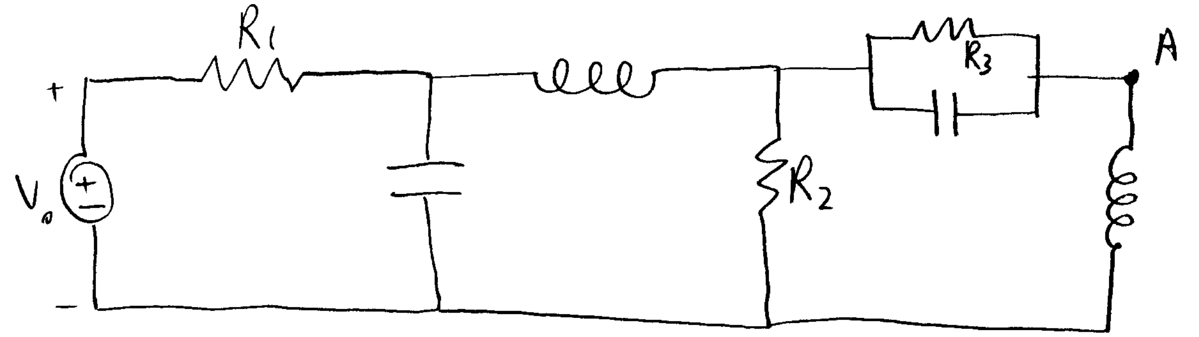
EE100 Problem Set 4

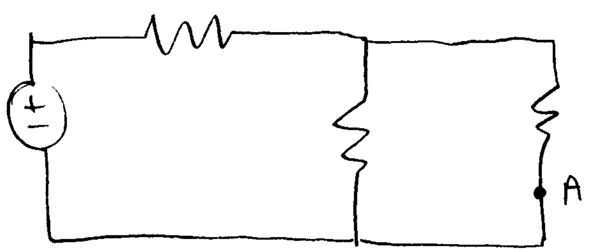
2)



$V_0 = 10V$, $R_1 = 4.7k\Omega$, $R_2 = 2.2k\Omega$, $R_3 = 3.3k\Omega$

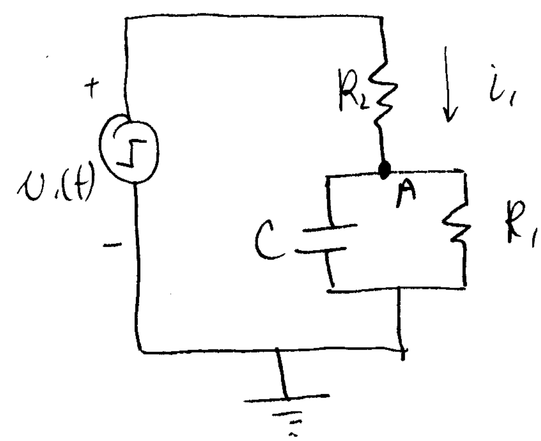
At DC Steady state, capacitors = open
inductors = short

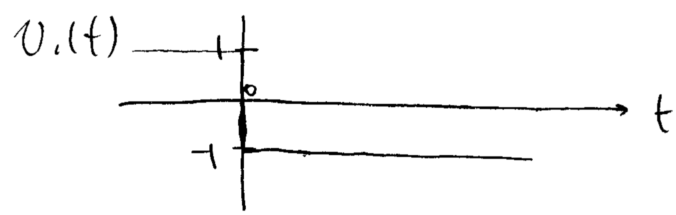
Circuit simplifies to:



$V_A = 0V$?

3)





$R_1 = 1k\Omega$, $R_2 = 2k\Omega$, $C = 10\mu F$

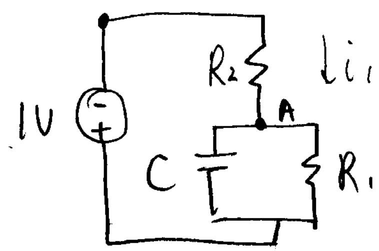
a) $V_A(t=0^-) \Rightarrow$ At steady state, C = open;
thus $V_A = \frac{R_1}{R_1+R_2} v_1 = \frac{1k}{1k+2k}(1) = \frac{1}{3}$ V

b) $V_A(t=0^+) \Rightarrow$ Since the voltage across a capacitor cannot change instantaneously,
$V_A(t=0^+) = V_A(t=0^-) = \frac{1}{3}V$

3) c) As the C is open, i_1 is simply:

$$\frac{v_1(t=0^-)}{R_1+R_2} = \frac{1V}{1k+2k} = \frac{1}{3}\,mA$$

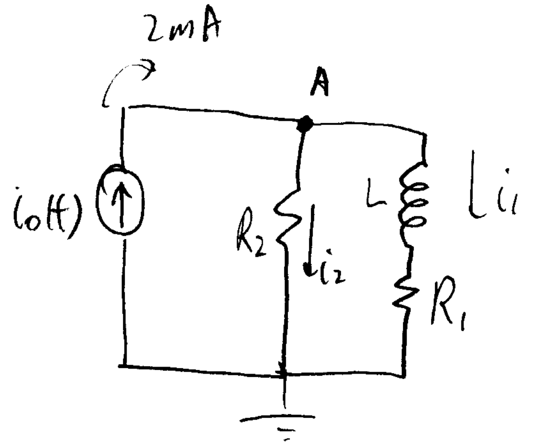
d)



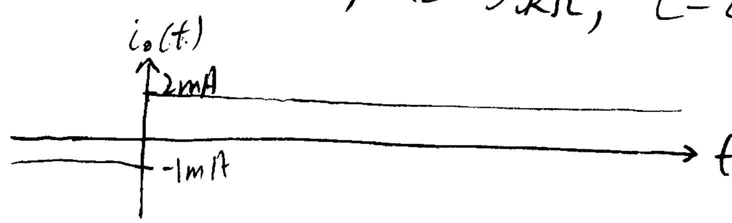
$V_A = \frac{1}{3}V$ as shown in part (a)

$$i_1 = \frac{1\frac{1}{3}V}{2k} = \frac{-2}{3}\,mA$$

4)



$R_1 = 2k\Omega$, $R_2 = 3k\Omega$, $L = 20mH$



a) $i_1(0^-) \Rightarrow$ L becomes short in steady state

$$= \frac{R_2}{R_1+R_2}\, i_0(0^-) = \frac{3k}{3k+2k}(-mA) = \frac{-3}{5}\,mA$$

b) $i_1(0^+) \Rightarrow$ Current across L cannot change instanteously $\Rightarrow i_1(0^+) = \frac{-3}{5}\,mA$

c) $i_2(0^-) \Rightarrow$ By KCL $= \frac{-2}{5}\,mA$

d) $i_2(0^+) \Rightarrow$ By KCL $= 2mA + \frac{3}{5}mA = 2\frac{3}{5}\,mA$

4) e) $V_A(0^-) \Rightarrow i_1 R_1 \doteq (\frac{-3}{5} mA)(2000\Omega)$
$= \frac{-6}{5} V$

f) $V_A(0^+) \Rightarrow i_2 R_2 = (2\frac{3}{5} mA)(3000\Omega)$
$= 6\frac{9}{5} V$

5) Want: $\frac{d}{dt}(V_A)$, $t = 0^+$

$V_A = i_2 R_2 = i_1 R_1 + L\frac{di_1}{dt} \Rightarrow \frac{dV_A}{dt} = R_1 \frac{di_1}{dt} + L\frac{d^2 i}{dt^2}$

We know $i_1(0^+) = \frac{-3}{5} mA$

and as $t \to \infty$ $i_1 = \frac{R_2}{R_1 + R_2}(2mA) = \frac{3k}{2k+3k} = \frac{6}{5} mA$

We have

$$i_L(t) = i_{Final} + (i_0 - i_{final}) e^{-t/\tau}$$
$$= \frac{6}{5} mA + (\frac{-3}{5} - \frac{6}{5}) mA\, e^{-t/\tau}$$
$$= \frac{6}{5} mA - \frac{9}{5} mA\, e^{-t/4\times10^{-6}s}$$

$$\tau = \frac{L}{R} = \frac{20mH}{2k + 3k} = 4\times10^{-6}$$

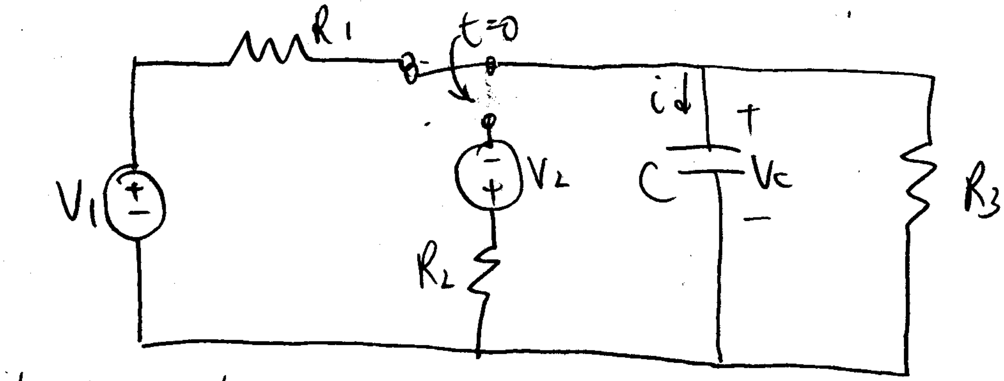
$$\frac{di}{dt} = \frac{-9}{5} mA \left(\frac{-1}{4\times10^{-6}}\right) e^{-t/4\times10^{-6}}$$

$$\frac{d^2 i}{dt^2} = \frac{-9}{5} mA \left(\frac{-1}{4\times10^{-6}}\right)^2 e^{-t/4\times10^{-6}}$$

$$\left.\frac{dV_A}{dt}\right|_{t=0} = 2k\left(\frac{-9}{5} mA \left(\frac{-1}{4\times10^{-6}}\right)\right) + 20mH\left(\frac{-9}{5} mA\right)\left(\frac{-1}{4\times10^{-6}}\right)^2$$

$$= -135.0000 = -1.35\ MV/s$$

6)



$V_1 = 4V$, $V_2 = 3V$, $R_1 = 50\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 200\,\Omega$, $C = 60\,\mu F$

a) In steady state, V_c is equal to voltage across R_3 because C is open circuit.

$$V_c(t<0) = \frac{R_3}{R_1+R_3} V_1 = \frac{200}{50+200}(4V)$$
$$= 3.2V$$

b) Capacitor voltage cannot change instanteously

$$V_c(0^+) = 3.2V$$

c) As $t \to \infty$, C is again an open circuit, therefore is equal to $-\left(\frac{R_3}{R_2+R_3}\right)V_2$

$$= -\left(\frac{200}{100+200}\right)3V = -2V$$

(negative because of sign reference)

d) The capacitor sees 2 resistors (R_2, R_3) in parallel, therefore $\tau = R_{eq}C$

$$= (R_2 \parallel R_3)\,C$$
$$= \frac{100(200)}{100+200}(60 \times 10^{-6})$$
$$= 0.004\,s$$

6) e)

Using formulas in the book / lecture notes, we have

Initial voltage across cap = 3.2 V

As $t \to 0$, $V_c(t) \to -2V$

Therefore, we have

$$v_c(t) = V_{final} + (V_0 - V_{final})\, e^{-t/RC}$$

$$= -2V + (3-(-2))\, e^{-t/RC}$$

$$= -2V + 5e^{-t/0.004} \quad \text{for } t \geq 0$$

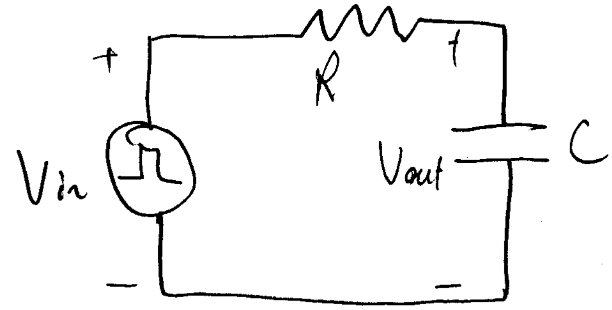
f) For Capacitor, $i_c = C \frac{dV_c}{dt}$

Therefore $i(t) = \frac{d}{dt} V_c(t)$

$$= 5e^{-t/0.004}\left(-\frac{1}{0.004}\right)$$

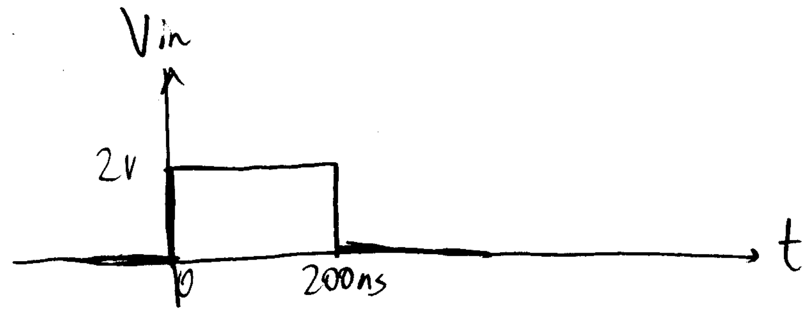
$$= -1250\, e^{-t/0.004}$$

7)



$R = 400\,\Omega$

$C = 0.001\,\mu F$



a) $\tau = RC = 400\,(0.001 \times 10^{-6})$

$= 4 \times 10^{-7}\,s$

Initially, $V_{out} = 0\,V$, therefore we have

$V_{out}(t) = V_{in}(1 - e^{-t/RC})$

$= 2V(1 - e^{-t/4\times10^{-7}s})$

b) We can do this in 2 steps;
we know that for $0 < t < 200\,ns$,

$V_{out}(t) = 2V(1 - e^{-t/4\times10^{-7}s})$

Since $200\,ns = 2 \times 10^{-7}\,s$ is not $\gg \tau$, we need to calculat $V_{out}(200ns)$

$V_{out}(200\,ns) = 2V(1 - e^{\frac{-2\times10^{-7}}{4\times10^{-7}}})$

$= 2V(1 - e^{-\frac{1}{2}}) \simeq 0.787V$

7) b) (Con't)

After the falling edge of the pulse, we have $V_{out}(200ns^+) = 0.787V$

for $t > 200ns$, we have

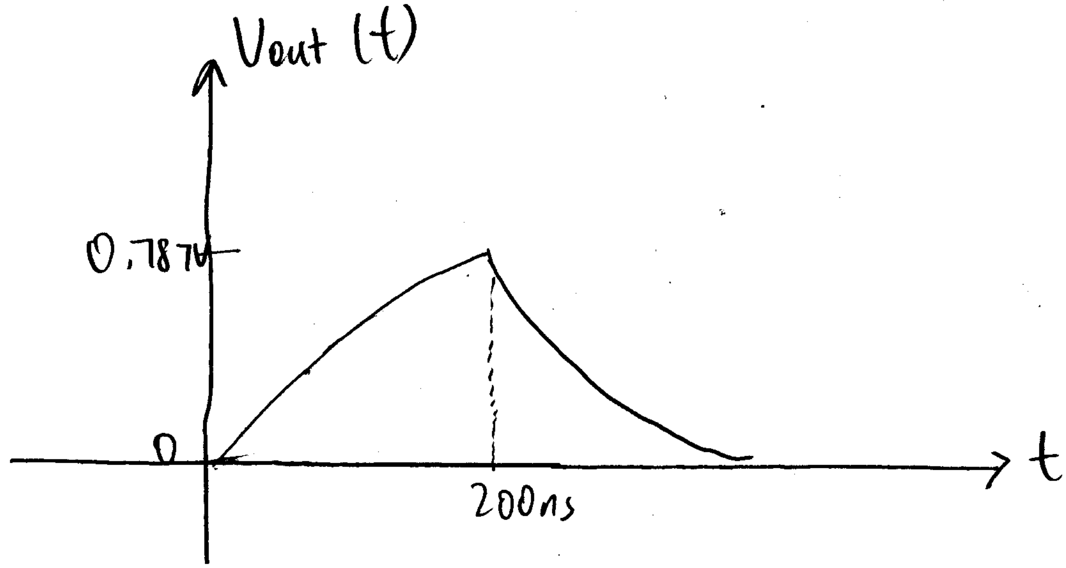
$$V_{out}(t) = 0 + (0.787 - 0)e^{-\frac{(t-200ns)}{400ns}}$$
$$= 0.787e^{-\frac{(t-200ns)}{400ns}}$$

Therefore the total voltage is

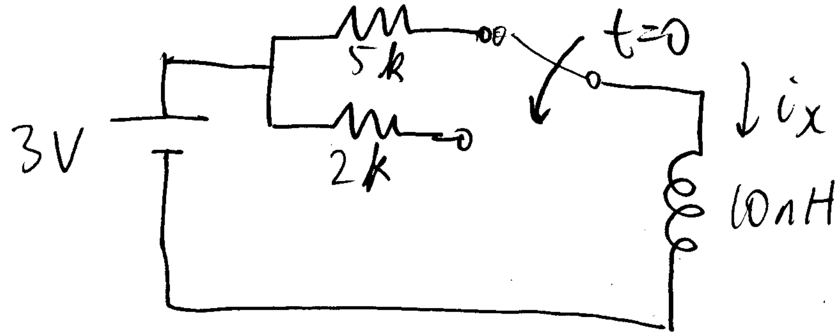
$$V_{out}(t) = \begin{cases} 2V(1-e^{-t/400ns} & , 0 \leq t \leq 200ns \\ 0.787e^{-(t-200ns)/400ns} & , t \geq 200ns \end{cases}$$

c) The maximum voltage occurs at $t = 200 ns$, $V_{out} = 0.787V$

d)



8)



a) In steady state, inductor is short circuit $\Rightarrow$ $i_x = \frac{3V}{5k} = 0.6\,mA$

b) Current through an inductor cannot change instanteously, therefore

$$i_x(0^+) = 0.6\ mA$$

c) for $t<0$, $\tau = \frac{L}{R_1} = \frac{10nH}{5k} = 2\times10^{-12}s$

d) for $t>0$, $\tau = \frac{L}{R_2} = \frac{10nH}{2k} = 5\times10^{-12}s$