CS61B Lecture #26

Today:

- Sorting algorithms: why?
- Insertion, Shell's, Heap, Merge sorts

Readings for Today:

DS(IJ), Chapter 8;

Public Service Announcement: Business Revolution: A 360 degree Overview on the Business of Technology. Speakers will present their careers in Venture Capital, Corporate Management, and Entrepreneurship. March 23, 2006, 7–9PM, 2060 VLSB. Complimentary Food. Hosted by the Xi Pledge Class of $AK\Psi$

Purposes of Sorting

- Sorting supports searching
- Binary search standard example
- Also supports other kinds of search:
 - Are there two equal items in this set?
 - Are there two items in this set that both have the same value for property X?
 - What are my nearest neighbors?
- Used in numerous unexpected algorithms, such as convex hull (smallest convex polygon enclosing set of points).

Some Definitions

- A sort is a permutation (re-arrangement) of a sequence of elements that brings them into order, according to some total order. A total order, \prec , is:
 - Total: $x \leq y$ or $y \leq x$ for all x, y.
 - Reflexive: $x \prec x$;
 - Antisymmetric: $x \leq y$ and $y \leq x$ iff x = y.
 - Transitive: $x \leq y$ and $y \leq z$ implies $x \leq z$.
- However, our orderings may allow unequal items to be equivalent:
 - E.g., can be two dictionary definitions for the same word: if entries sorted only by word, then sorting could put either entry first
 - A sort that does not change the relative order of equivalent entries is called stable.

Classifications

- Internal sorts keep all data in primary memory
- External sorts process large amounts of data in batches, keeping what won't fit in secondary storage (in the old days, tapes).
- Comparison-based sorting assumes only thing we know about keys is order
- Radix sorting uses more information about key structure.
- Insertion sorting works by repeatedly inserting items at their appropriate positions in the sorted sequence being constructed.
- Selection sorting works by repeatedly selecting the next larger (smaller) item in order and adding it one end of the sorted sequence being constructed.

Sorting by Insertion

- Simple idea:
 - starting with empty sequence of outputs.
 - add each item from input, inserting into output sequence at right point.
- Very simple, good for small sets of data.
- With vector or linked list, time for find + insert of one item is at worst $\Theta(k)$, where k is # of outputs so far.
- ullet So gives us $O(N^2)$ algorithm. Can we say more?

Inversions

- ullet Can run in $\Theta(N)$ comparisons if already sorted.
- Consider a typical implementation for arrays:

```
for (int i = 1; i < A.length; i += 1) {
 int j;
 Object x = A[i];
 for (j = i-1; j >= 0; j -= 1) {
   if (A[i].compareTo(x) <= 0) /* (1) */
    break;
  A[j+1] = A[j];
A[j+1] = x;
```

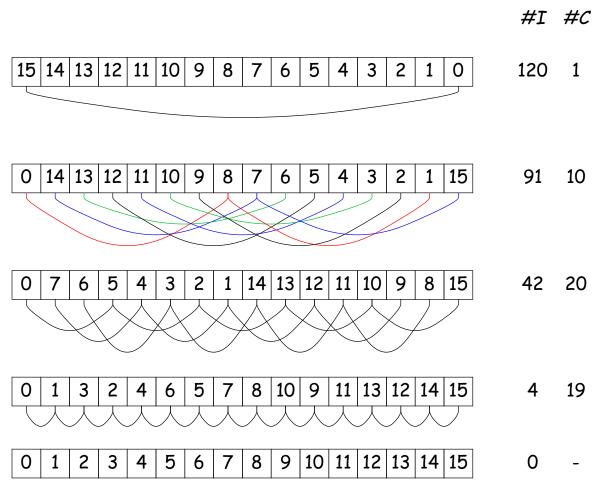
- \bullet #times (1) executes \approx how far x must move.
- ullet If all items within K of proper places, then takes O(KN) operations.
- Thus good for any amount of nearly sorted data.
- One measure of unsortedness: # of inversions: pairs that are out of order (= 0 when sorted, N(N-1)/2 when reversed).
- Each step of j decreases inversions by 1.

Shell's sort

Improve insertion sort by first sorting distant elements: Idea:

- First sort subsequences of elements $2^k 1$ apart:
 - sort items #0, $2^k 1$, $2(2^k 1)$, $3(2^k 1)$, ..., then
 - sort items #1, $1+2^k-1$, $1+2(2^k-1)$, $1+3(2^k-1)$, ..., then
 - sort items #2, $2+2^k-1$, $2+2(2^k-1)$, $2+3(2^k-1)$, ..., then
 - etc.
 - sort items $\#2^k-2,\ 2(2^k-1)-1,\ 3(2^k-1)-1,\ \ldots$
 - Each time an item moves, can reduce #inversions by as much as $2^{k} + 1$.
- Now sort subsequences of elements $2^{k-1}-1$ apart:
 - sort items #0, $2^{k-1}-1$, $2(2^{k-1}-1)$, $3(2^{k-1}-1)$, ..., then
 - sort items #1, $1 + 2^{k-1} 1$, $1 + 2(2^{k-1} 1)$, $1 + 3(2^{k-1} 1)$, ...,
 - -:
- \bullet End at plain insertion sort ($2^0=1$ apart), but with most inversions gone.
- Sort is $\Theta(N^{1.5})$ (take CS170 for why!).

Example of Shell's Sort



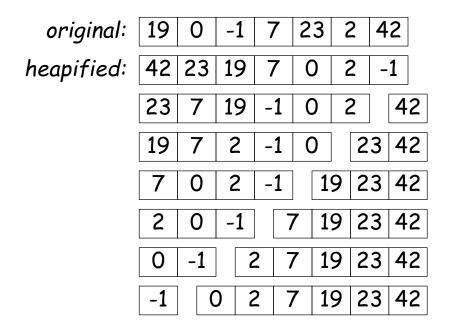
I: Inversions left.

C: Comparisons needed to sort subsequences.

Sorting by Selection: Heapsort

Keep selecting smallest (or largest) element. Idea:

- Really bad idea on a simple list or vector.
- But we've already seen it in action: use heap.
- Gives $O(N \lg N)$ algorithm (N remove-first operations).
- Since we remove items from end of heap, we can use that area to accumulate result:



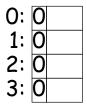
Merge Sorting

Divide data in 2 equal parts; recursively sort halves; merge re-Idea: sults.

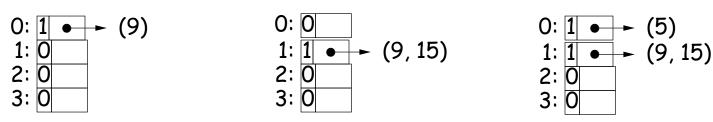
- Already seen analysis: $\Theta(N \lg N)$.
- Good for external sorting:
 - First break data into small enough chunks to fit in memory and sort.
 - Then repeatedly merge into bigger and bigger sequences.
 - Can merge K sequences of arbitrary size on secondary storage using $\Theta(K)$ storage.
- For internal sorting, can use binomial comb to orchestrate:

Illustration of Internal Merge Sort

L: (9, 15, 5, 3, 0, 6, 10, -1, 2, 20, 8)



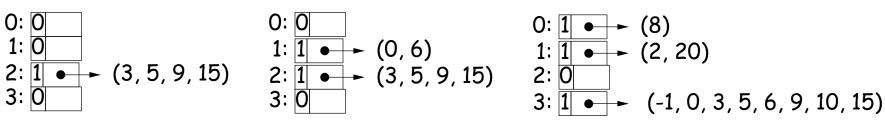
O elements processed



1 element processed

2 elements processed

3 elements processed



4 elements processed

6 elements processed

11 elements processed