Today.

Finish Counting.

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...and then Professor Walrand.

But first..

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Alice: 0, Bob: 1, Eve: 4.

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Each split "is" a sequence of stars and bars.

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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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7 positions in which to place the 2 bars.

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Bars in first and third position.

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$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

6 or 7???

An alternative counting.

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$\star \quad \star \ \star \ \star \ \star$

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Ways to add up n numbers to sum to k?

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In general, k stars $n-1$ bars.

$$\star\star|\star|\cdots|\star\star.$$

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$$\binom{n+k-1}{n-1}$$

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$n+k-1$ positions from which to choose $n-1$ bar positions.

$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Quick review of the basics.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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k Samples with replacement from n items: n^k.

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Sample without replacement and order doesn't matter: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.
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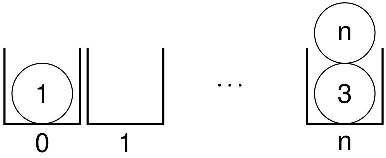
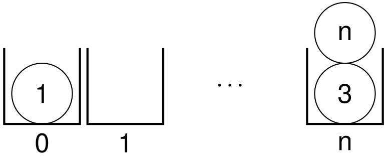
Sample with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Balls in bins.

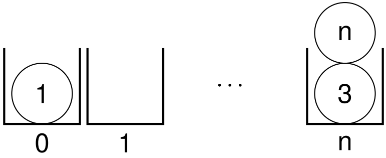


Balls in bins.



"*k* Balls in *n* bins" $\equiv$ "*k* samples from *n* possibilities."

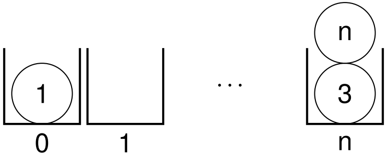
Balls in bins.



"k Balls in n bins" $\equiv$ "k samples from n possibilities."

"indistinguishable balls" $\equiv$ "order doesn't matter"

Balls in bins.

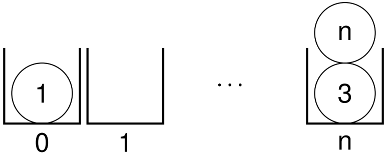


"*k* Balls in *n* bins" $\equiv$ "*k* samples from *n* possibilities."

"indistinguishable balls" $\equiv$ "order doesn't matter"

"only one ball in each bin" $\equiv$ "without replacement"

Balls in bins.



"k Balls in n bins" $\equiv$ "k samples from n possibilities."

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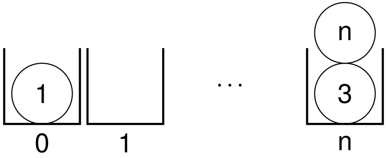
5 balls into 10 bins

Balls in bins.



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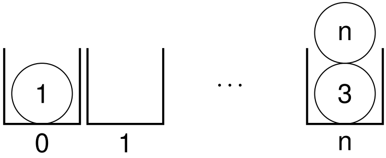
5 balls into 10 bins
5 samples from 10 possibilities with replacement

Balls in bins.



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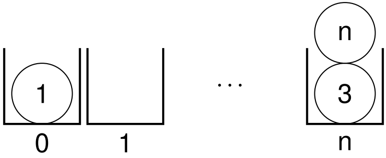
5 balls into 10 bins
5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

Balls in bins.



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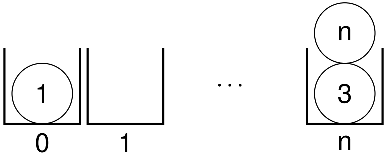
5 balls into 10 bins
5 samples from 10 possibilities with replacement
  Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin

Balls in bins.



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5 balls into 10 bins
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Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
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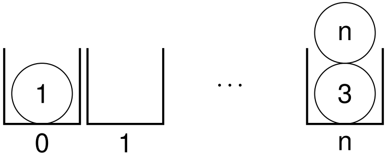
Balls in bins.



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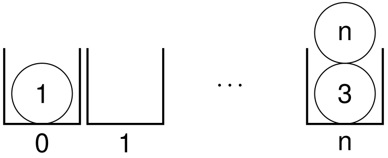
5 balls into 10 bins
5 samples from 10 possibilities with replacement
 Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
 Example: Poker hands.

Balls in bins.



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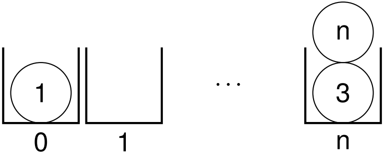
5 balls into 10 bins
5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
Example: Poker hands.

5 indistinguishable balls into 3 bins

Balls in bins.



"k Balls in n bins" $\equiv$ "k samples from n possibilities."

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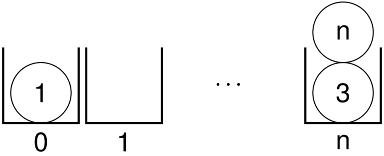
"only one ball in each bin" $\equiv$ "without replacement"

5 balls into 10 bins
5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order

Balls in bins.



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5 balls into 10 bins
5 samples from 10 possibilities with replacement
  Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
  Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order
  Dividing 5 dollars among Alice, Bob and Eve.

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?

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Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?
Sum rule: Can sum over disjoint sets.

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Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?
Sum rule: Can sum over disjoint sets.
No jokers

$$\binom{52}{5}$$

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?
Sum rule: Can sum over disjoint sets.
No jokers "exclusive" or One Joker

$$\binom{52}{5} + \binom{52}{4}$$

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?
Sum rule: Can sum over disjoint sets.
No jokers “exclusive” or One Joker “exclusive” or Two Jokers

$$\binom{52}{5} + \binom{52}{4} + \binom{52}{3}.$$

Sum Rule

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$$\binom{52}{5} + \binom{52}{4} + \binom{52}{3}.$$

Two distinguishable jokers in 54 card deck.
How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

$$\binom{52}{5} + 2 * \binom{52}{4} +$$

Sum Rule

Two indistinguishable jokers in 54 card deck.
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Wait a minute!

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Wait a minute! Same as choosing 5 cards from 54 or

$$\binom{54}{5}$$

Theorem: $\binom{54}{5}$

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Two distinguishable jokers in 54 card deck.
How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

$$\binom{52}{5} + 2 * \binom{52}{4} + \binom{52}{3}$$

Wait a minute! Same as choosing 5 cards from 54 or

$$\binom{54}{5}$$

Theorem: $\binom{54}{5} = \binom{52}{5} + 2 * \binom{52}{4} + \binom{52}{3}$.

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

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Proof: How many subsets of size k?

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Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$
and what's left out

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$
and what's left out is a subset of size k.

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$
and what's left out is a subset of size k.
Choosing a subset of size k is same

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$
and what's left out is a subset of size k.
Choosing a subset of size k is same
as choosing $n-k$ elements to not take.

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
Choose a subset of size $n-k$
and what's left out is a subset of size k.
Choosing a subset of size k is same
as choosing $n-k$ elements to not take.
$\Longrightarrow$ $\binom{n}{n-k}$ subsets of size k.

Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

Proof: How many subsets of size k? $\binom{n}{k}$

How many subsets of size k?
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and what's left out is a subset of size k.
Choosing a subset of size k is same
as choosing $n-k$ elements to not take.
$\implies \binom{n}{n-k}$ subsets of size k.

$\square$

Pascal’s Triangle

Pascal's Triangle

1

1 1

Pascal's Triangle

		1		
	1		1	
1		2		1

Pascal's Triangle

			1			
		1		1		
	1		2		1	
1		3		3		1

Pascal's Triangle

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms)

Pascal's Triangle

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:

2^n terms:

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

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Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

Simplify: collect all terms corresponding to x^k.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

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Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

Simplify: collect all terms corresponding to x^k.
Coefficient of x^k is $\binom{n}{k}$: choose k factors where x is in product.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:
2^n terms: choose 1 or x froom each factor of $(1+x)$.

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$$\begin{array}{ccc} & \binom{0}{0} & \\ \binom{1}{0} & & \binom{1}{1} \end{array}$$

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

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Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

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  Coefficient of x^k is $\binom{n}{k}$: choose k factors where x is in product.

$$\begin{array}{ccccccc} & & & \binom{0}{0} & & & \\ & & \binom{1}{0} & & \binom{1}{1} & & \\ & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{array}$$

Pascal's rule $\implies \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.
Proof: How many size k subsets of $n+1$?

Combinatorial Proofs.

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Proof: How many size k subsets of $n+1$? $\binom{n+1}{k}$.

How many size k subsets of $n+1$?
How many contain the first element?

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

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How many size k subsets of $n+1$?
How many contain the first element?
Chose first element,

Combinatorial Proofs.

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Proof: How many size k subsets of $n+1$? $\binom{n+1}{k}$.

How many size k subsets of $n+1$?
How many contain the first element?
Chose first element, need to choose $k-1$ more from remaining n elements.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Proof: How many size k subsets of $n+1$? $\binom{n+1}{k}$.

How many size k subsets of $n+1$?
How many contain the first element?
Chose first element, need to choose $k-1$ more from remaining n elements.
$\implies \binom{n}{k-1}$

Combinatorial Proofs.

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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