Due April 1

1. Introductions

It's the first discussion section and the GSI is trying to come up with a clever method to make students form groups and introduce themselves to each other. As there are always shy people in the class, the GSI cannot just ask them to do these things, so s/he comes up with this method: s/he prints the names of all n students, each name on a separate card, and then distributes these cards to all students in a completely random order. So by the end, each student is holding the name of another student (or possibly his/her own name). Assume that there are no absentees, so the number of cards is equal to the number of card-holders.

Then the GSI asks students to form groups this way: each person should find the person whose name s/he holds, and then be in the same group with that person. For example if A is holding B's name, B is holding C's name and C is holding A's name, then one of the groups will be $\{A,B,C\}$.

1. Prove that in each group you can always arrange people in a way that each person holds the name of the next person, and the last person holds the name of the first person. Hint: consider starting from any person, and going to the person whose name s/he holds, and again going to the person whose name s/he holds, and so on. What happens if at the end the last person holds the name of someone other than the first person?
2. One of the students who just loves probability, wants to find out the size of the group s/he ends up in. For each $1 \leq k \leq n$, what is the probability that this particular student ends up in a group of size k? Hint: count the number of orderings which result in this person being in a group of size k.

2. Round the clock

This problem was entirely moved to the next homework.

3. Fair bet?

Your friend proposes the following game. She will roll a fair dice six times. If the number of different numbers that show up is exactly four, then you win 1 dollar. Otherwise, she wins 1 dollar. Would you bet on this game? Justify your answer with a calculation.

4. Box of marbles

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

1. If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?
2. If we see that the marble is blue, what is the probability that it is chosen from box 1?
3. Suppose we pick one marble from box 1 and without looking at its color we put it side. Then we pick another marble from box 1. What is the probability that the second marble is blue?

5. Disease diagnosis

You have a high fever and go to the doctor to identify the cause. 1% of the people have H1N1, 10% of the people have the flu, and 89% have neither. Assume that no person has both. Suppose that 100% of the H1N1 people have a high fever, 30% of the flu people have a high fever, and 2% of the people who have neither, have a high fever. Is it more likely that you have H1N1, the flu, or neither?

6. Futurama Question

There is a test to determine whether one has boneitis, but the test is not always accurate. For those who do have boneitis, the test has a 4 in 5 chance of coming out positive. For those who don't have boneitis, the test has a 1 in 9 chance of coming out positive. Overall, about 1 in 10 people have boneitis. Suppose the test comes out positive for That Guy. What is the probability That Guy has boneitis?

7. The Simpson's Question

When he's not shaping the keen young minds who will be our nation's future, Professor Sahai likes to kick back and relax with one of the many programs he's saved up in his Netflix[1] queue.

Professor Sahai classifies each program in his queue as either a comedic TV show, a dramatic TV show, a comedic movie, or a dramatic movie, with none being assigned more than one of these categories. He has at least one program from each of these categories saved up.

Furthermore, each program in his queue is either good or bad, but not both (Professor Sahai does not believe in the notion of "so bad, it's good", any more than he believes in "acceptably late homework" or "dramedy").

Alas, the professor is unable to ever completely rest his mind from probability. Recently, he excitedly noted, "It seems to me that, when I restrict myself to the TV shows in my queue, I have a better chance of watching something good if I pick a random comedy than if I pick a random drama. And it also seems to me that, when I restrict myself to the movies in my queue, I have a better chance of watching something good if I pick a random comedy than if I pick a random drama. Yet, all the same, I think I'd have a better chance of watching something good if I picked a random drama from my queue than if I picked a random comedy."

While this GSI exhaustedly explained that the professor had to stop calling so late at night, Sahai finished with this cryptic remark:

"...Or have I miscounted something? **Give either a rigorous proof that I must have made a mistake, or an explicit example of how I could be correct.**"[2]

8. Your Own Problem Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. An example is the computation of the expected number of empty bins when throwing n balls into n bins (in note 14). Of course you should not use this problem again. What is the problem? How do you formulate it? How do you approach it? What is your solution?

[1] This question is sponsored by Netflix. Watch TV shows & movies anytime, anywhere, for only $7.99 a month! Visit http://www.netflix.com today to start your 1 month free trial.

[2] After heated argument between the readers and the professor, it's been decided that it will not suffice simply to note "Professor Sahai does not make mistakes".