```



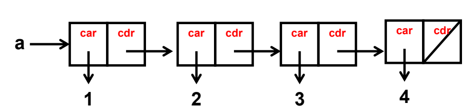
How can you make the list (1 2)?

```
a) (define a (cons 1 2 '()))
b) (define a (cons 1 (cons 2)))
c) (define a (cons 1 (cons 2 '())))
d) (define a (cons (cons 2 '()) 1)))
e) ???
```



How many calls to cons are made?

```
STk> (define a (list 1 2 3 4))
```



A) 2 B) 3 C) 4 D) 5 E) 6

How many calls to cons are made?

```
STk> (define a (list 1 2 (list 3 4) 5))
```

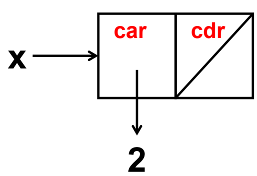
A) 2 B) 3 C) 4 D) 5 E) 6

Accessing Elements

Using `car` and `cdr`

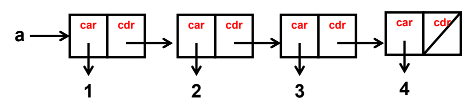
The Empty List w/ `car` & `cdr`

```
STk> (define x (cons 2 '())
x
STk> x
(2)
STk> (car x)
2
STk> (cdr x)
()
```



How do you get the 2?

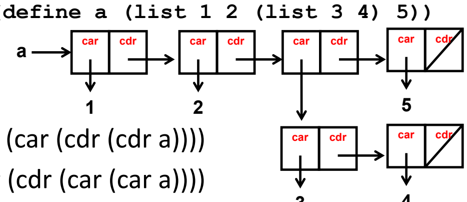
```
STk> (define a (list 1 2 3 4))
```



A) (car (cdr a))
B) (cdr (car a))
C) (cdr (cdr (car a)))
D) (car (cdr (cdr a)))
E) (cdr (car (car a)))

How do you get the 3?

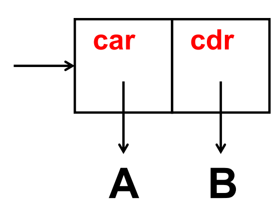
```
STk> (define a (list 1 2 (list 3 4) 5))
```



A) (car (car (cdr (cdr a))))
B) (cdr (cdr (car (car a))))
C) (cdr (car (cdr (car a))))
D) (car (cdr (car (cdr a))))
E) ???

Cons makes a pair

```
(cons a b)
```

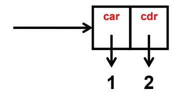


Dots

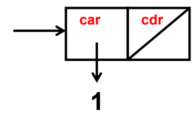
Demo

Dots

```
STk> (cons 1 2)
(1 . 2)
```

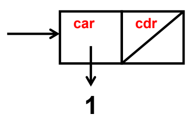



```
STk> (cons 1 '())
(1)
```

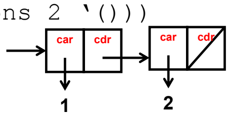



```
STk> (cons 1 '())
(1 . ( ))
```

Dots




```
STk> (cons 1 '())
(1 . ())
(1)
STk> (cons 1 (cons 2 '()))
(1 . (2 . ( )))
(1 2)
```



CONSTRUCTOR SOLUTION

```
(define (make-card rank suit)
  (cond
    ((equal? suit 'heart) rank)
    ((equal? suit 'spade) (+ rank 13))
    ((equal? suit 'diamond) (+ rank 26))
    ((equal? suit 'club) (+ rank 39))
    (else (error "say what?")) ))
```

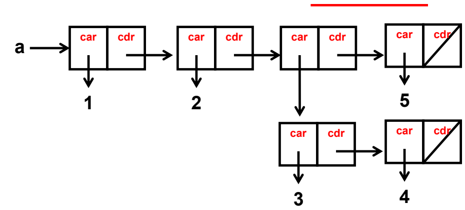
SELECTOR SOLUTION

```
(define (card-rank card)
  (remainder card 13))

(define (suit card)
  (cond
    ((> 14 card) 'heart)
    ((> 27 card) 'spade)
    ((> 40 card) 'diamond)
    (else 'club)))
```

How many calls to cons are made?

STk> (define a (list 1 2 (list 3 4) 5))



A) 2 B) 3 C) 4 D) 5 E) 6

Solution: (cons 1 (cons 2 (cons (cons 3 (cons 4 '())) (cons 5 '()))))