

1. Sock etiquette

In your second week of Charm School you learn that you should only wear matching pair of socks. In each pair, both socks must be of the same color and pattern. But all of them are in one big basket and now you have to take a pair out. Let's say you own n pairs of socks which are all perfectly distinguishable (no two pairs have the same color and pattern). You are now randomly picking one sock after the other without looking at which one you pick.

- (a) How many distinct subsets of k socks are there?
- (b) How many distinct subsets of k socks which do not contain a pair are there?
- (c) What is the probability of forming at least one pair when picking k socks out of the basket?
- (d) Now, in a different experiment, suppose there is exactly one sock of each pair in the basket (so there are n socks in the basket) and we sample (with replacement) k socks from the basket. What is the probability that we pick the same sock at least twice in the course of the experiment?

2. Company Selection

Company A produces a particular device consisting of 10 components. Company A can either buy all the components from Company S or Company T, and then uses them to produce the devices without testing every individual component. After that, each device will be tested before leaving the factory. The device works only if every component works properly. Each working device can be sold for x dollars, but each non-working device must be thrown away. Products from Company S have a failure probability of $q = 0.01$ while Company T has a failure probability of $q/2$. However, every component from Company S costs \$10 while it costs \$30 from Company T. Should Company A build the device with components from Company S or Company T in order to maximize its expected profit per device? (Your answer will depend on x .)

3. Dinner Time

Now let's move on to the actual dinner. Each person has all sorts of plates, flatwares, and glasses in front of them, as shown in figure 1.¹ The basic rule is to start using utensils furthest from your plate and end with the closest ones. Table 1 lists the menu and the corresponding utensils.

- (a) Ron is confused what utensils to use (*'Wait, I think I'm at the wrong Charm School.'*). Fortunately, he can wait for his server to select the right plates and glasses. He just needs to pick flatwares. All he sees are, 4 forks (B, C, D, and N), 3 knives (H, I, and L), and 2 spoons (J and

¹Source: <http://damoneroberts.tumblr.com/post/51078389219/home-tip-of-the-day-proper-place-setting>

Menu	Plates	Flatwares	Glasses
Water	-	-	O
Red wine	-	-	P
White wine	-	-	Q
Bread	K	L	-
Soup	-	J	-
Salad	E	B	-
Fish	F	C, I	-
Meat	G	D, H	-
Dessert	-	M, N	-

Table 1: Courses and utensils



Figure 1: Formal dinner setting

M). So, for each course served, he mimics what other people are using. For example, if other people are using a fork and a knife, he picks one fork and one knife. (He can't tell the difference between each fork, but can separate forks from knives and spoons just fine.) What is the probability he uses all utensils correctly? Each utensil is collected after each course and can't be used twice.

- (b) Luna just doesn't care. For each course she just picks one or two random flatwares so that all of them are used at the end, and forces the server to serve on one random plate. For each drink she picks a random glass. What is the probability she used at least two things wrong? (If a utensil isn't used in the course it is matched with, then it is used wrongly.)
- (c) (*Optional*) What is the probability Hermione used all correct plates, flatwares, and glasses?