CS 70 Discrete Mathematics and Probability Theory
Spring 2016 Walrand and Rao

Discussion 12a

1. **Playing Pollster**

   As an expert in probability, the staff members at the Daily Californian have recruited you to help them conduct a poll to determine the percentage p of Berkeley undergraduates that plan to participate in the student sit-in. They've specified that they want your estimate $\hat{p}$ to have an error of at most ε with confidence $1-\delta$. That is,

   $$P(|\hat{p}-p| \leq \varepsilon) \geq 1-\delta.$$

   Assume that you've been given the bound

   $$P(|\hat{p}-p| \geq \varepsilon) \leq \frac{1}{4n\varepsilon^2},$$

   where n is the number of students in your poll.

   (a) Using the formula above, what is the smallest number of students n that you need to poll so that your poll has an error of at most ε with confidence $1-\delta$?

   (b) At Berkeley, there are about 26,000 undergraduates and about $10,000$ graduate students. Suppose you only want to understand the frequency of sitting-in for the undergraduates. If you want to obtain an estimate with error of at most 5% with 98% confidence, how many undergraduate students would you need to poll? Does your answer change if you instead only want to understand the frequency of sitting-in for the graduate students?

   (c) It turns out you just don't have as much time for extracurricular activities as you thought you would this semester. The writers at the Daily Californian insist that your poll results are reported with at least 95% confidence, but you only have enough time to poll 500 students. Based on the bound above, what is the worst-case error with which you can report your results?

2. **Vegas**

   On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

   (a) Given the results of your experiment, how should you estimate p?

   (b) How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

3. **Working with the Law of Large Numbers**

(a) A fair coin is tossed and you win a prize if there are more than 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed and you win a prize if there are more than 40% heads. Which is better: 10 tosses or 100 tosses? Explain.

(c) A coin is tossed and you win a prize if there are between 40% and 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(d) A coin is tossed and you win a prize if there are exactly 50% heads. Which is better: 10 tosses or 100 tosses? Explain.