CS 70 Discrete Mathematics and Probability Theory
Spring 2016 Walrand and Rao Discussion 14B

1. Chebyshev's Inequality vs. Central Limit Theorem Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with the following distribution:

$$\Pr[X_i = -1] = 1/12; \quad \Pr[X_i = 1] = 9/12; \quad \Pr[X_i = 2] = 2/12.$$

1. What are the expectations and variances of X_i, $(\sum_{i=1}^n X_i)$, $((\sum_{i=1}^n X_i) - n)$, and $\left(\frac{(\sum_{i=1}^n X_i)-n}{\sqrt{n/2}}\right)$?

2. If $Z = \frac{(\sum_{i=1}^n X_i)-n}{\sqrt{n/2}}$, use Chebyshev's Inequality to find an upper bound b for $\Pr[|Z| \geq 2]$.

3. Can you use b to bound $\Pr[Z \geq 2]$ and $\Pr[Z \leq -2]$?

4. As $n \to \infty$, what is the distribution of Z?

5. A normal distribution N has the following property: $\Pr[|N - \mu| \leq 2\sigma] \approx 0.9545$. As $n \to \infty$, can you provide approximations for $\Pr[Z \geq 2]$ and $\Pr[Z \leq -2]$?

2. The average jump of a certain frog is 3 inches. However, because of the wind, the frog does not always go exactly 3 inches. A zoologist tells you that the distance the frog travels is normally distributed with mean 3 and variance $\frac{1}{4}$. Recall that given a random variable X distributed normally with mean μ and variance σ^2, the random variable $Z = \frac{X-\mu}{\sigma}$ is distributed normally with mean 0 and variance 1. Observe that this implies that $\Pr[Z \leq z] = \Pr[X \leq \sigma z + \mu]$. Let us define the function $\Phi(z)$ as the CDF of the standard normal, i.e.,

$\Phi(z) = \Pr[Z \leq z]$ where $Z = N(0,1)$. For the following problems, write your answer in terms of $\Phi(z)$. Then use the table on the back of this sheet to get a numerical answer.

1. What is the probability that the frog jumps more than 4 inches?

2. What is the probability that the distance the frog jumps is between 2 and 4 inches?

3. The Cal basketball team plays 100 independent games, each of which they have probability 0.8 of winning. Use the Central Limit Theorem and the table on the back of this sheet to estimate the probability that they win at least 90 games.

Here is a table of approximations of the CDF of the standard normal distribution. To look up the Pr $\left[N(0,1) \leq x\right]$, you look up the row of the first two digits of x and then the column of the third digit. For example, to look up $x = 1.34$, first find the row labeled 1.3 and then the column 0.04. The result is the entry in that cell, 0.9099.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990