Lecture 5: Graphs.

Graphs!

Lecture 5: Graphs.

Graphs!
Euler

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Lecture 5: Graphs.

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Lecture 5: Graphs.

Graphs!
  Euler
  Definitions: model.
  Fact!
  Euler Again!!
Planar graphs.

Lecture 5: Graphs.

- Graphs!
  - Euler
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- Planar graphs.
  - Euler Again!!!!

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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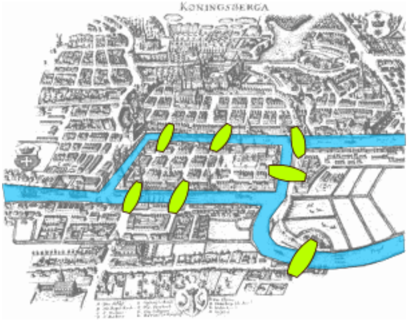

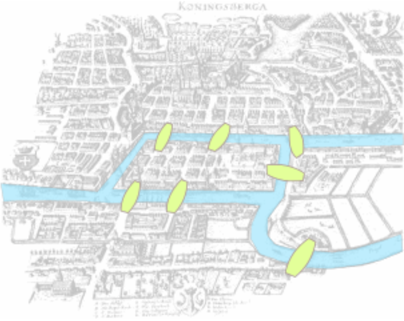

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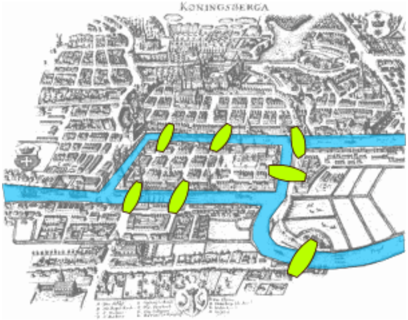

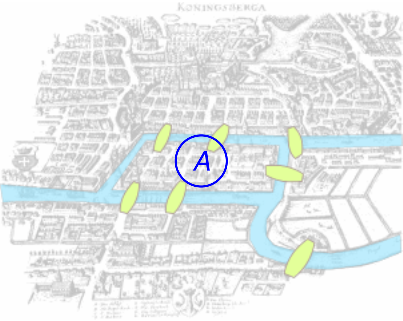

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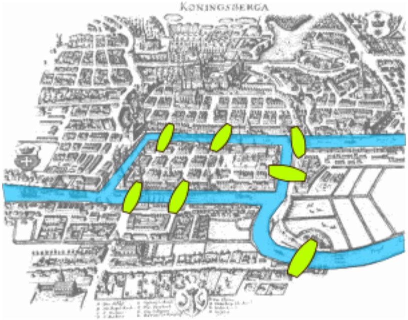

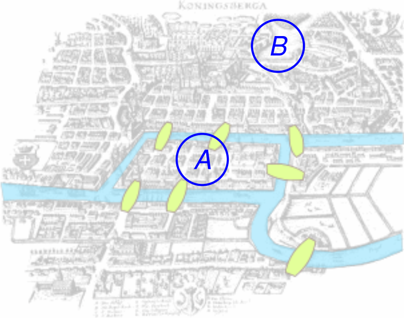

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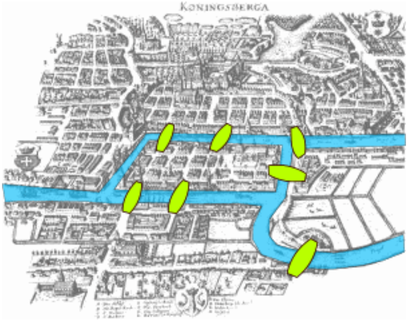

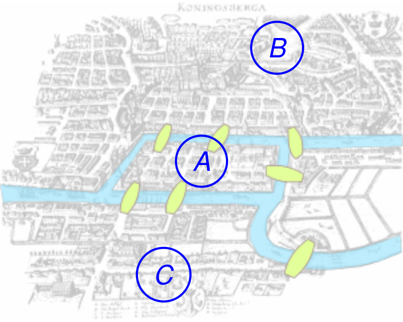

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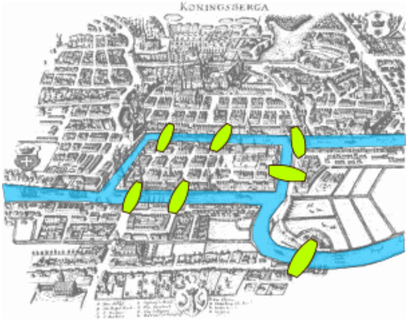

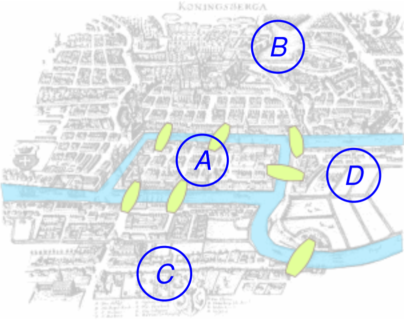

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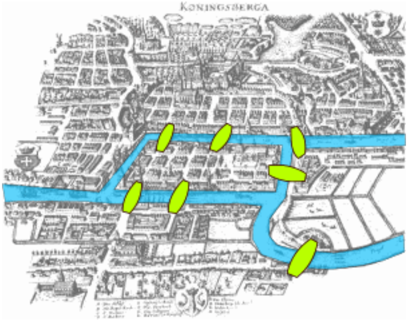

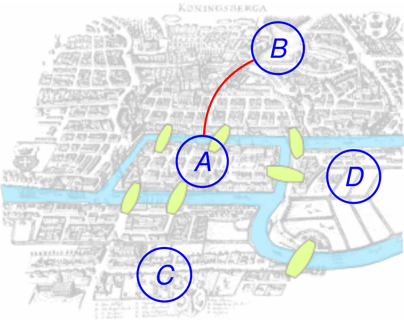

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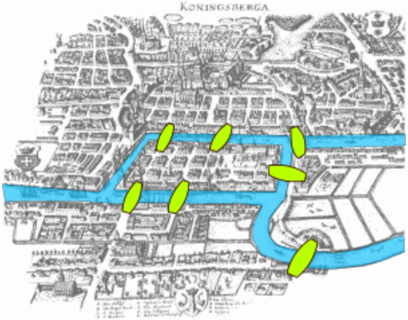

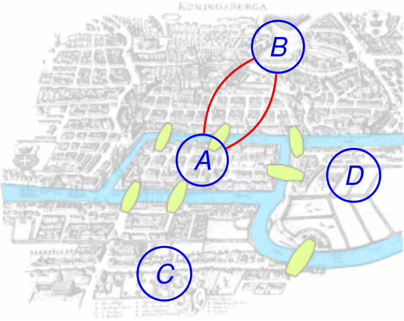

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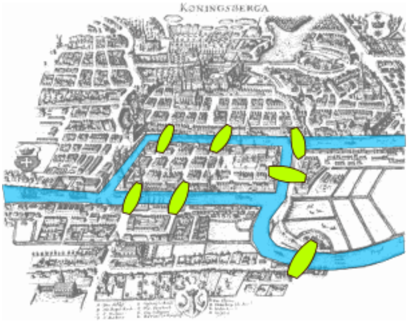

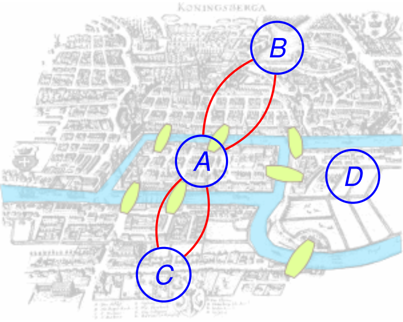

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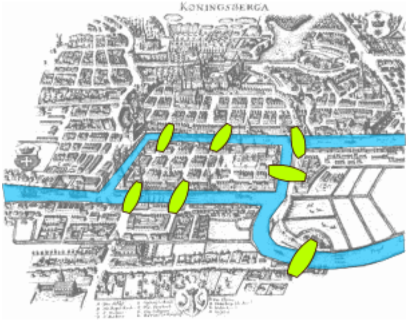

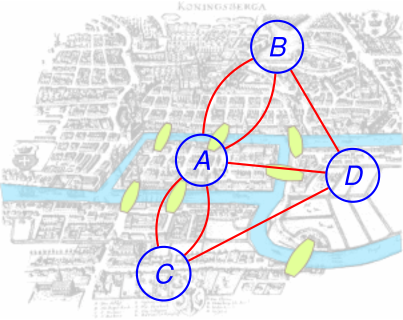

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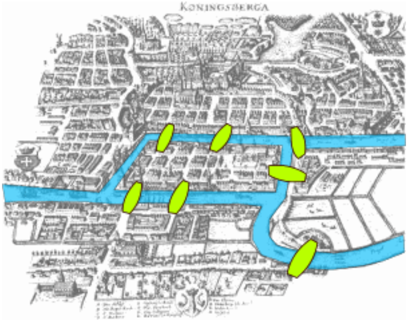

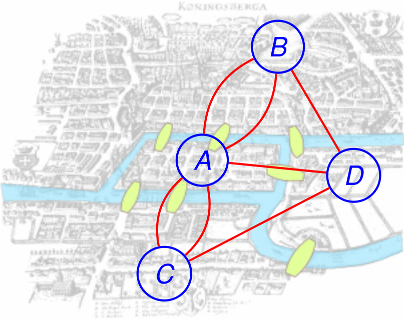

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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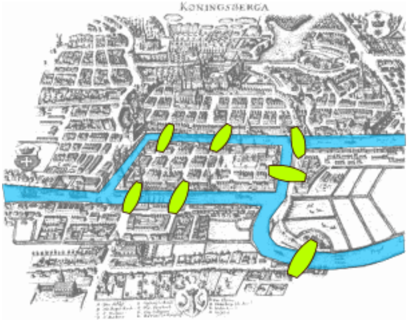

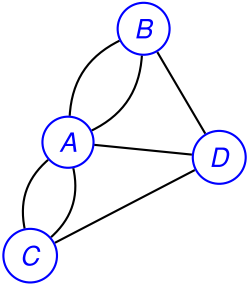

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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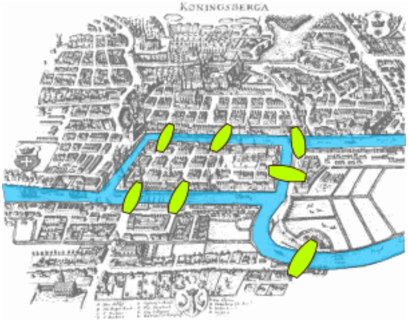

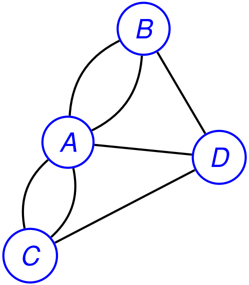

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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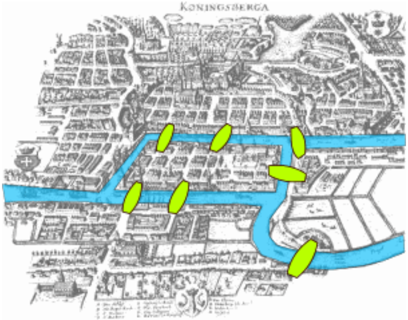
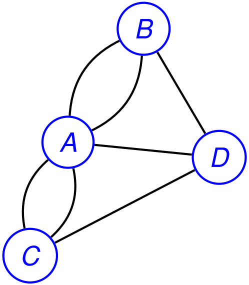
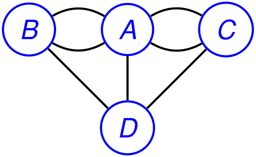




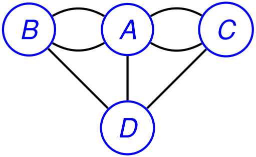
Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Graphs: formally.



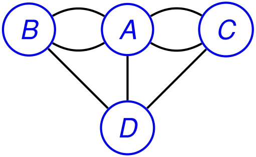
Graph:

Graphs: formally.



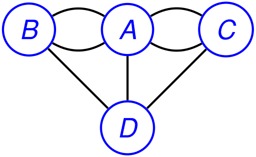
Graph: $G = (V, E)$.

Graphs: formally.



Graph: $G = (V, E)$.
V - set of vertices.

Graphs: formally.

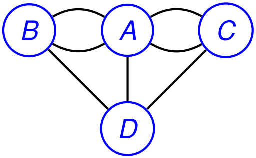


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



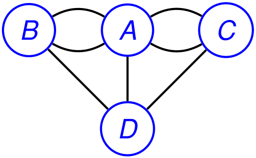
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



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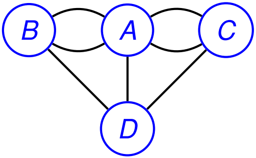


Graph: $G = (V, E)$.

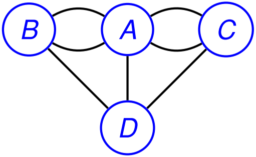
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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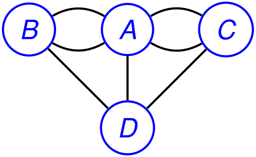
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

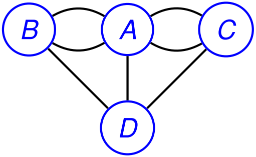
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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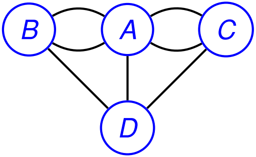
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

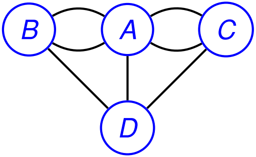
$\{A, B, C, D\}$

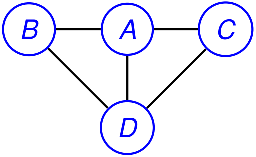
$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.

Graph: $G = (V, E)$.

V - set of vertices.

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$E \subseteq V \times V$ - set of edges.

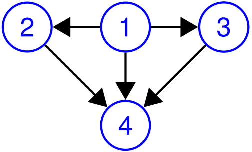
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

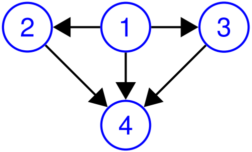
Multigraph above.

Directed Graphs



$G = (V, E)$.

Directed Graphs



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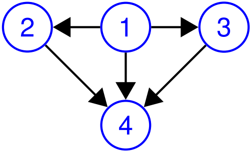
V - set of vertices.

Directed Graphs

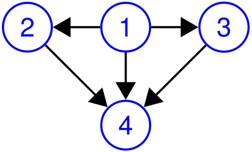


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



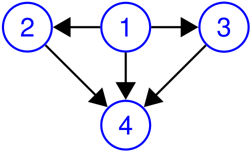
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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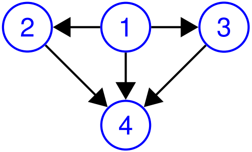
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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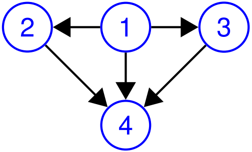
E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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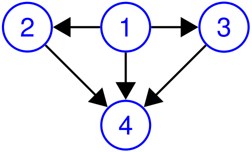

$G = (V, E)$.

V - set of vertices.

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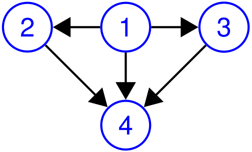
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

Directed Graphs



$G = (V, E)$.

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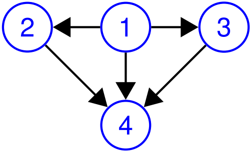
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.

Directed Graphs



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V - set of vertices.

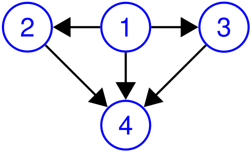
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

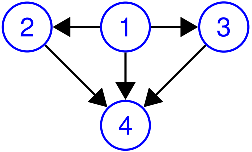
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Tournament: 1 beats 2,

Directed Graphs



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V - set of vertices.

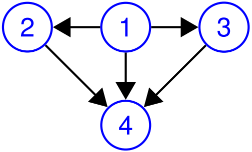
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

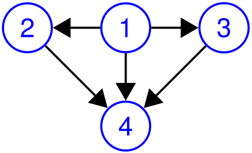
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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E ordered pairs of vertices.

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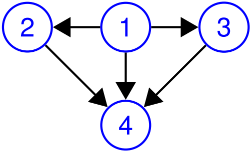
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

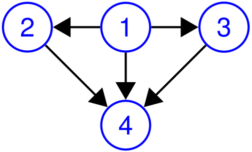
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



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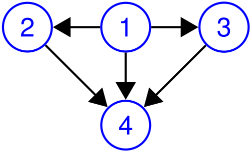

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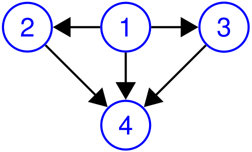
Directed Graphs



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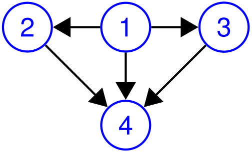
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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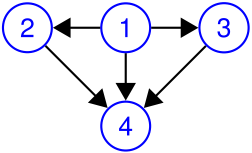
Directed Graphs



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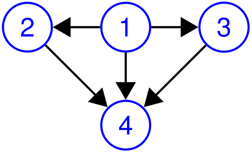
Likes.

Directed Graphs



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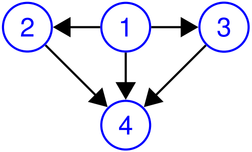
Directed Graphs



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Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

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neighbors, adjacent, degree, incident, in-degree, out-degree

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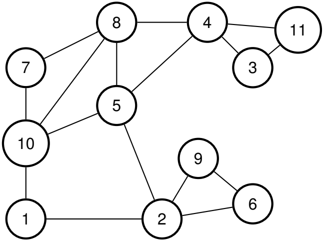
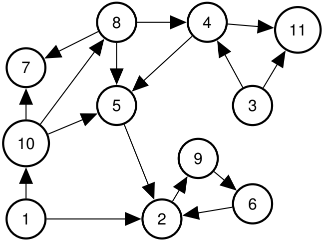


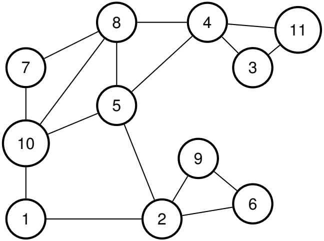


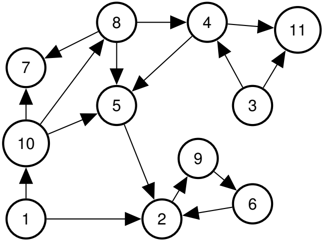
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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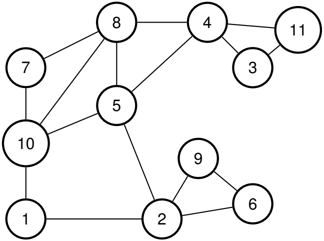

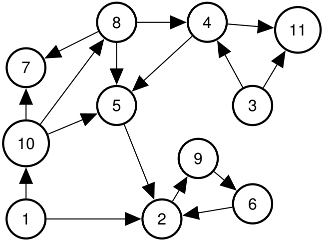

Neighbors of 10? 1,5,

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Graph: $G = (V, E)$

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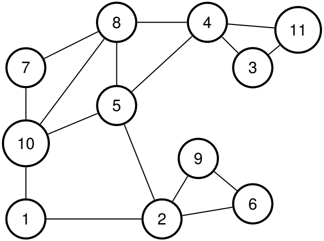

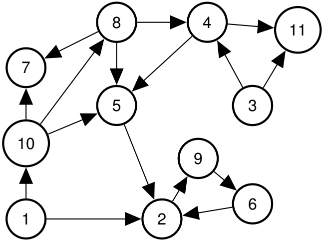

Neighbors of 10? 1,5,7,

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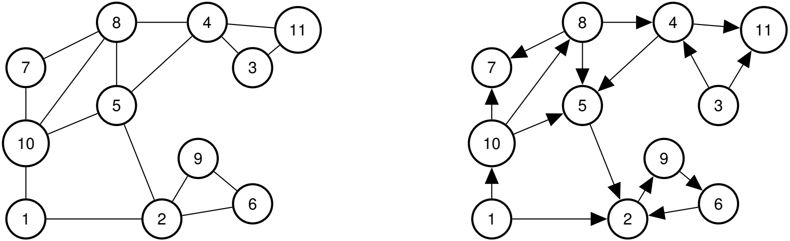

Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

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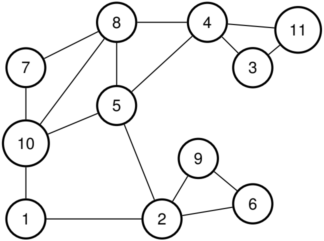

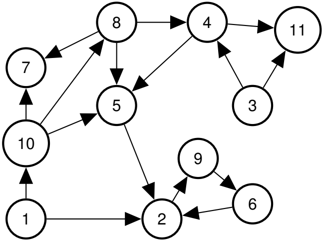

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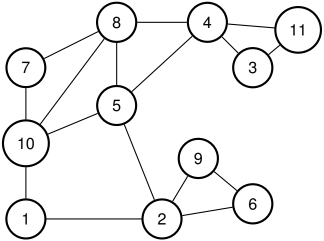

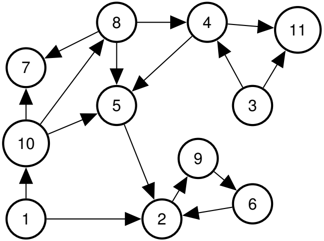

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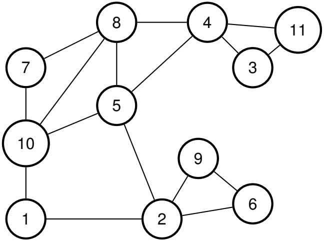

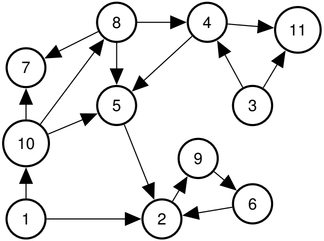

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Edge (u, v) is incident to u and v.

Degree of vertex 1?

Graph Concepts and Definitions.

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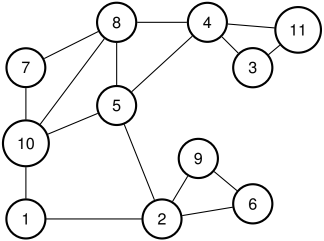

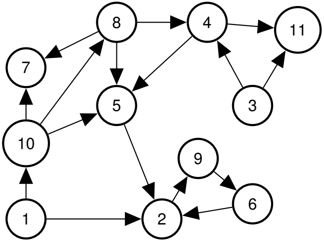

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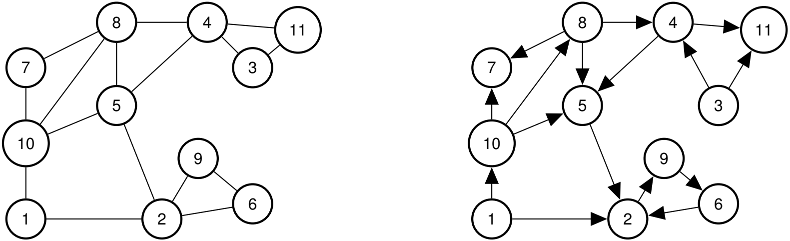

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Degree of vertex u is number of incident edges.

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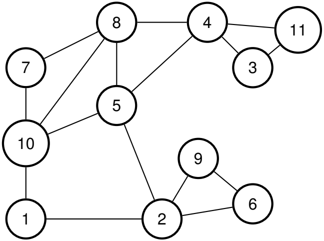

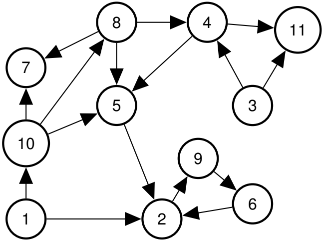

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Equals number of neighbors in simple graph.

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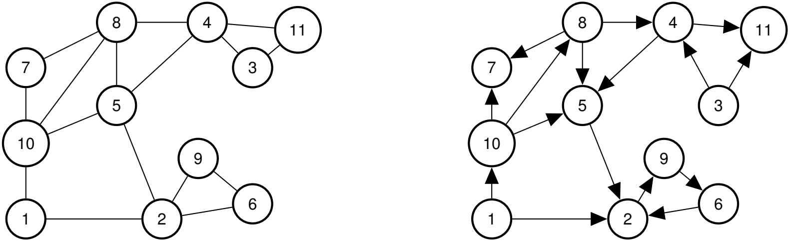

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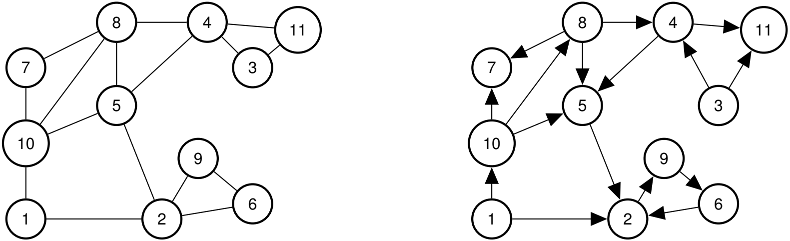

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Graph Concepts and Definitions.

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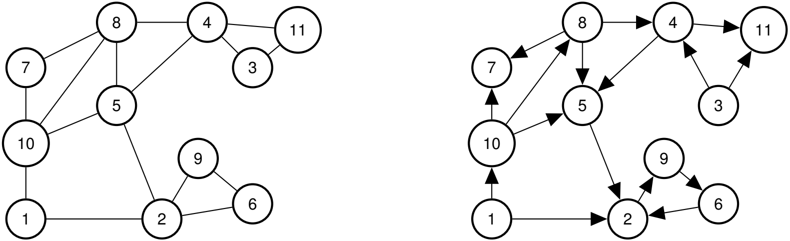
neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10?

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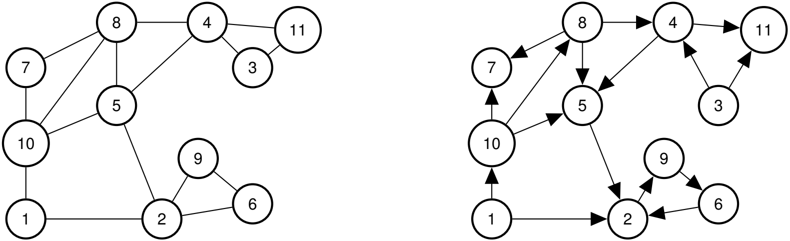

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Directed graph?

In-degree of 10? 1

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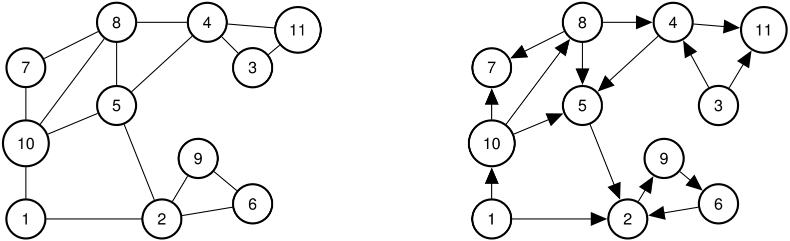

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In-degree of 10? 1   Out-degree of 10?

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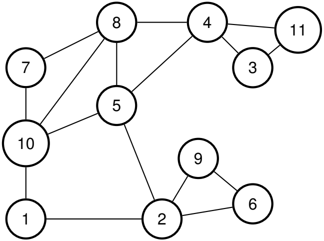

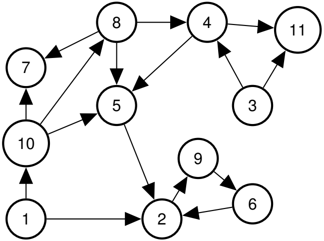

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Directed graph?

In-degree of 10? 1 Out-degree of 10? 3

Graph Concepts and Definitions.

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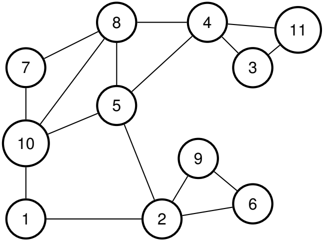

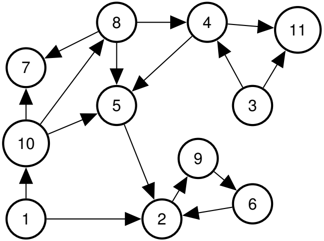

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The sum of the vertex degrees is equal to

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(A) the total number of vertices, $|V|$.

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(B) the total number of edges, $|E|$.

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(C) What?

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Not (A)!

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Not (A)! Triangle.

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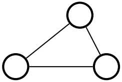

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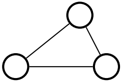

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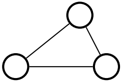

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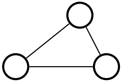

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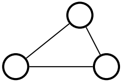

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What? For triangle number of edges is 3, the sum of degrees is 6.

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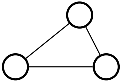

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What? For triangle number of edges is 3, the sum of degrees is 6.

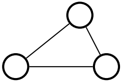
Could it always be...

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

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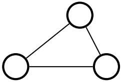

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How many incidences does each edge contribute?

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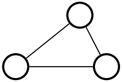

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

How many incidences does each edge contribute? 2.

Quick Proof.

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(B) the total number of edges, $|E|$.
(C) What?

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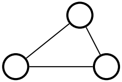

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$2|E|$ incidences are contributed in total!

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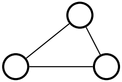

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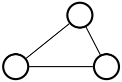

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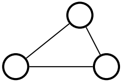

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sum of degrees is total incidences

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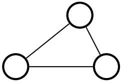

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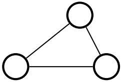

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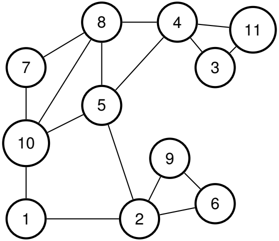
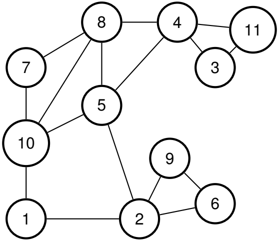
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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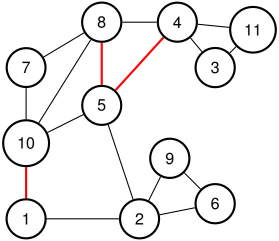

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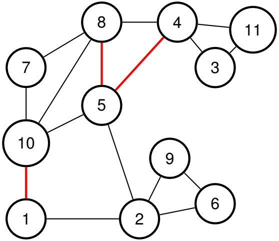
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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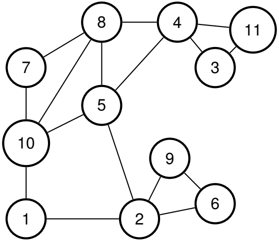

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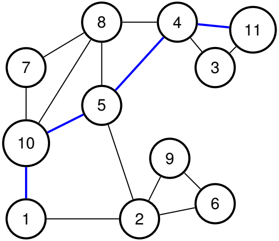

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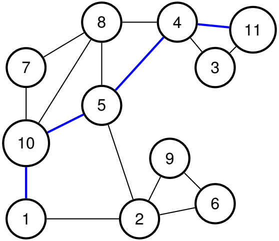
Paths, walks, cycles, tour.



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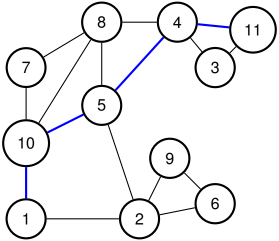

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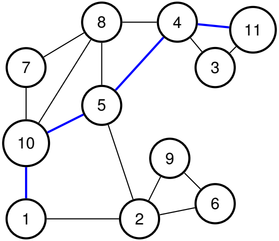
Paths, walks, cycles, tour.



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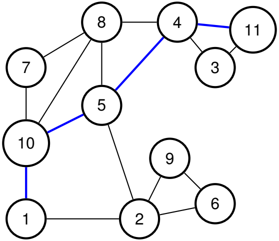
Quick Check!

Paths, walks, cycles, tour.



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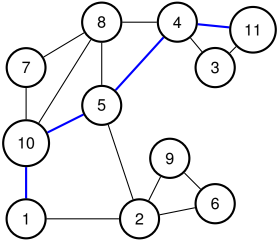
Quick Check! Length of path?

Paths, walks, cycles, tour.



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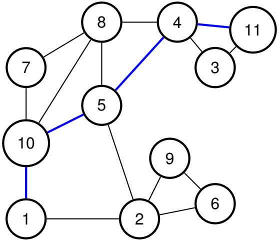
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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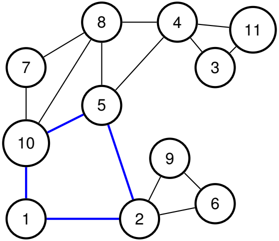
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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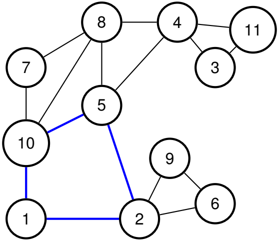
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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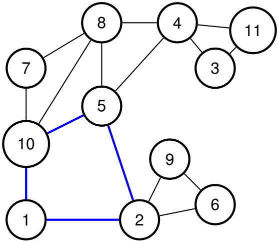
Paths, walks, cycles, tour.



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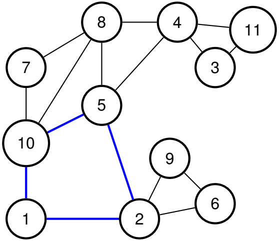
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Paths, walks, cycles, tour.



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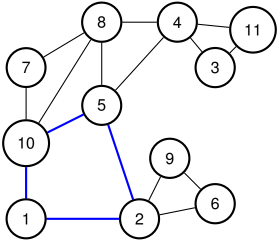
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Path is usually simple.

Paths, walks, cycles, tour.



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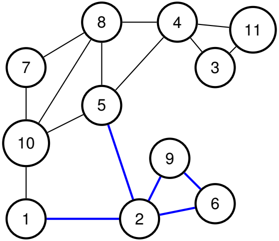
Paths, walks, cycles, tour.



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Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

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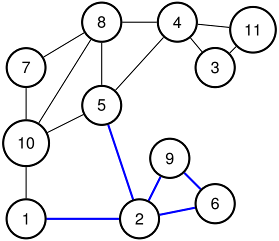
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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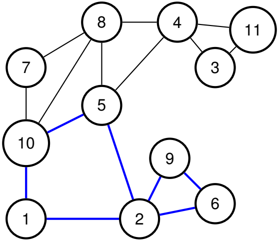

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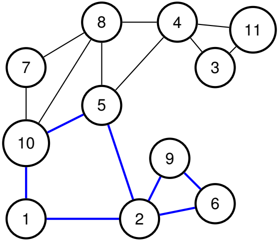
Tour is walk that starts and ends at the same node.

Paths, walks, cycles, tour.



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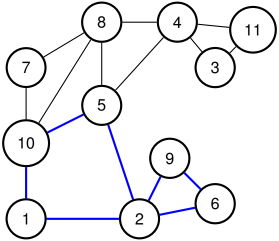

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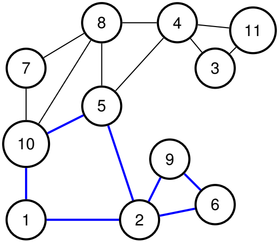
Paths, walks, cycles, tour.



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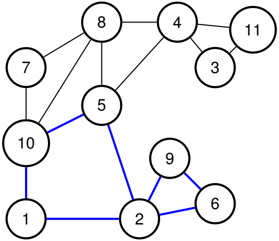
Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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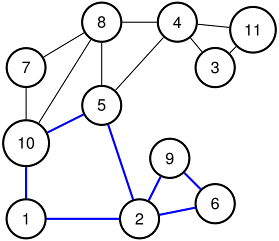
Paths, walks, cycles, tour.



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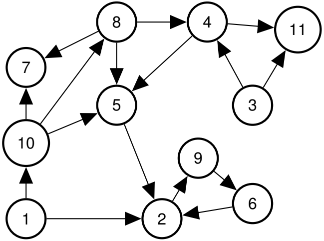
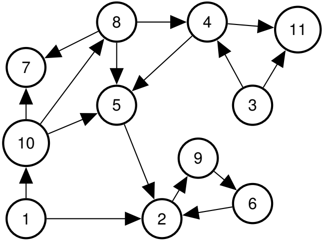
Directed Paths.



Directed Paths.



Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

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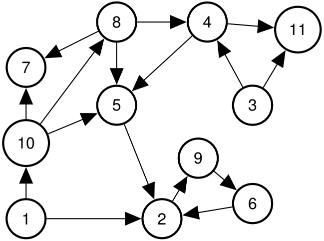

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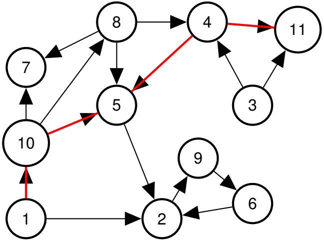

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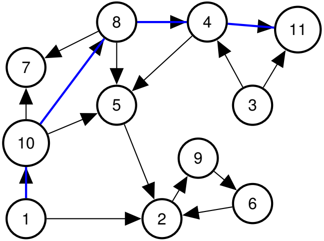

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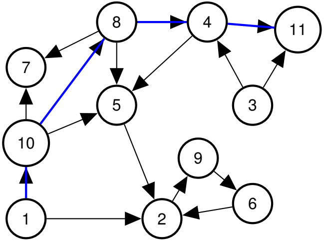

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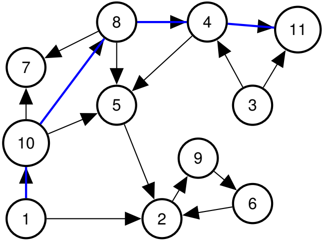

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Paths, walks,

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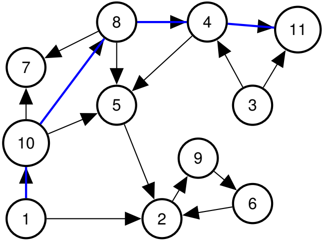

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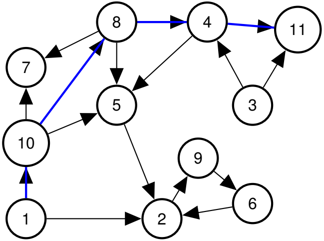

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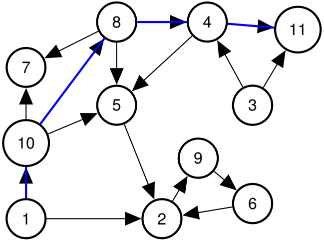

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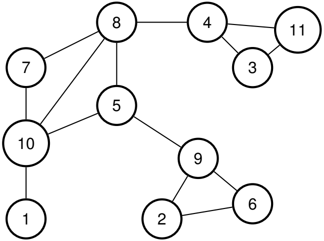
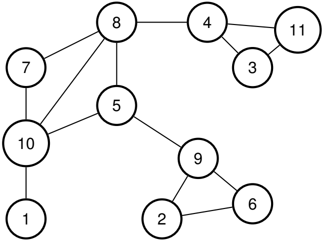
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity



u and v are connected if there is a path between u and v.

Connectivity



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A connected graph is a graph where all pairs of vertices are connected.

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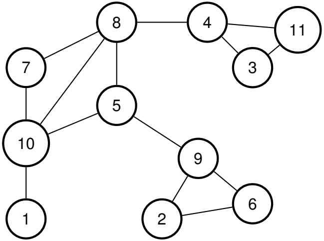

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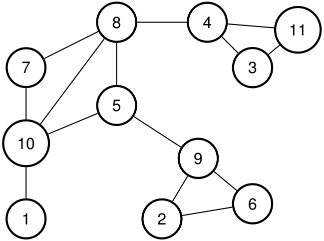

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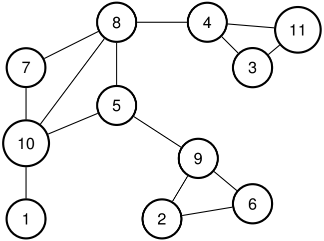

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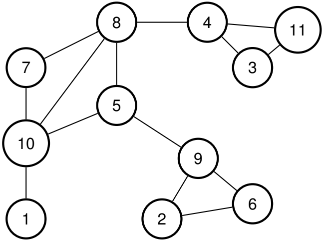

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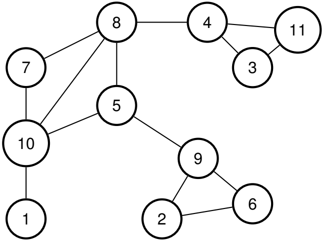

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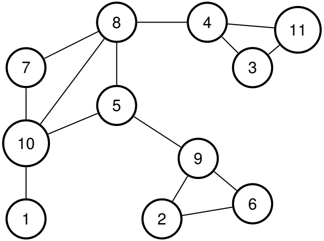

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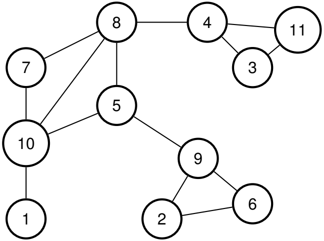

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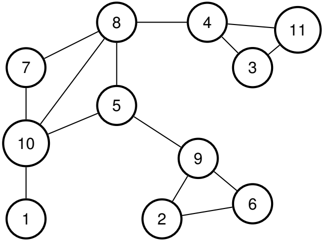

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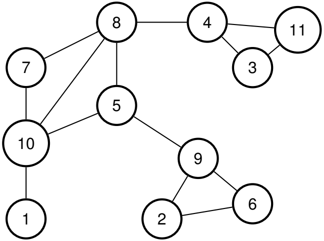

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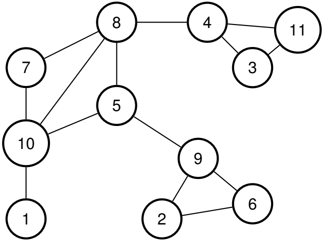

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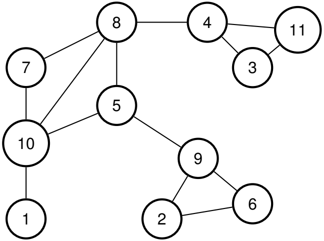

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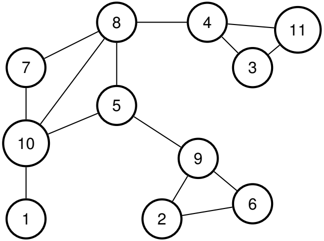

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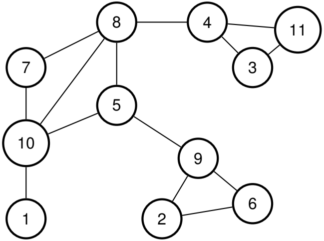
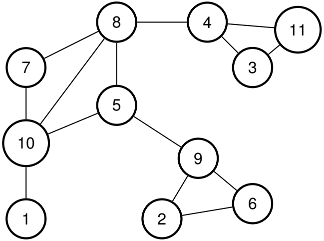


Is graph above connected?



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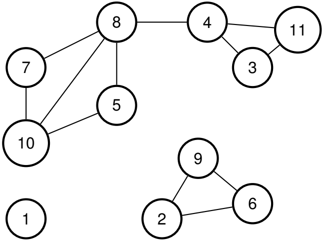
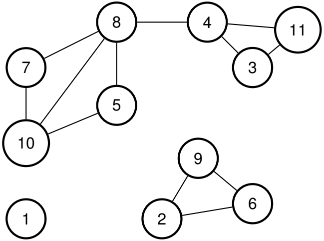


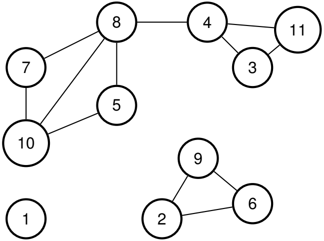
Is graph above connected? Yes!

How about now?



Is graph above connected? Yes!

How about now? No!



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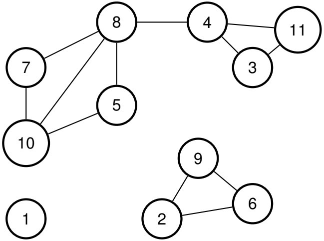
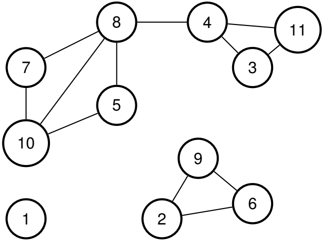
Connected Components?



Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.



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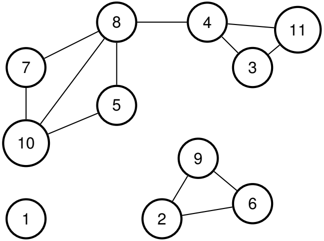
Connected component - maximal set of connected vertices.



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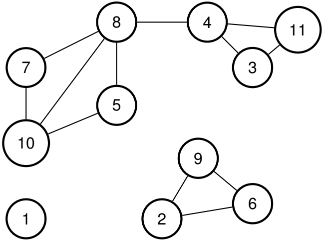
Quick Check: Is $\{10, 7, 5\}$ a connected component?



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Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.

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Quick Check: Is $\{10,7,5\}$ a connected component? No.

Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

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Eulerian Tour is connected so graph is connected.

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Tour enters and leaves vertex v on each visit.

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When you enter, you leave.
For starting node, tour leaves first ....then enters at end.

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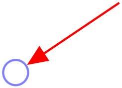

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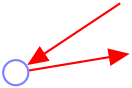

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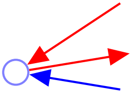

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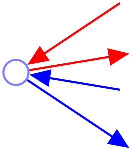

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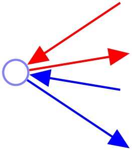

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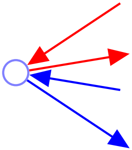

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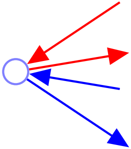

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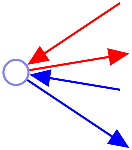

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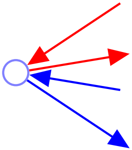
An Eulerian Tour is a tour that visits each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Proof of only if: Eulerian $\Longrightarrow$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore v has even degree. □



When you enter, you leave.
For starting node, tour leaves first ....then enters at end.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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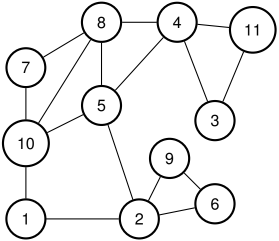

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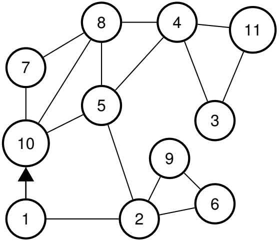

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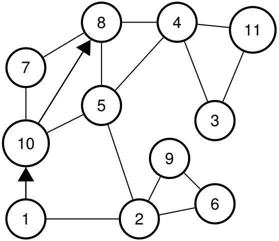

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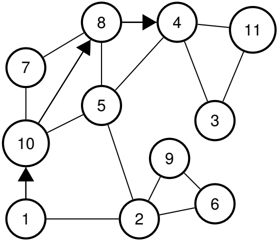

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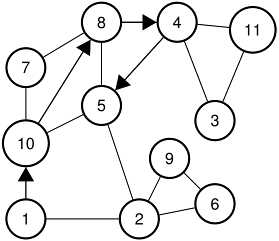

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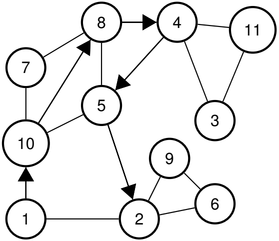

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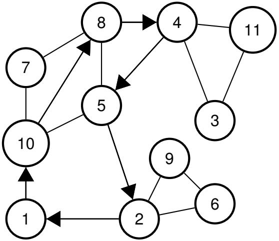

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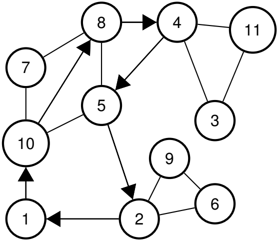

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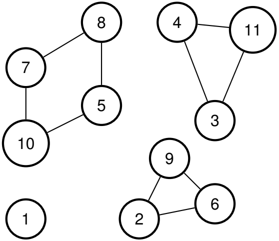

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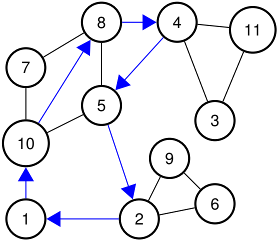

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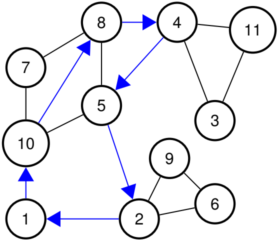

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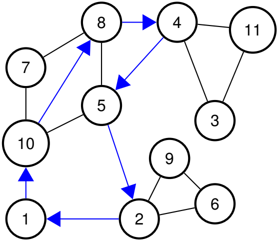

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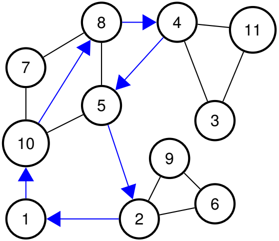

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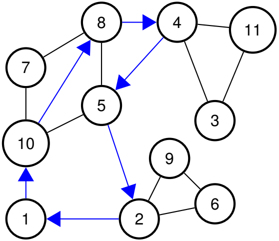

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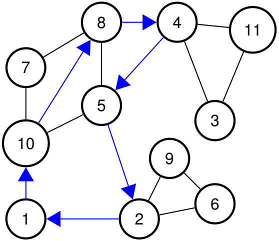

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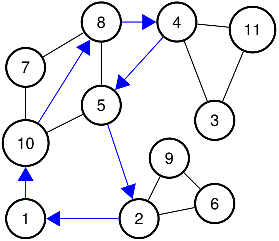

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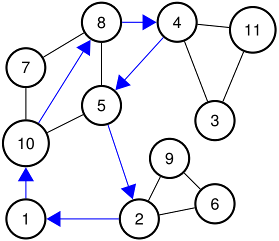

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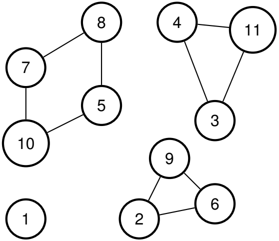

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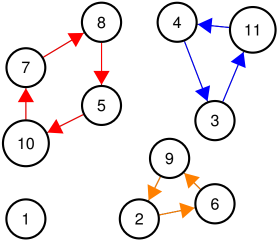

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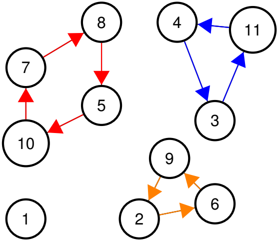

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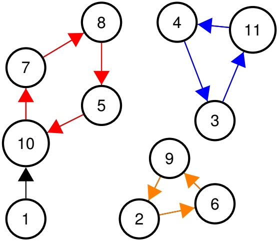

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   1,10

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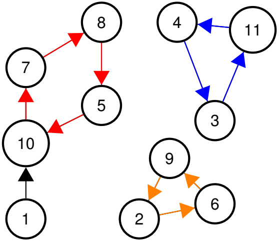

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   1,10,7,8,5,10

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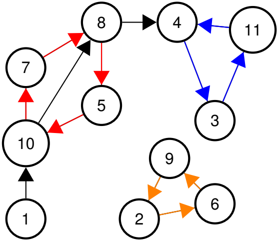

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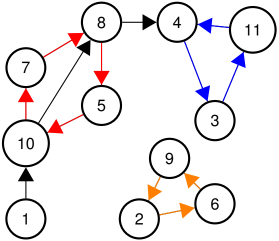

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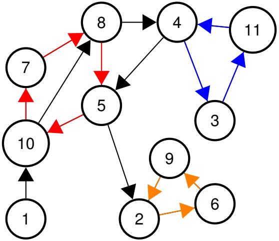

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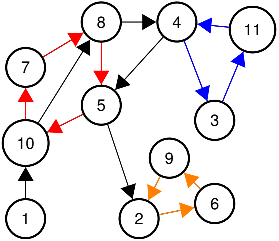

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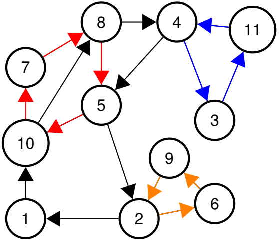

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Finding a tour: in general.

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Claim: Do get back to v!

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Proof of Claim: Even degree.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave

Finding a tour: in general.

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Proof of Claim: Even degree. If enter, can leave except for v.

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Proof of Claim: Even degree. If enter, can leave except for v. □

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2. Remove cycle, C, from G.

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Resulting graph may be disconnected. (Removed edges!)

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Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

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Let v_i be first vertex of C that is in G_i.

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Why is there a v_i in C?

Finding a tour: in general.

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G was connected $\implies$

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Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
  Why is there a v_i in C?
    G was connected $\Longrightarrow$
      a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

Finding a tour: in general.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

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3. Find tour T_i of G_i starting/ending at v_i.

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3. Find tour T_i of G_i starting/ending at v_i. Induction.

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Visits every edge once:
Visits edges in C

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By induction for all edges in each G_i.

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Break time!

Well admin time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

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Test Option: don't have to do homework.

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Should do homework.

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Should do homework. Can turn in.

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How will I do?

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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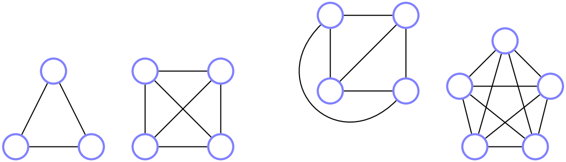
A graph that can be drawn in the plane without edge crossings.



Planar?

Planar graphs.

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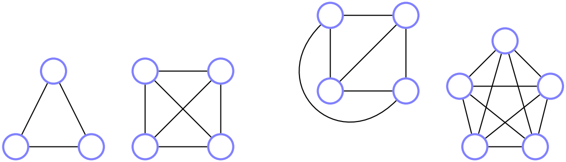

Planar? Yes for Triangle.

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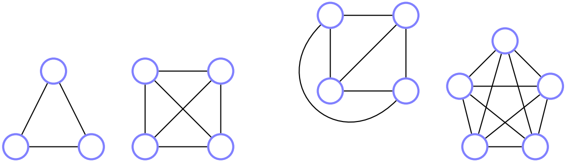

Planar? Yes for Triangle.

Four node complete?

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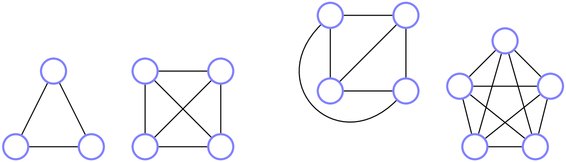

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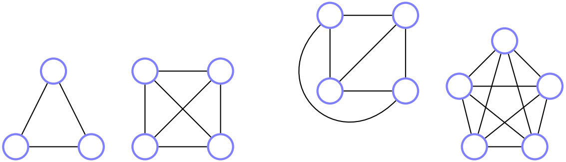

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5?

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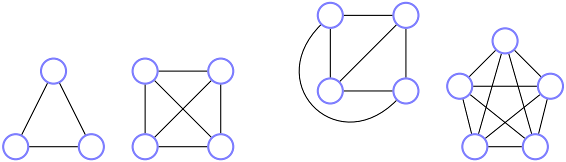

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Five node complete or K_5? No!

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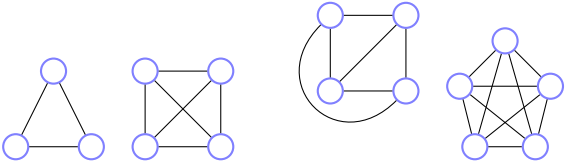

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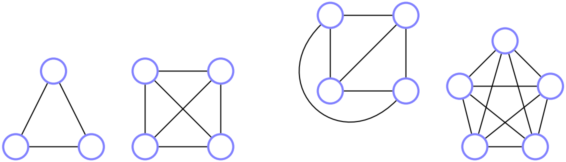

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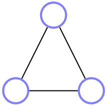
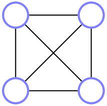
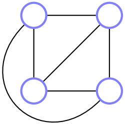
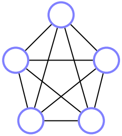

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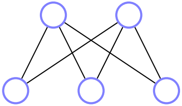
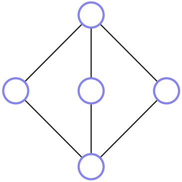
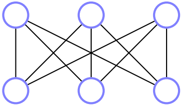

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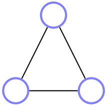 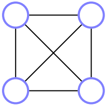 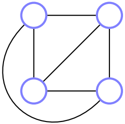 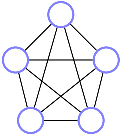

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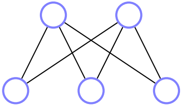
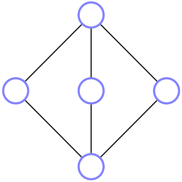
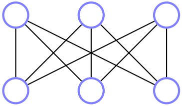
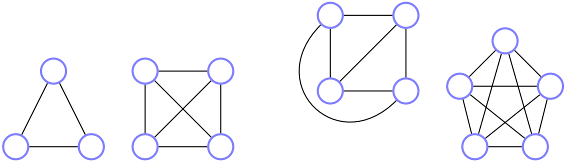
  

Two to three nodes, bipartite?

Planar graphs.

A graph that can be drawn in the plane without edge crossings.



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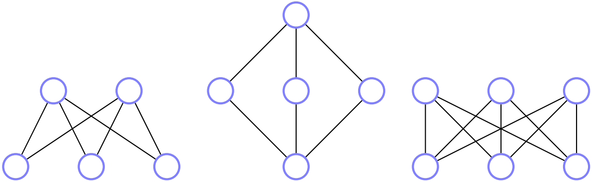

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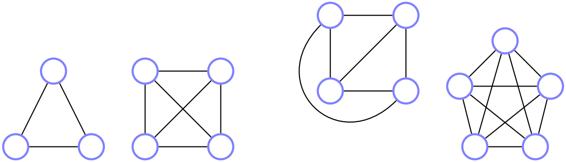

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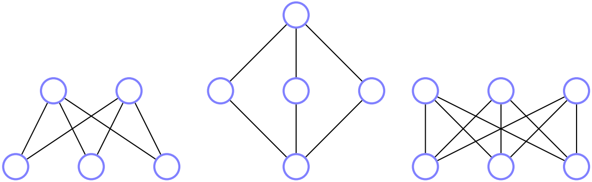

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$.

Planar graphs.

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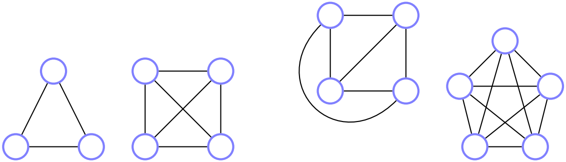

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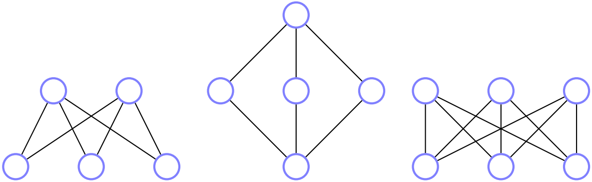

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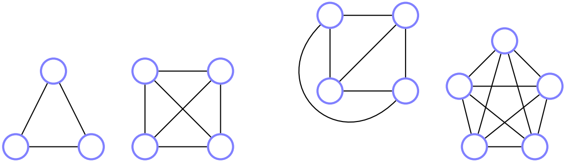

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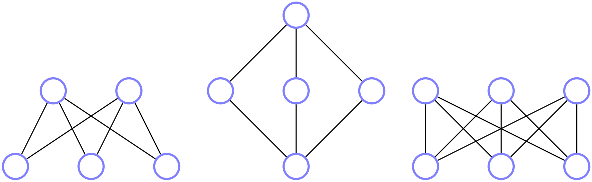

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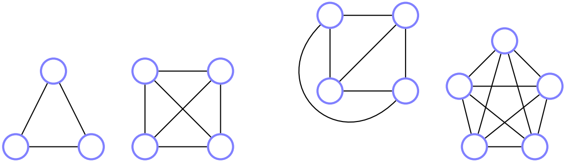

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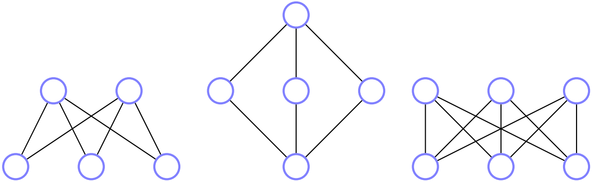

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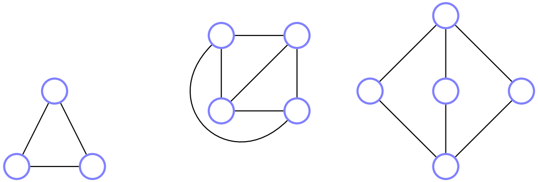
Euler's Formula.



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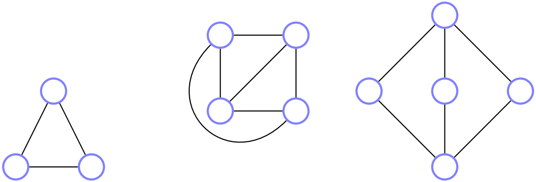
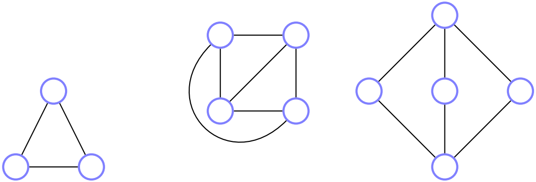


Faces: connected regions of the plane.

Euler's Formula.



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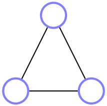
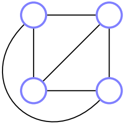
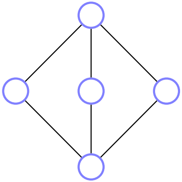
How many faces for

Euler's Formula.



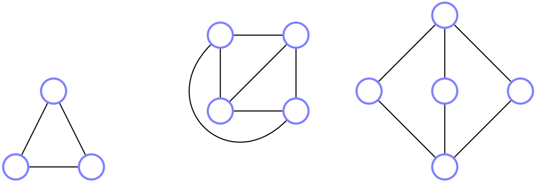




Faces: connected regions of the plane.

How many faces for
triangle?

Euler's Formula.

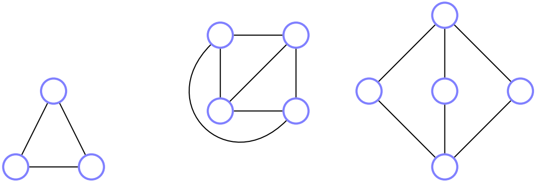


Faces: connected regions of the plane.

How many faces for

triangle? 2

Euler's Formula.



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How many faces for
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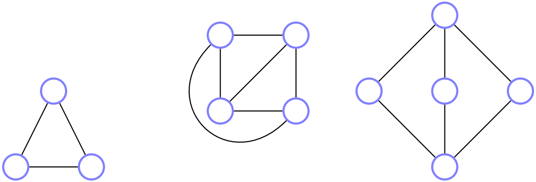

Faces: connected regions of the plane.

How many faces for

triangle? 2

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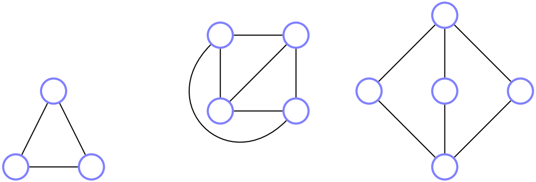

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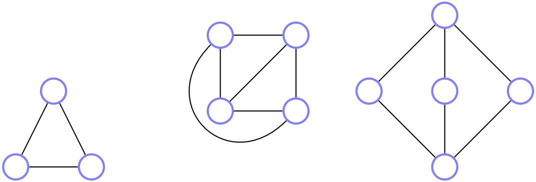

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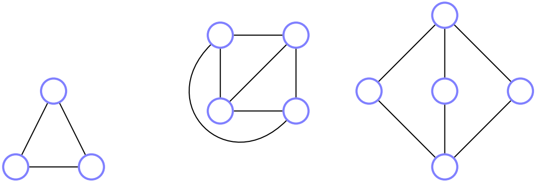

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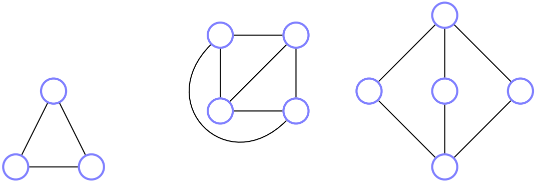

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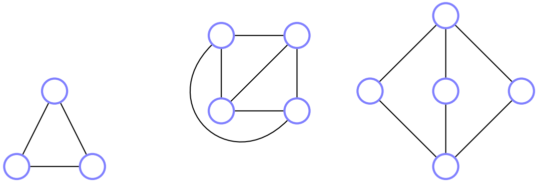

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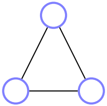 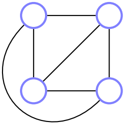 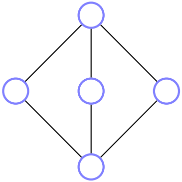

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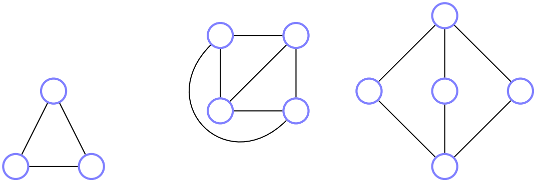

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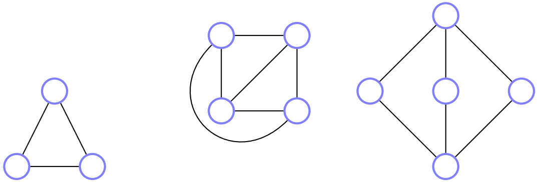

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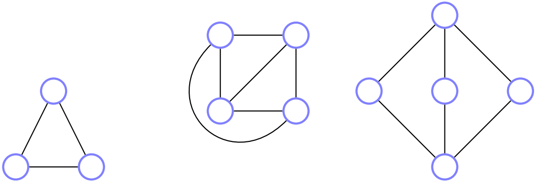
Triangle: $3 + 2 = 3 + 2$!

Euler's Formula.



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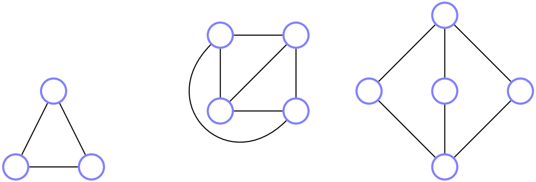

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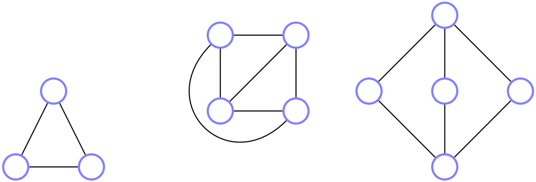
K_4: $4+4=6+2$!

Euler's Formula.



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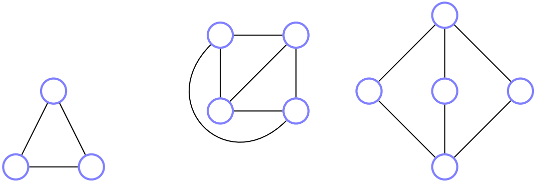

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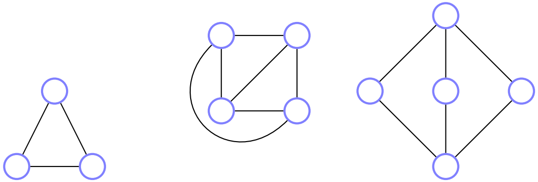

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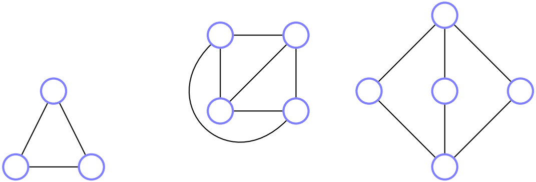

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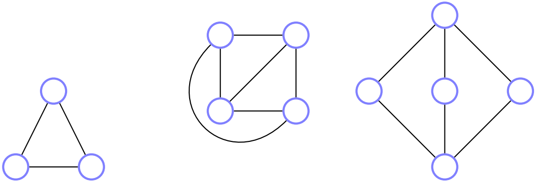
Examples = 3!

Euler's Formula.



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Examples = 3! Proven!

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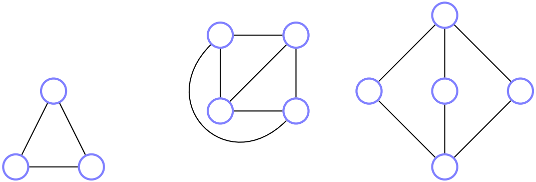

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Examples = 3! Proven! Not!!!!

Euler and Polyhedron.

Greeks knew formula for polyhedron.

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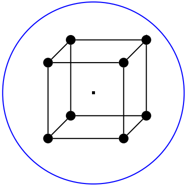

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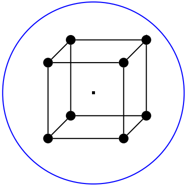

Faces?

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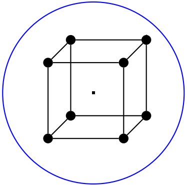

Faces? 6. Edges?

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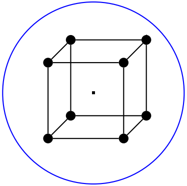

Faces? 6.   Edges? 12.

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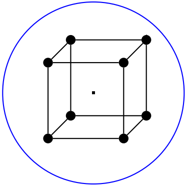

Faces? 6.   Edges? 12.   Vertices?

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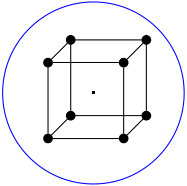

Faces? 6. Edges? 12. Vertices? 8.

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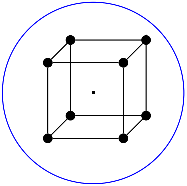

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Euler: Connected planar graph: $v + f = e + 2$.

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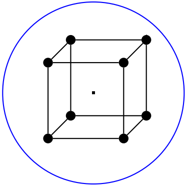

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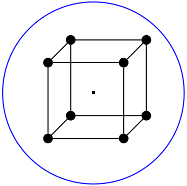

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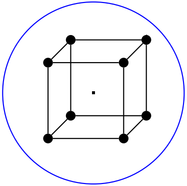

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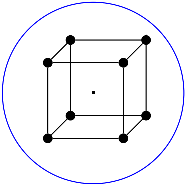

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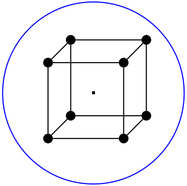

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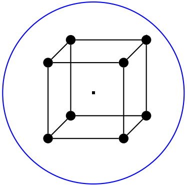

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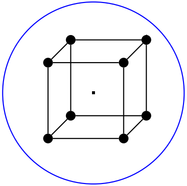

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Polyhedron without holes

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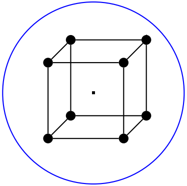

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Polyhedron without holes $\equiv$

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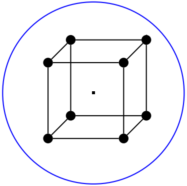

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Polyhedron without holes $\equiv$ Planar graphs.

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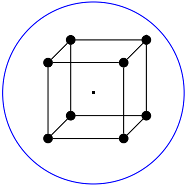

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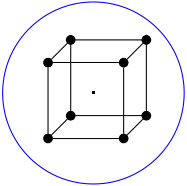
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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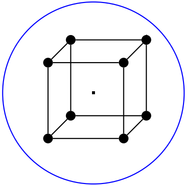
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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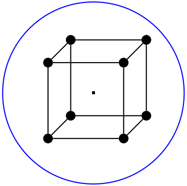
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6.   Edges? 12.   Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.
$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

Euler and Polyhedron.

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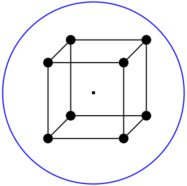

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Euler: Connected planar graph: $v+f=e+2$.

$8+6=12+2$.

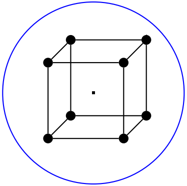
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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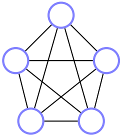
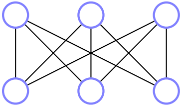
Euler proved formula thousands of years later!

Euler and planarity of K_5 and $K_{3,3}$





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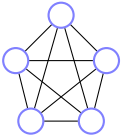

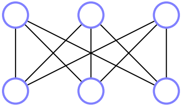

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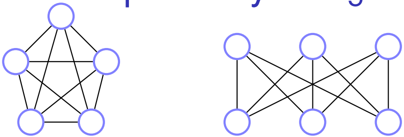

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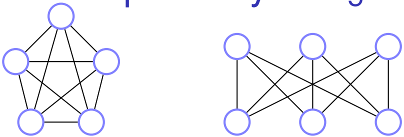

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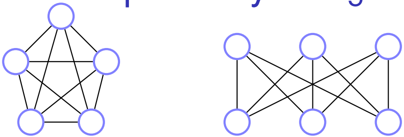

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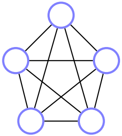

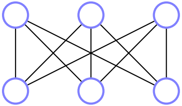

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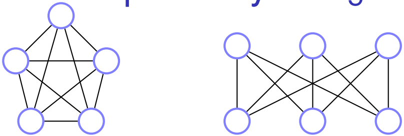

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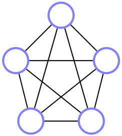

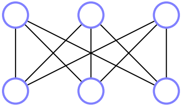

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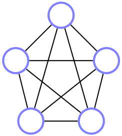

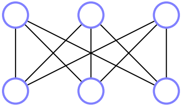

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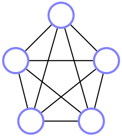

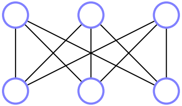

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K_5

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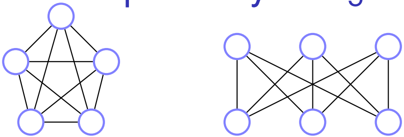

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K_5 Edges?

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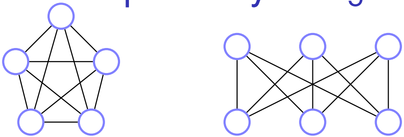

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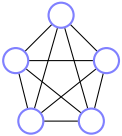 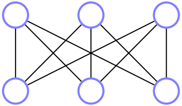

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K_5 Edges? $4 + 3 + 2 + 1 = 10$.

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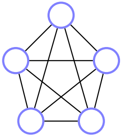

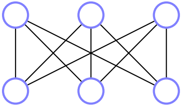

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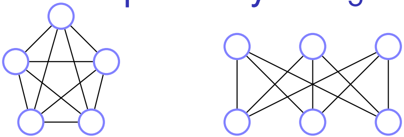

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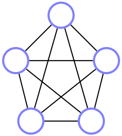 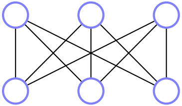

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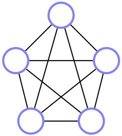
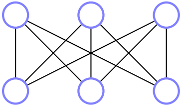
$10 \not\leq 3(5) - 6 = 9$.

Euler and planarity of K_5 and $K_{3,3}$





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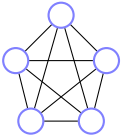

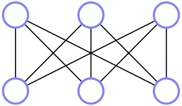

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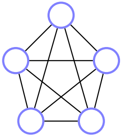

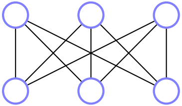

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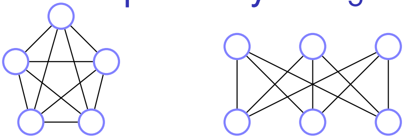

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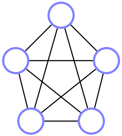

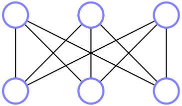

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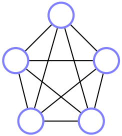

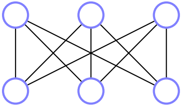

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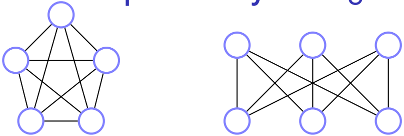

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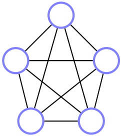 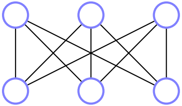

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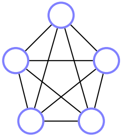
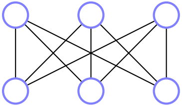
$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!
But no cycles that are triangles.

Euler and planarity of K_5 and $K_{3,3}$





Euler: $v + f = e + 2$ for connected planar graph.

Each face is adjacent to at least three edges. $\geq 3f$ face-edge adjacencies.
Each edge is adjacent to (at most) two faces. $\leq 2e$ face-edge adjacencies.
$\implies 3f \leq 2e$

Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

K_5 Edges? $4+3+2+1 = 10$. Vertices? 5.
$10 \not\leq 3(5) - 6 = 9$. $\implies$ K_5 is not planar.

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!
But no cycles that are triangles. Face is of length ≥ 4.

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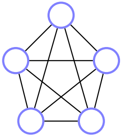

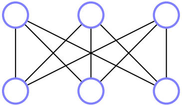

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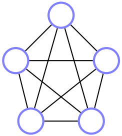

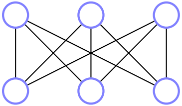

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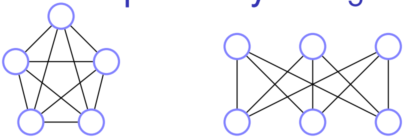

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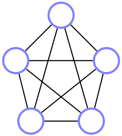

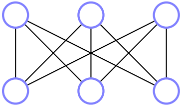

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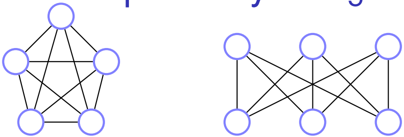

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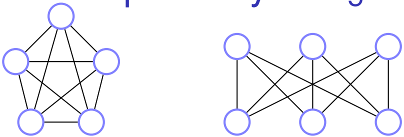

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Tree.

A tree is a connected acyclic graph.

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To tree or not to tree!

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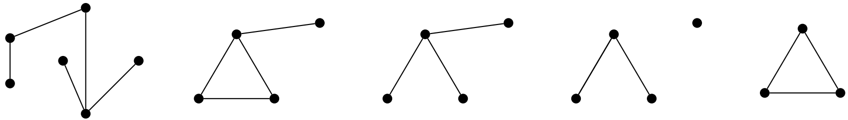

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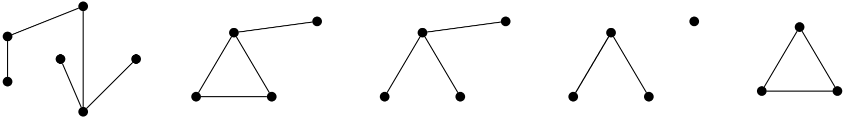

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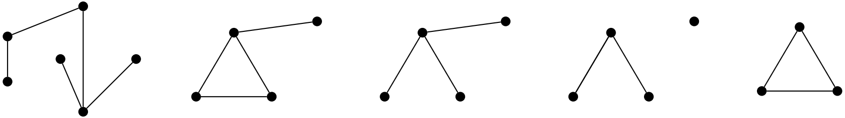

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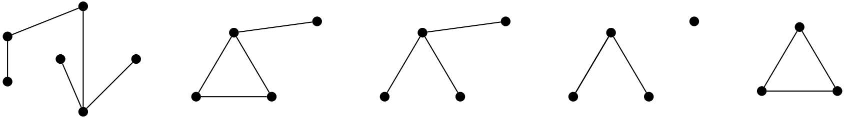

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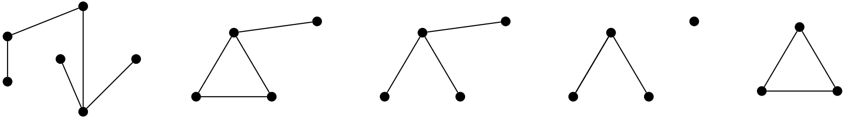

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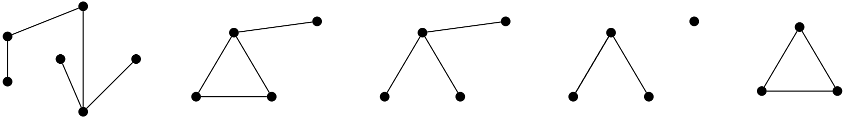

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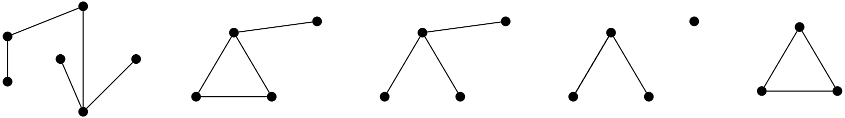

Yes. No. Yes. No. No.

Faces?

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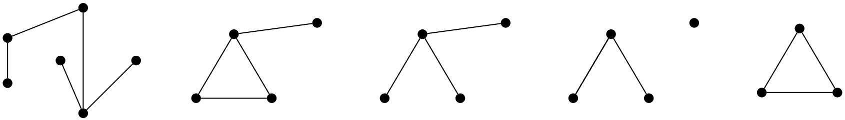

Yes. No. Yes. No. No.

Faces? 1.

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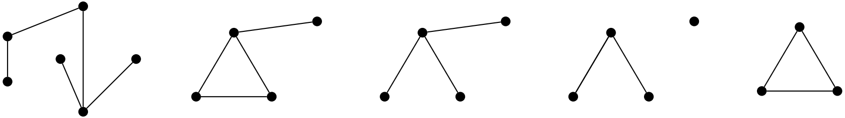

Yes. No. Yes. No. No.

Faces? 1. 2.

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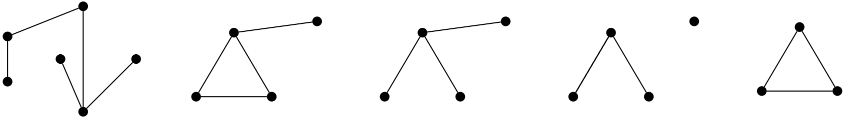

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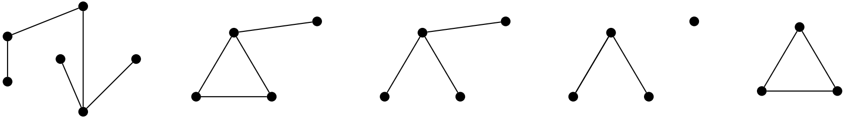

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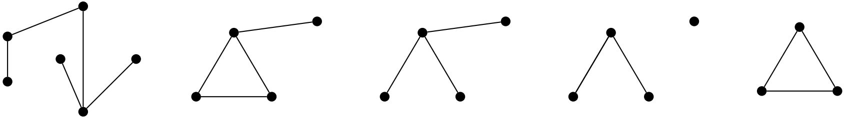

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Faces? 1. 2. 1. 1. 2.

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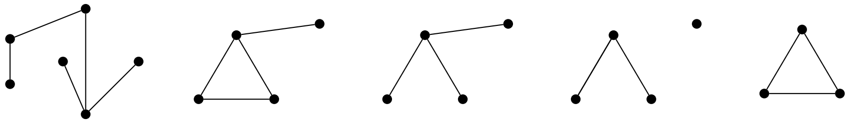

Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges.

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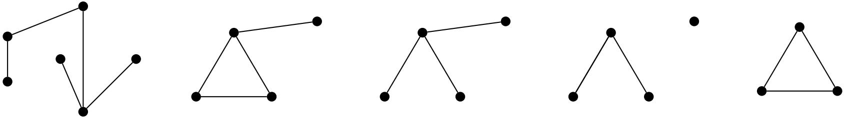

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Faces? 1. 2. 1. 1. 2.
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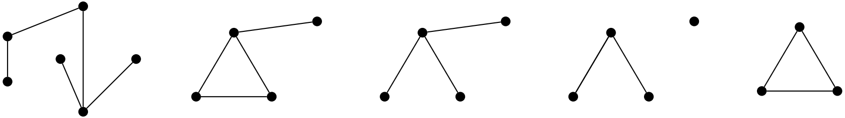

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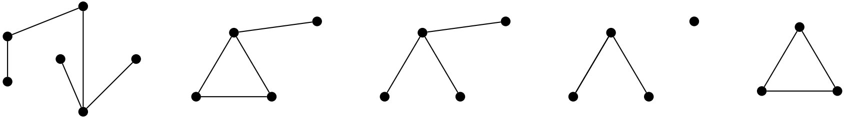

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One face for trees!

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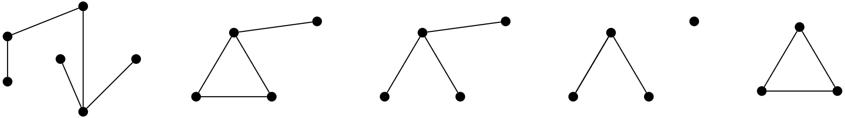

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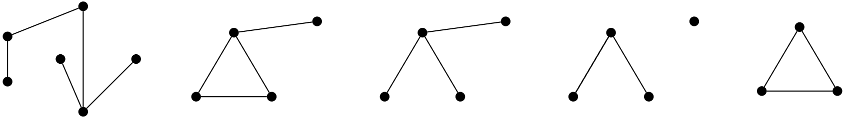

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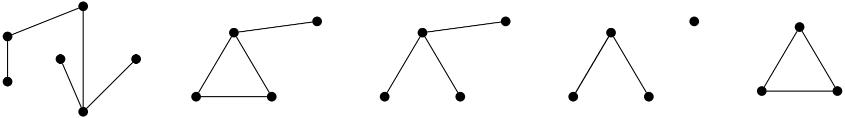

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Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

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Proof sketch:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$,

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree.

Euler's formula.

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Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.

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Base: $e = 0$, $v = f = 1$.
Induction Step:
  If it is a tree. Done.
  If not a tree.
    Find a cycle.

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Induction Step:
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    Find a cycle. Remove edge.

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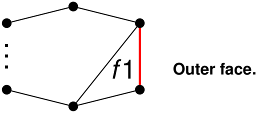

Joins two faces.

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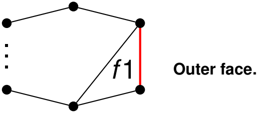

Joins two faces.
New graph: v-vertices.

Euler's formula.

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Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.
Induction Step:
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If not a tree.
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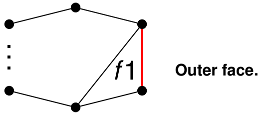

Joins two faces.
New graph: v-vertices. $e-1$ edges.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
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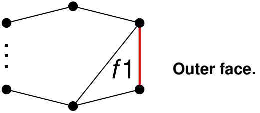
Find a cycle. Remove edge.



Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces.

Euler's formula.

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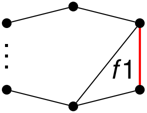

Outer face.

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.

Euler's formula.

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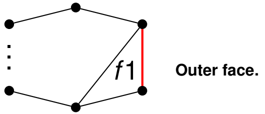

Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v+(f-1)=(e-1)+2$ by induction hypothesis.

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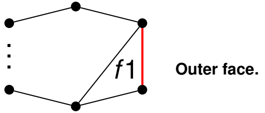
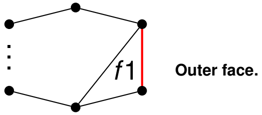


Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.
$v + (f - 1) = (e - 1) + 2$ by induction hypothesis.
Therefore $v + f = e + 2$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.



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Oh my goodness..what have we done!

Graphs.

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  Basics.

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Graphs.
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Connectivity.

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Graphs.
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Connectivity.
Algorithm for Eulerian Tour.

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Also Euler's formula.

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Yay!