CS 61B Discussion 7: Exam Prep Spring 2020

1 Asymptotic analysis

In the following questions, give the runtime of the following functions in $\Theta(.)$ or $O(.)$ notation as requested. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms. (MT2, Spring 2015)

(a)
```
public static void f1(int n) {
    for (int i = 0; i < 2*n; i += 1) {
        System.out.println("hello");
    }
}
```
Runtime: ___$\Theta(n)$____

(b)
```
public static void f3(int n) {
    if (n == 0) { return; }
    f3(n/3);
    f1(n);
    f3(n/3);
    f1(n);
    f3(n/3);
}
```
Runtime: ___$\Theta(nlogn)$____

(c)
```
public static void f4(int n) {
    if (n == 0) { return; }
    f4(n-1);
    f1(17);
    f4(n-1);
}
```
Runtime: ___$\Theta(2^n)$____

(d)
```
public static void f5(int n, int m) {
    if (m <= 0) {
        return;
    } else {
        for (int i = 0; i < n; i += 1) {
            f5(n, m-1);
        }
    }
}
```
Runtime: ___$\Theta(n^m)$____

(e)
```
public static void f6(int n) {
    if (n == 0) { return; }
    f6(n / 2);
    f6(n / 2);
    f6(n / 2);
    f6(n / 2);
    g(n);           // runs in Θ(N^2) time
}
```
Runtime: ___$\Theta(n^2logn)$____

(f)
```
public static void f7(int n, int m) {
    if (n < 10) { return; }
    for (int i = 0; i <= n % 10; i++) {
        f7(n / 10, m / 10);
        System.out.println(m);
    }
}
```
Runtime: ___$O(n)$___

2 More analysis

For each problem, give the best and worst-case runtimes in $\Theta(.)$ notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.

(a)
```
public static void removeIndex(int[] arr, int i) {
    // Assume i > 0
    int N = arr.length;
    for (int j = i; j < N; j += 1) {
        arr[j - 1] = arr[j];
    }
}
```
Best Case: ___$\Theta(1)$___ Worst Case: ___$\Theta(N)$___

(b)
```
public static int recurse(int N) {
    return helper(N, N / 2);
}
private static int helper(int N, int M) {
    if (N <= 1) {
        return N;
    }
    for (int i = 1; i < M; i *= 2) {
        System.out.println(i);
    }
    return helper(N - 1, M) + helper(N - 1, M);
}
```
Best Case: ___$\Theta(2^N logN)$___ Worst Case: ___$\Theta(2^N logN)$___

(c)
```
public static boolean find(int tgt, int[] arr) {
    int N = arr.length;
    return find(tgt, arr, 0, N);
}
private static boolean find(int tgt, int[] arr, int lo, int hi) {
    if (lo == hi || lo + 1 == hi) {
        return arr[lo] == tgt;
    }
    int mid = (lo + hi) / 2;
    for (int i = 0; i < mid; i += 1) {
        System.out.println(arr[i]);
    }
    return arr[mid] == tgt || find(tgt, arr, lo, mid) || find(tgt, arr,
       mid, hi);
}
```
Best Case: ___$\Theta(N)$___ Worst Case: ___$\Theta(N^2)$___

3 Bit Operations

In the following questions, use bit manipulation operations to achieve the intended functionality and fill out the function details -

(a) Implement a function `isPalidrome` which checks if the binary representation of a given number is palindrome. The function returns true if and only if the binary representation of `num` is a palindrome.

For example, the function should return true for `isPalindrome(9)` since binary representation of 9 is `1001` which is a palindrome.

```
/**
* Returns true if binary representation of num is a palindrome
*/
public static boolean isPalindrome(int num)
{
    // stores reverse of binary representation of num
    int reverse = 0;

    // do till all bits of num are processed
    int k = num;
    while (k > 0)
    {
        // add rightmost bit to reverse
        reverse = (reverse << 1) | (k & 1);
        k = k >> 1;             // drop last bit
    }
    return num == reverse;
}
```

(b) Implement a function `swap` which for a given integer, swaps two bits at given positions. The function returns the resulting integer after bit swap operation.

For example, when the function is called with inputs `swap(31, 3, 7)`, it should reverse the 3rd and 6th bit from the right and return 91 since 31 (00011111) would become 91 (01011011).

```
/**
* Function to swap bits at position a and b (from right) in integer num
*/
public static int swap(int num, int a, int b)
{
    int p = a-1;
    int q = b-1;

    int bit_a = (num >> p) & 1;
    int bit_b = (num >> q) & 1;

    if (bit_a != bit_b) {       // if the bits are different
        num ^= (1 << p);
        num ^= (1 << q);
    }
    return num;
}
```