1. **Tournament**

A *tournament* is defined to be a directed graph such that for every pair of distinct nodes v and w, exactly one of (v,w) and (w,v) is an edge (representing which player beat the other in a round-robin tournament). Prove that every tournament has a Hamiltonian path. In other words, you can always arrange the players in a line so that each player beats the next player in the line.

2. **Leaves in a tree**

A *leaf* in a tree is a vertex with degree 1.

(a) Prove that every tree on $n \geq 2$ vertices has at least two leaves.

(b) What is the maximum number of leaves in a tree with $n \geq 3$ vertices?

3. Edge-disjoint paths in hypercube

Prove that between any two distinct vertices x, y in the n-dimensional hypercube graph, there are at least n edge-disjoint paths from x to y (i.e., no two paths share an edge, though they may share vertices).

4. Planarity

Consider graphs with the property T: For every three distinct vertices v_1, v_2, v_3 of graph G, there are at least two edges among them. Prove that if G is a graph on ≥ 7 vertices, and G has property T, then G is nonplanar.

5. Graph Coloring

Prove that a graph with maximum degree at most k is $(k+1)$-colorable.