1. Let $P(x)$ and $Q(x)$ denote some propositions involving x. For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

(a) $\forall x\,(P(x) \vee Q(x))$ is equivalent to $(\forall x, P(x)) \vee (\forall x, Q(x))$.

(b) $\forall x\,(P(x) \wedge Q(x))$ is equivalent to $(\forall x, P(x)) \wedge (\forall x, Q(x))$.

(c) $\exists x\,(P(x) \vee Q(x))$ is equivalent to $(\exists x, P(x)) \vee (\exists x, Q(x))$.

(d) $\exists x\,(P(x) \wedge Q(x))$ is equivalent to $(\exists x, P(x)) \wedge (\exists x, Q(x))$.

2. A *perfect square* is an integer n of the form $n = m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.

3. Prove that $2^{1/n}$ is not rational for any integer $n > 3$. [Hint : Fermat's Last Theorem]

4. Prove that if you put $n+1$ apples into n boxes, any way you like, then at least one box must contain at least 2 apples. This is known as the *pigeonhole principle*.

5. Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.