Bayes' Rule: $P(A|B) = P(A \cap B)/P(B)$

Total probability rule: $P(A) = P(A \cap B) + P(A \cap \bar{B})$

Independence: $P(A \cap B) = P(A) * P(B)$

1. **Probability Practice**

   1. If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?

   2. A message source M of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0, 1, 2\}$, and all such words are equally probable. What is the probability that M produces a word that looks like a byte (*i.e.*, no appearance of '2')?

   3. If five numbers are selected at random from the set $\{1, 2, 3, \ldots, 20\}$ with replacement, what is the probability that their minimum is larger than 5?

2. **Disease diagnosis**

   You have a high fever and go to the doctor to identify the cause. 1% of the people have H1N1, 10% of the people have the flu, and 89% have neither. Assume that no person has both. Suppose that 100% of the H1N1 people have a high fever, 30% of the flu people have a high fever, and 2% of the people who have neither, have a high fever. Is it more likely that you have H1N1, the flu, or neither?

3. **Company Selection**

   Company A produces a particular device consisting of 10 components. Company A can either buy all the components from Company S or Company T, and then uses them to produce the devices without testing every individual component. After that, each device will be tested before leaving the factory. The device works only if every component works properly. Each working device can be sold for x dollars, but each non-working device must be thrown away. Products from Company S have a failure probability of $q = 0.01$ while Company T has a failure probability of $q/2$. However, every component from Company S costs \$10 while it costs \$30 from Company T. Should Company A build the device with components from Company S or Company T in order to maximize its expected profit per device? (Your answer will depend on x.)