1. Set Operations

- $\mathbb{N}$ denotes the set of all natural numbers: $\{0,1,2,3,...\}$.
- $\mathbb{Z}$ denotes the set of all integer numbers: $\{\ldots,-2,-1,0,1,2,\ldots\}$.
- $\mathbb{Q}$ denotes the set of all rational numbers: $\{\frac{a}{b} \mid a,b \in \mathbb{Z}, b \neq 0\}$.
- $\mathbb{R}$ denotes the set of all real numbers.

(a) $\mathbb{N} \cap \mathbb{Z} =$

(b) $\mathbb{Z} \cap \mathbb{Q} =$

(c) $\mathbb{Q} \cap \mathbb{R} =$

(d) $\mathbb{N} \cup \mathbb{Z} =$

(e) $\mathbb{Z} \cup \mathbb{Q} =$

(f) $\mathbb{Q} \cup \mathbb{R} =$

(g) $\mathbb{N} \setminus \mathbb{Z} =$

(h) $\mathbb{Z} \setminus \mathbb{N} =$

(i) $\mathbb{Z} \setminus \mathbb{Q} =$

(j) $\mathbb{Q} \setminus \mathbb{Z} =$

(k) $\mathbb{Q} \setminus \mathbb{R} =$

(l) $\mathbb{R} \setminus \mathbb{Q} =$

2. Summation and Product

(a) $\sum_{i=1}^{3} i =$

(b) $\prod_{i=1}^{3} i =$

(c) $\sum_{i=1}^{3} \sum_{j=1}^{3} j =$

(d) $\prod_{i=1}^{3} \sum_{j=1}^{i} j =$

3. (Truth table) Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

4. (Writing in propositional logic.)

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

1. The square of a nonzero integer is positive.
2. There are no integer solutions to the equation $x^2 - y^2 = 10$.

3. There is one and only one real solution to the equation $x^3 + x + 1 = 0$.
4. For any two distinct real numbers, we can find a rational number in between them.

5. (Implication) Which of the following implications are true? Give a counterexample for each false assertion.

1. $\forall x, \forall y, P(x,y) \implies \forall y, \forall x, P(x,y)$.
2. $\exists x, \exists y, P(x,y) \implies \exists y, \exists x, P(x,y)$.
3. $\forall x, \exists y, P(x,y) \implies \exists y, \forall x, P(x,y)$.
4. $\exists x, \forall y, P(x,y) \implies \forall y, \exists x, P(x,y)$.