CS 70   Discrete Mathematics and Probability Theory
Spring 2016   Rao and Walrand   Discussion 7A

1. **Review of Halting Problem**

   Ask your TA any question about the Halting Problem.

2. **Code reachability is impossible**

   Consider triplets (M, x, L) where

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   M is a java program
   x is some input
   L is an integer
   ```

   and the question of: if we execute $M(x)$, do we ever hit line L?

   Prove this problem is undecidable.

3. **Compute this**

   (a) Can you write a program that gets n (a natural number) as input and finds the shortest formula that computes n? A formula is a valid sequence consisting of decimal digits, the operators $+$, $\times$, ^ (raising to the power), and parentheses. The length of a formula is simply the number of characters you need to use to type it (i.e. each operator, decimal digit, or paranthesis counts as one character).

   (b) Now assume that you want to write a computer program that given the input n (a natural number) finds another computer program (in a specific language, e.g. C or Python) that prints n. The program that is found has to have the minimum length plus execution time amongst all programs that print n, where length is measured by the number of characters in the source code and execution time is measured by a concrete number such as the number of CPU instructions executed. Can this be done?

   (c) Consider the set of programs (or functions) that take a single natural number n as input and output a natural number in at most $10^6 + 2^n$ steps (i.e. they always terminate after $10^6 + 2^n$ steps). Let this set be L. A member of L is called **thorough** if every natural number m can be produced as its output (by an appropriate input). As an example, a member of L that always returns values mod n (for some n in natural numbers) would not be thorough. Can you write a program that takes a member of L as input and determines whether that member is thorough? The given member of L is guaranteed to be in L, there is no need for your program to verify the membership.

   (HINT: If you had such a program, could you somehow use it to solve the halting problem? If so, what would that mean?)