Lecture 14

What’s to come?

Lecture 14

What’s to come? Probability.

Lecture 14

What's to come? Probability.

A bag contains:

Lecture 14

What's to come? Probability.

A bag contains:

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Lecture 14

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A bag contains:

What is the chance that a ball taken from the bag is blue?

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Today:

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

Later: Probability.

Lecture 14

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

Later: Probability. Professor Walrand.

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..

How many 3-bit strings?

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How many 3-bit strings?

How many different sequences of three bits from $\{0, 1\}$?

Using a tree..

How many 3-bit strings?
  How many different sequences of three bits from $\{0, 1\}$?
  How would you make one sequence?

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How would you make one sequence?

How many different ways to do that making?

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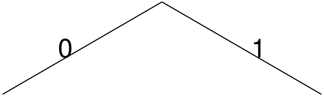

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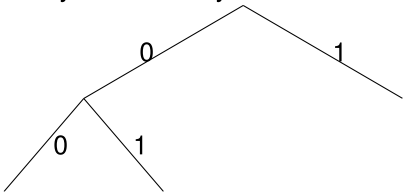

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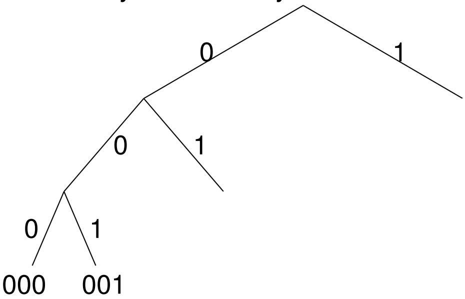

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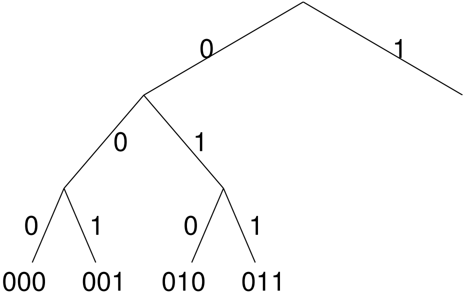

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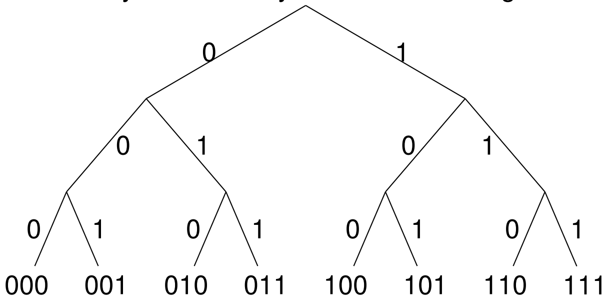

8 leaves which is $2 \times 2 \times 2$.

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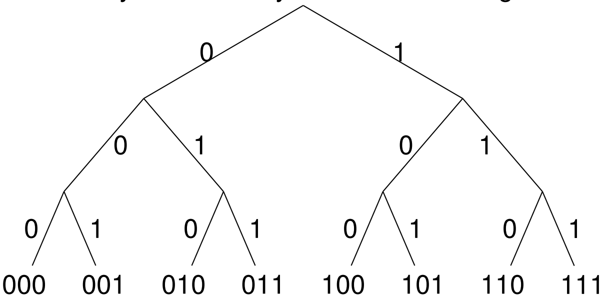

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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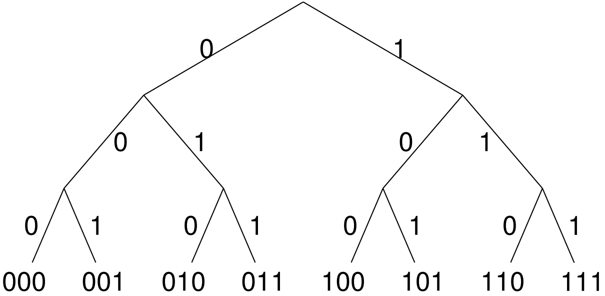

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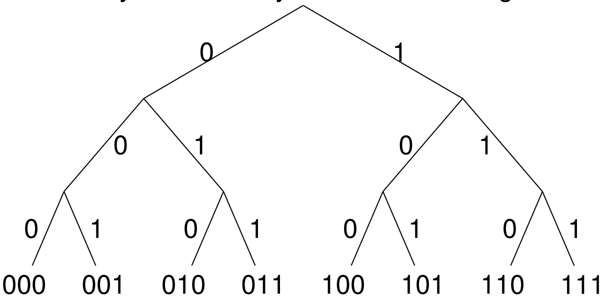

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8 3-bit srings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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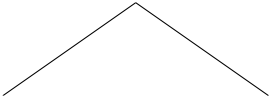

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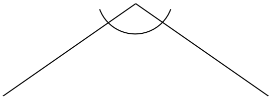

n_1

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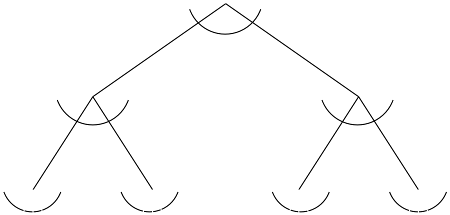

n_1

$\times n_2$

$\times n_3$

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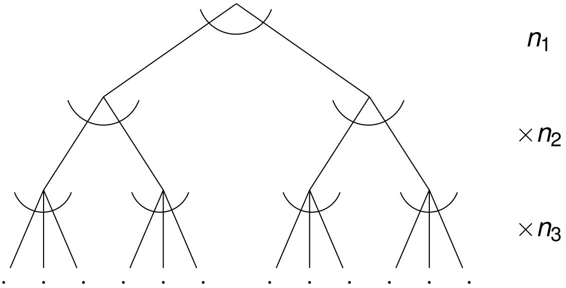

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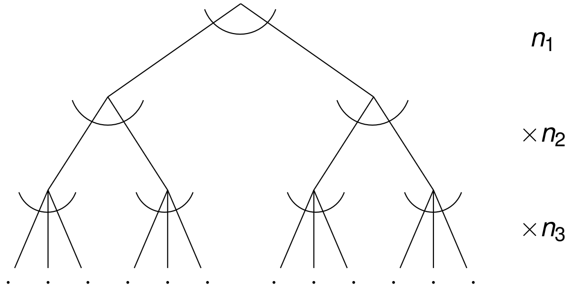

In picture, $2 \times 2 \times 3 = 12$!

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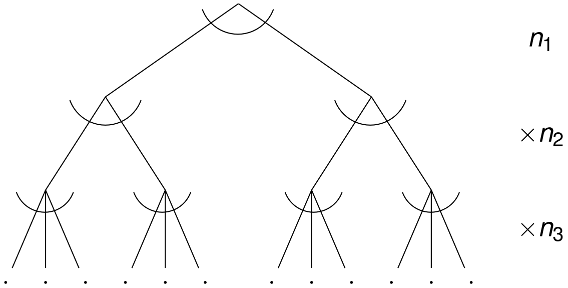

In picture, $2 \times 2 \times 3 = 12$!

Using the first rule..

How many outcomes possible for k coin tosses?

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2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

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Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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How many 10 digit numbers?

Using the first rule..

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10 ways for first choice, 10 ways for second choice, ...
$10 \times$

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Functions, polynomials.

How many functions f mapping S to T?

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p ways to choose for first coefficient,

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Permutations.

[1] By definition: $0! = 1$.

Permutations.

How many 10 digit numbers **without repeating a digit**?

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second,

---

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How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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One-to-One Functions.

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How many one-to-one functions from $|S|$ to $|S|$.

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One-to-One Functions.

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$|S|$ choices for $f(s_1)$, $|S|-1$ choices for $f(s_2)$, ...

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$|S|$ choices for $f(s_1)$, $|S|-1$ choices for $f(s_2)$, ...

So total number is $|S| \times |S|-1 \cdots 1 = |S|!$

One-to-One Functions.

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$|S|$ choices for $f(s_1)$, $|S|-1$ choices for $f(s_2)$, ...

So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

---

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

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Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: $5!$.

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: 5!.

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: $5!$.

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: 5!.

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

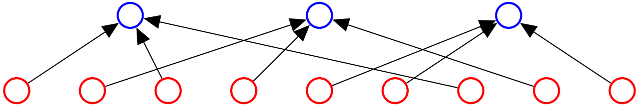
[2]When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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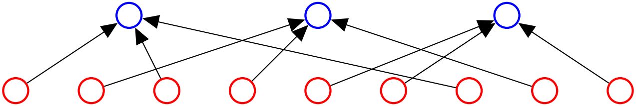

How many red nodes (ordered objects)?

Ordered to unordered.

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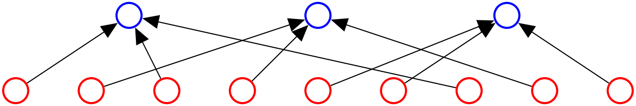
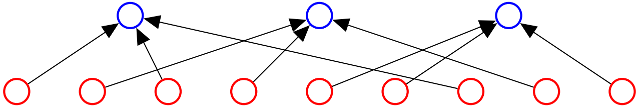


How many red nodes (ordered objects)? 9.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

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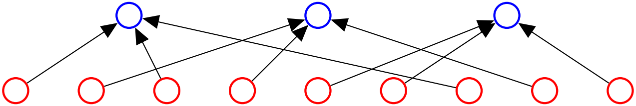

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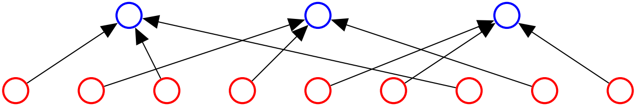

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How many blue nodes (unordered objects)?

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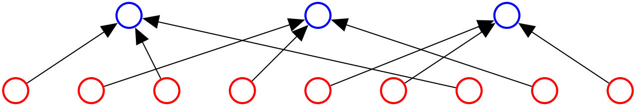

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How many blue nodes (unordered objects)? $\frac{9}{3}$

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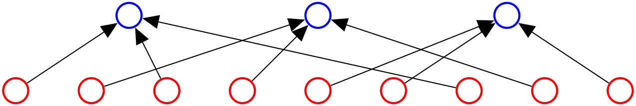

How many red nodes (ordered objects)? 9.

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Ordered to unordered.

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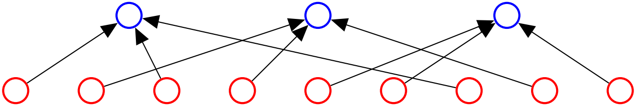

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Ordered to unordered.

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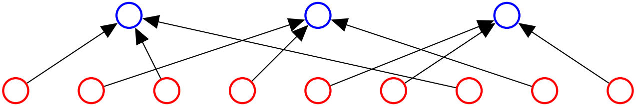

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

Ordered to unordered.

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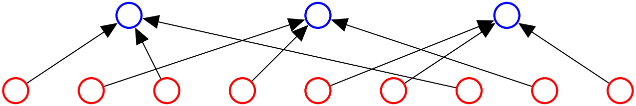

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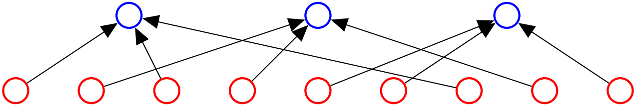
How many poker hands per deal?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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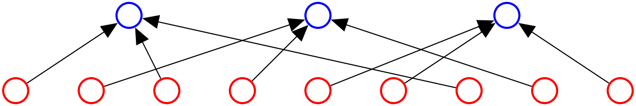
How many poker hands per deal? Map each deal to ordered deal.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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How many poker hands per deal? Map each deal to ordered deal. $5!$

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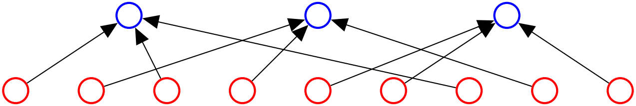

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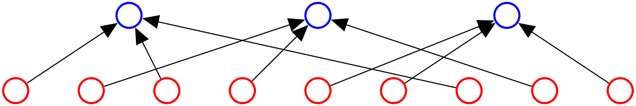

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How many poker hands? $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$

..order doesn’t matter.

..order doesn't matter.

Choose 2 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\underline{n \times (n-1)}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of n?

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$$\frac{n \times (n-1) \times (n-2)}{}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

..order doesn't matter.

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Choose k **out of** n?

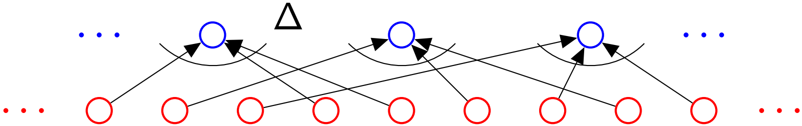
$$\frac{n!}{(n-k)! \times k!}$$

Notation: $\binom{n}{k}$ and pronounced "n choose k."

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

Second rule: when order doesn't matter divide..when possible.



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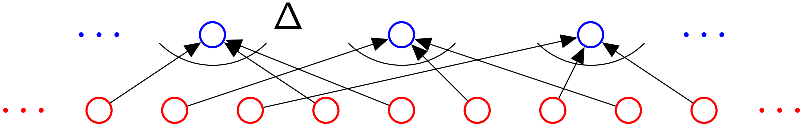

3 card Poker deals: 52

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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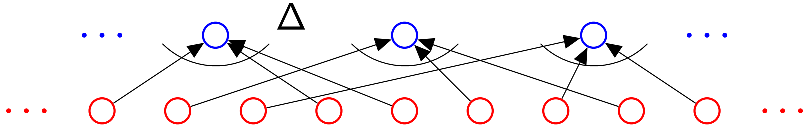

3 card Poker deals: 52×51

Example: Visualize the proof..

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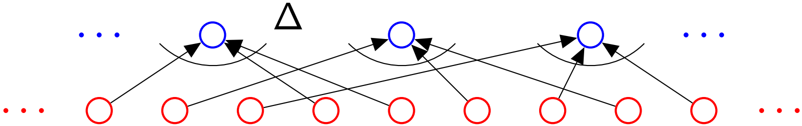

3 card Poker deals: $52 \times 51 \times 50$

Example: Visualize the proof..

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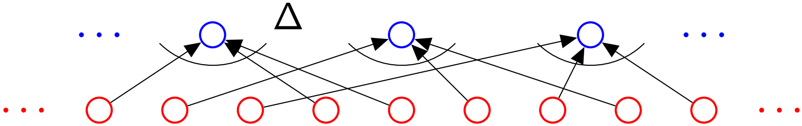

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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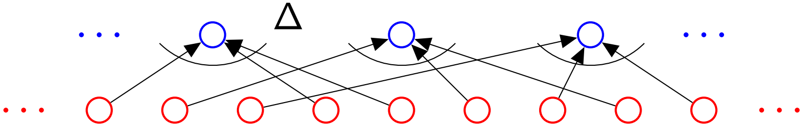

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

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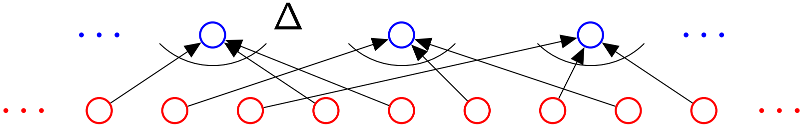

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

Example: Visualize the proof..

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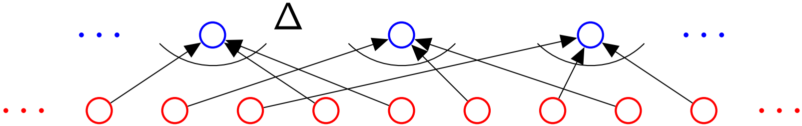

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.

Example: Visualize the proof..

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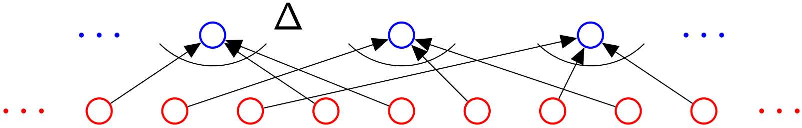

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  Hand: Q, K, A.
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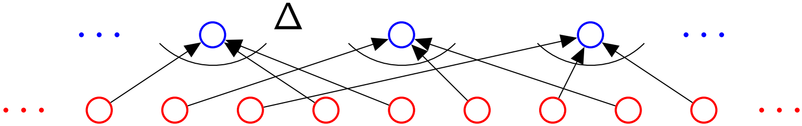

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Poker hands: Δ?

Hand: Q, K, A.

Deals: Q, K, A, Q, A, K,

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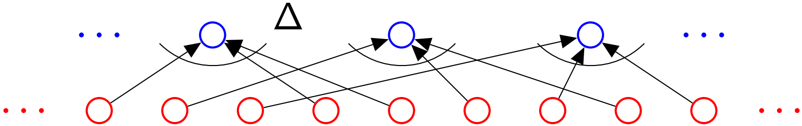

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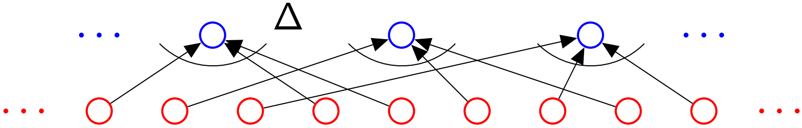

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.
Deals: Q, K, A, Q, A, K, K, A, Q, K, A, Q, A, K, Q, A, Q, K.
$\Delta = 3 \times 2 \times 1$

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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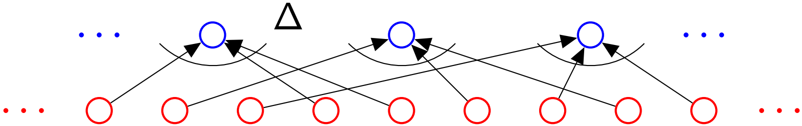

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Poker hands: Δ?

Hand: Q, K, A.

Deals: Q,K,A, Q,A,K, K,A,Q, K,A,Q, A,K,Q, A,Q,K.

$\Delta = 3 \times 2 \times 1$ First rule again.

Example: Visualize the proof..

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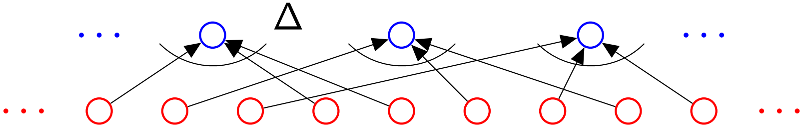

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Total:

Example: Visualize the proof..

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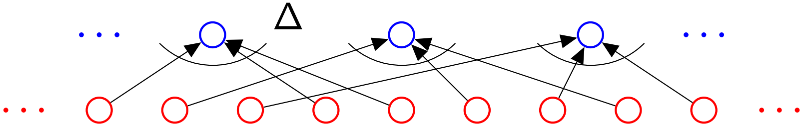

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  Hand: Q, K, A.
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$\Delta = 3 \times 2 \times 1$ First rule again.
Total: $\frac{52!}{49!3!}$

Example: Visualize the proof..

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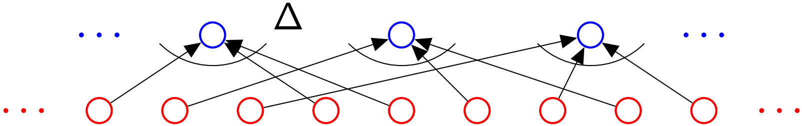
Second rule: when order doesn't matter divide..when possible.



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Poker hands: Δ?

Hand: Q, K, A.

Deals: Q, K, A, Q, A, K, K, A, Q, K, A, Q, A, K, Q, A, Q, K.

$\Delta = 3 \times 2 \times 1$ First rule again.
Total: $\frac{52!}{49!3!}$ Second Rule!

Example: Visualize the proof..

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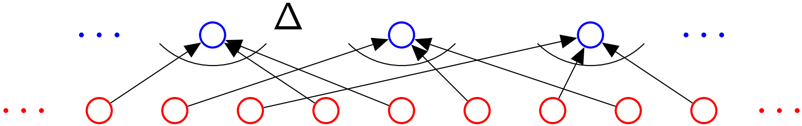

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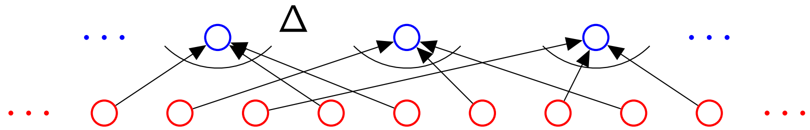

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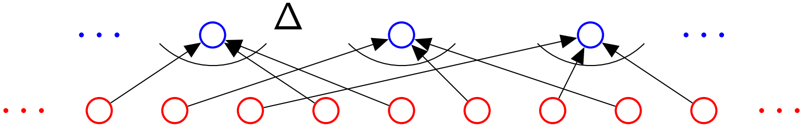

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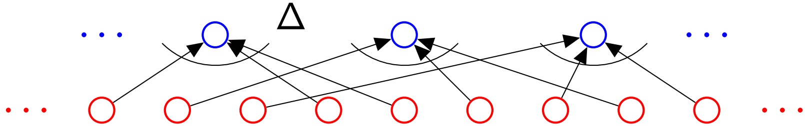

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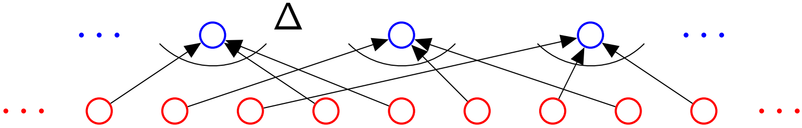

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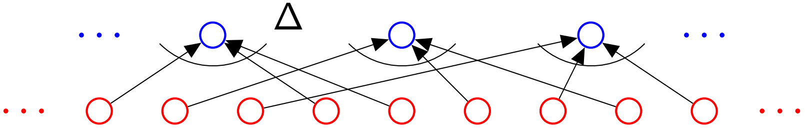

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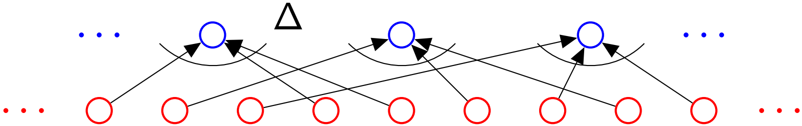

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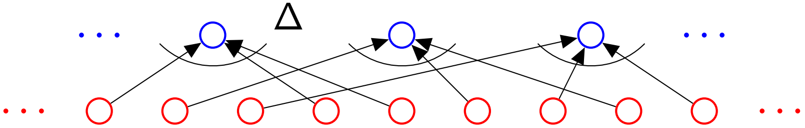

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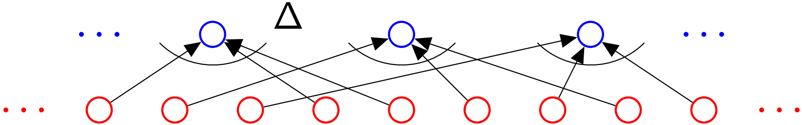

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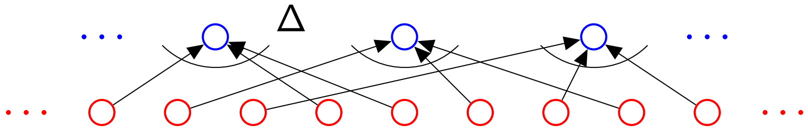

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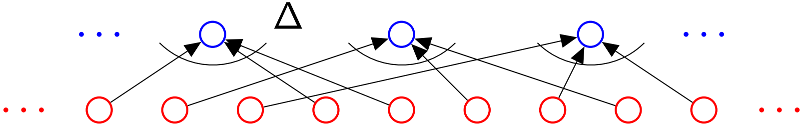

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Ordered Set: 7!

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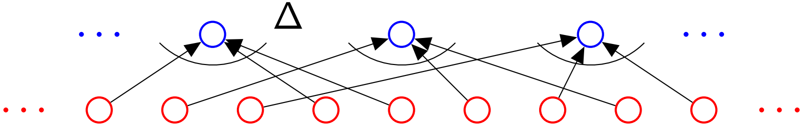

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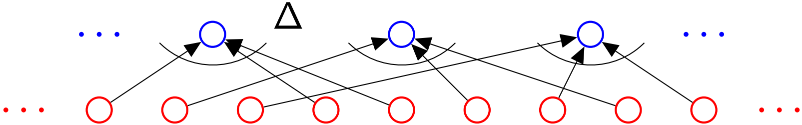

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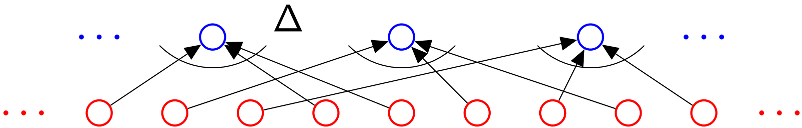

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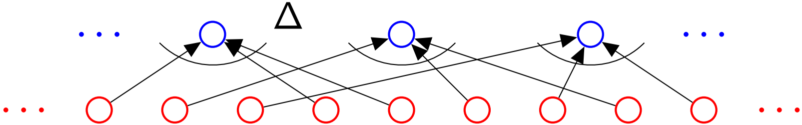

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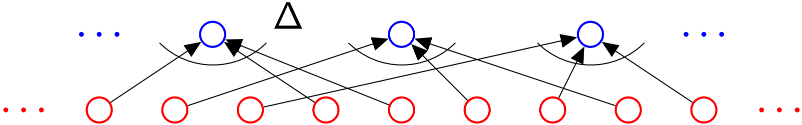

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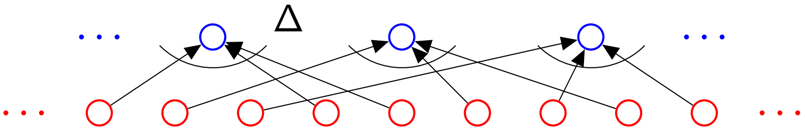

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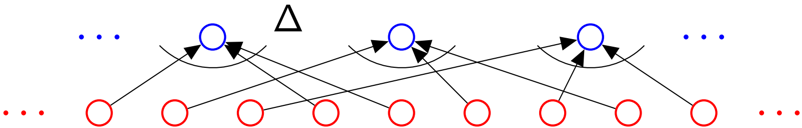

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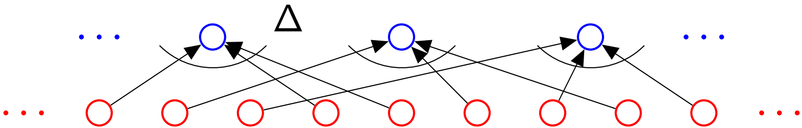

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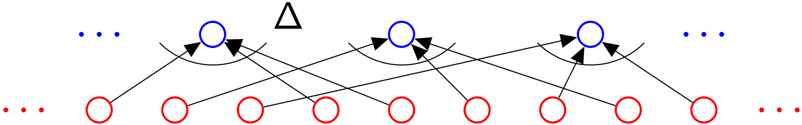

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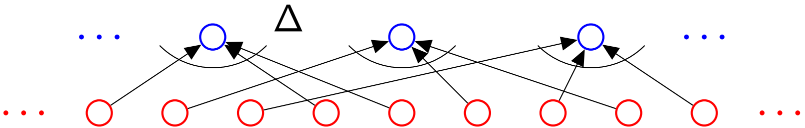

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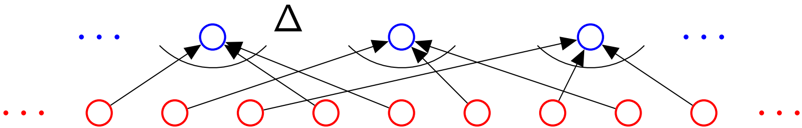

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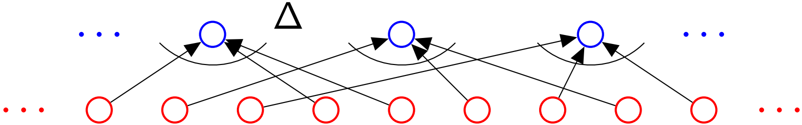

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Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways to choose second, 1 for last.

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How many orderings of MISSISSIPPI?

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11 letters total!

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11! ordered objects!

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Sample k items out of n

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Without replacement:

Sampling...

Sample k items out of n

Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

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With Replacement.

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Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

$\implies \frac{n!}{(n-k)!k!}$.

"n choose k"

With Replacement.

Order matters: $n \times n$

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Unordered elt: $1, 2, 3$

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How do we deal with this mess?!?!

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B)
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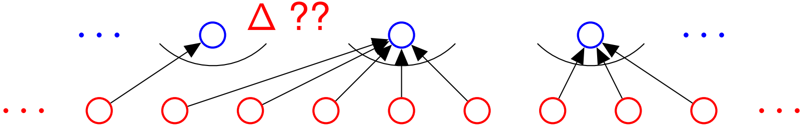

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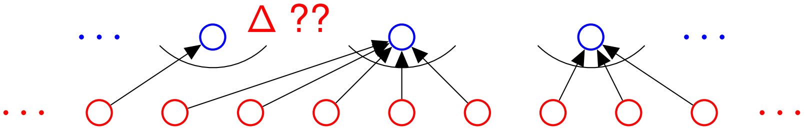

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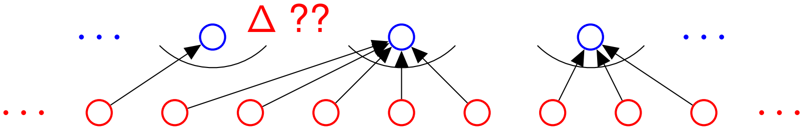
(B, B, B, B, B)
(A, B, B, B, B) (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B)

and so on.



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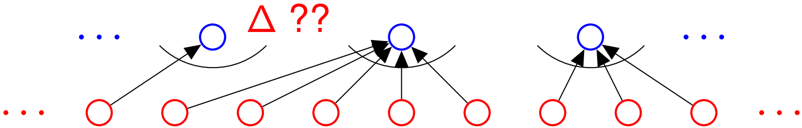

Second rule of counting is no good here!

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

Stars and Bars.

Ways to add up n numbers to sum to k?

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Ways to add up n numbers to sum to k? or

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$n+k-1$ positions from which to choose $n-1$ bar positions.

$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n}$.

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5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

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Example: Poker hands.

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Example: Poker hands.

5 indistinguishable balls into 3 bins

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5 balls into 10 bins
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5 indistinguishable balls into 52 bins only one ball in each bin
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Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order

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Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order
Dividing 5 dollars among Alice, Bob and Eve.

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?

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How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

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Above is combinatorial proof.

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Choosing a subset of size k is same

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$\square$

Pascal's Triangle

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0

1 1

Pascal's Triangle

$$\begin{array}{ccccc} & & 0 & & \\ & 1 & & 1 & \\ 1 & & 2 & & 1 \end{array}$$

Pascal's Triangle

$$\begin{array}{ccccccc} & & & 0 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

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				0				
			1		1			
		1		2		1		
	1		3		3		1	
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Pascal's Triangle

$$\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
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Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
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Foil (4 terms)

Pascal's Triangle

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Foil (4 terms) on steroids:

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\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
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2^n terms:

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
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Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

Pascal's Triangle

$$\begin{array}{ccccccccc} &&&& 0 &&&& \\ &&& 1 && 1 &&& \\ && 1 && 2 && 1 && \\ & 1 && \color{blue}{3} && \color{blue}{3} && 1 & \\ 1 && 4 && \color{blue}{6} && 4 && 1 \end{array}$$

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$$\begin{array}{ccccccc} & & & \binom{0}{0} & & & \\ & & \binom{1}{0} & & \binom{1}{1} & & \\ & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{array}$$

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

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Pascal's rule $\implies \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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$n_1 \times n_2 \times \cdots \times n_k$ objects.

Second Rule of counting: If order does not matter.
Count with order. Divide by number of orderings/sorted object.
Typically: $\binom{n}{k}$.

Stars and Bars: Sample k objects with replacement from n.
Order doesn't matter.
Typically: $\binom{n+k-1}{k-1}$.

Inclusion/Exclusion: two sets of objects.
Add number of each subtract intersection of sets.
Sum Rule: If disjoint just add.

Combinatorial Proofs: Identity from counting same in two ways.
Pascal's Triangle Example: $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$.
RHS: Number of subsets of $n+1$ items size k.
LHS: $\binom{n}{k-1}$ counts subsets of $n+1$ items with first item.
$\binom{n}{k}$ counts subsets of $n+1$ items without first item.

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Disjoint – so add!