CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Add $99a + 11b$ to both sides.

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Direct proof of $P \implies Q$:
Assumed P: $11|a-b+c$. Proved Q: $11|n$.

The Converse

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Proof: Assume $\neg Q$: d is even.

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Contrapositive: True $\implies P$.

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Did we prove?

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- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
-
$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}}$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

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Thus, we have irrational x and y with a rational x^y (i.e., 2).

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One of the cases is true so theorem holds. □

Question: Which case holds?

Proof by cases.

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One of the cases is true so theorem holds. □

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

Be careful.

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Proof: Assume $3 = 4$.

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Start with $12 = 12$.

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Divide one side by 3 and the other by 4 to get
$4 = 3$.

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By commutativity

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Be careful.

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Proof: Assume $3 = 4$.

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Divide one side by 3 and the other by 4 to get
$4 = 3$.

By commutativity theorem holds. $\square$

Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
Proof:

Be really careful!

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Proof: For $x = y$, we have

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Theorem: $1 = 2$
Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$
$$x(x - y) = (x + y)(x - y)$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \end{aligned}$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x-y) &= (x+y)(x-y) \\
x &= (x+y) \\
x &= 2x
\end{aligned}
$$

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□

Dividing by zero is no good.

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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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To Prove: $P \implies Q$.

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Direct Proof:
To Prove: $P \implies Q$. Assume P.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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By Contraposition:

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By Contraposition:
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Summary: Note 2.

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To Prove: P

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Don't assume the theorem.

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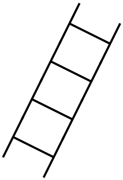
And finally. Have a nice weekend!!

CS70: Note 3. Induction!

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

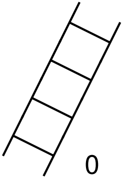
The naturals.

The naturals.



The naturals.

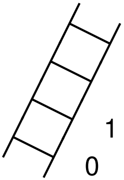
0,



0

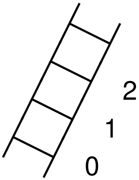
The naturals.

0, 1,



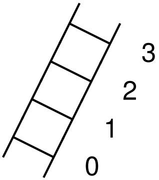
The naturals.

0, 1, 2,



The naturals.

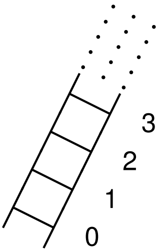
0, 1, 2, 3,



The naturals.

0, 1, 2, 3,

. . . ,



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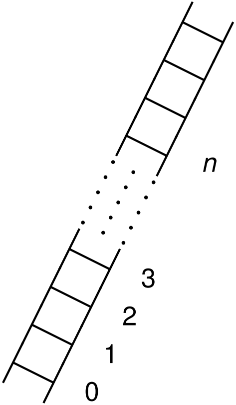
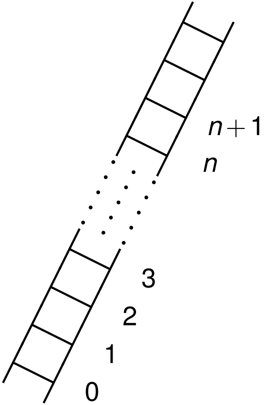


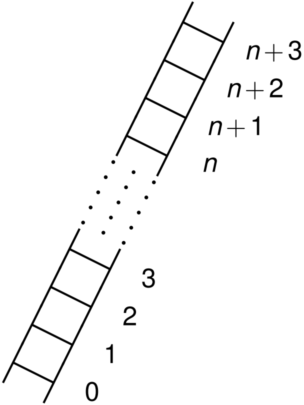
0, 1, 2, 3,
$\ldots, n,$

The naturals.



$0, 1, 2, 3,$
$\ldots, n, n+1,$

The naturals.



0, 1, 2, 3,

$\ldots, n, n+1, n+2, n+3,$

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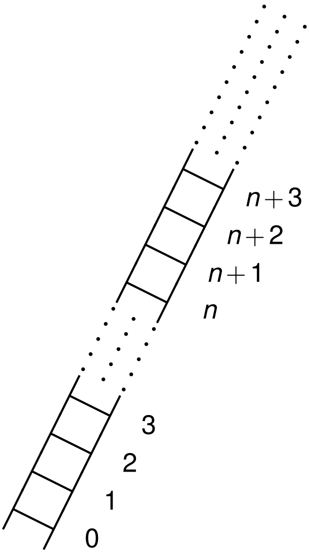

$0, 1, 2, 3,$

$\ldots, n, n+1, n+2, n+3, \ldots$

A formula.

A formula.

Teacher: Hello class.

A formula.

Teacher: Hello class.
Teacher:

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Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$

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Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

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Idea: assume predicate $P(n)$ for $n = k$.

Gauss and Induction

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Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

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Is predicate, $P(n)$ true for $n = k + 1$?

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$\sum_{i=1}^{k+1} i$

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$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1)$

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$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

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How about $k+2$.

Gauss and Induction

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How about $k+2$. Same argument starting at $k+1$ works!

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Predicate, $P(n)$, True for all natural numbers!

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