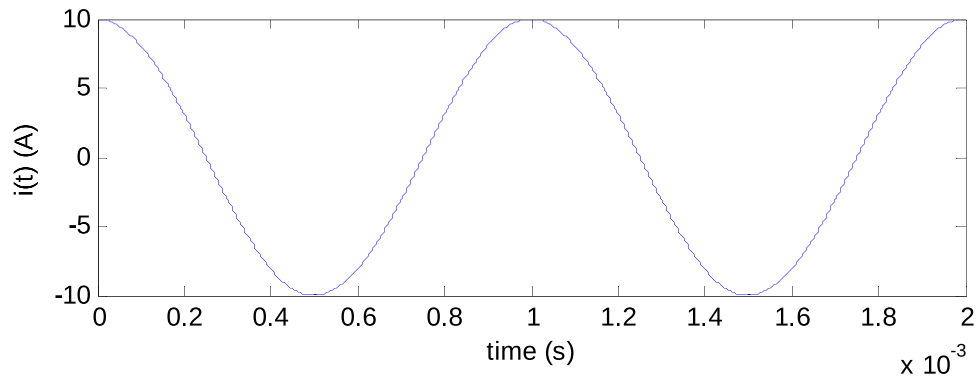
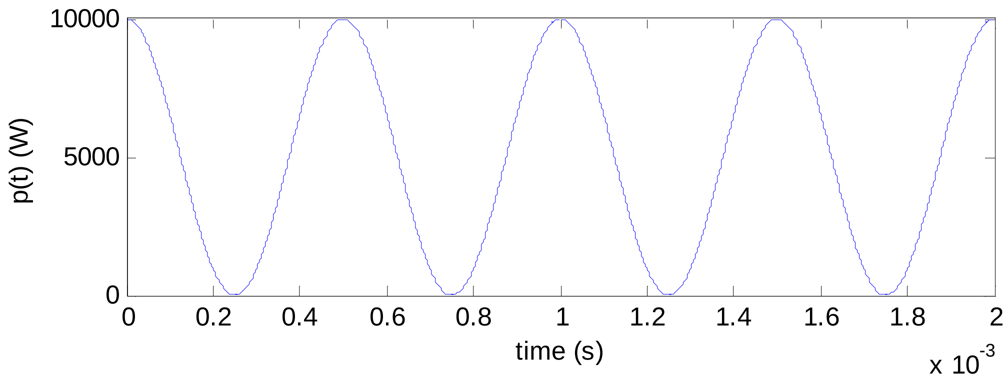
Problem 2

$$i(t) = 10 \cdot \cos(2000 \cdot \pi \cdot t)$$
$$R = 100$$

$$p(t) = i^2(t) \cdot R = 10^4 \cdot \cos^2(2000 \cdot \pi \cdot t) = \frac{10^4}{2}\left(1 + \cos(4000 \cdot \pi \cdot t)\right)$$





$$P_{avg} = \frac{1}{T}\int_T p(t)dt = \frac{1}{0.5 \times 10^{-3}} \int_0^{0.5\times10^{-3}} \frac{10^4}{2}\left(1 + \cos(4000 \cdot \pi \cdot t)\right)dt$$
$$= \frac{10^4}{2 \cdot 0.5 \times 10^{-3}}\left(0.5 \times 10^{-3}\right) = 5000 \ W$$

Problem 3

$$v(t) = 5 + 10 \cdot \cos(20 \cdot \pi \cdot t)$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_T v^2(t)dt} = \sqrt{\frac{1}{0.1}\int_0^{0.1}(5+10\cdot\cos(20\cdot\pi\cdot t))^2 dt}$$
$$= \sqrt{10}\cdot\sqrt{\int_0^{0.1}25dt + 100\cdot\int_0^{0.1}\cos(20\cdot\pi\cdot t) + \frac{100}{2}\int_0^{0.1}(1+\cos(40\cdot\pi\cdot t))dt}$$
$$= \sqrt{10}\cdot\sqrt{25\cdot 0.1 + 0 + (50\cdot 0.1 + 0)} = \sqrt{75} = 5\sqrt{3}\ V_{rms}$$

Problem 4

$$v(t) = A\cdot\cos(2\cdot\pi\cdot t) + B\cdot\sin(2\cdot\pi\cdot t)$$

$$v^2(t) = A^2\cdot\cos^2(2\cdot\pi\cdot t) + 2AB\cdot\cos(2\cdot\pi\cdot t)\cdot\sin(2\cdot\pi\cdot t) + B^2\cdot\sin^2(2\cdot\pi\cdot t)$$
$$= \frac{A^2}{2}(1+\cos(4\cdot\pi\cdot t)) + AB\cdot\sin(4\cdot\pi\cdot t) + \frac{B^2}{2}\cdot(1-\cos(4\cdot\pi\cdot t))$$

Now, the definition of root-mean-squared:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_T v^2(t)dt} = \sqrt{\int_0^1 \frac{A^2}{2}(1+\cos(4\cdot\pi\cdot t)) + AB\cdot\sin(4\cdot\pi\cdot t) + \frac{B^2}{2}\cdot(1-\cos(4\cdot\pi\cdot t))dt}$$
$$= \sqrt{\frac{A^2}{2}+\frac{B^2}{2}}\ V_{rms}$$

Problem 5

The two voltages can be expressed as:

$$v_a(t) = 8\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_{\mathrm{a}})$$
$$v_b(t) = 3\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_{\mathrm{b}})$$

If $\phi_a = \phi_b = \phi$, then our voltages have the same phase and they add to give us our maximum voltage:

$$v_a(t) + v_b(t) = 8\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi) + 3\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi) = 11\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi)$$
$$\Rightarrow \mathrm{V}_{\mathrm{RMS,max}} = 11$$

If $\phi_b = \phi_b \pm \pi$, then our signals are 180° out of phase, and thus our signal subtract directly from each other to give us our minimum voltage:

$$v_a(t) + v_b(t) = 8\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_a) + 3\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_a + \pi)$$
$$= 8\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_a) - 3\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_a) = 5\sqrt{2}\cos(2\cdot\pi\cdot \mathrm{t} + \phi_a)$$
$$\Rightarrow \mathrm{V}_{\mathrm{RMS,min}} = 5$$

<u>Problem 6</u>

$$v_1(t) = 10\cdot\cos\left(2\cdot\pi\cdot 200\times 10^3\cdot t + 30°\right)$$
$$v_2(t) = 5\cdot\cos\left(2\cdot\pi\cdot 200\times 10^3\cdot t + 150°\right)$$
$$v_3(t) = 10\cdot\cos\left(2\cdot\pi\cdot 200\times 10^3\cdot t + 90°\right)$$

v_1 lags v_2 by 120° and lags v_3 by 60°. v_2 leads v_1 by 120° and leads v_3 by 60°. v_3 leads v_1 by 60° and lags v_2 by 60°.

<u>Problem 7</u>

Rewrite the sine term as a cosine:

$$4\cdot\sin(\omega\cdot t) = 4\cdot\cos(\omega\cdot t - \pi/4)$$

Now we have:

$$5\cdot\cos(\omega t + 75°) - 3\cdot\cos(\omega t - 75°) + 4\cdot\cos(\omega t - 90°)$$
$$= \Re\left(5\cdot e^{j(\omega t + 75°)}\right) - \Re\left(3\cdot e^{j(\omega t - 75°)}\right) + \Re\left(4\cdot e^{j(\omega t - 90°)}\right)$$
$$= \Re\left(e^{jwt}\left(5e^{j75°} - 3e^{-j75°} + 4e^{-j90°}\right)\right)$$

Thus we need only convert the term multiplying the $e^{j\omega t}$ into a single complex number with magnitude and phase:

$$5e^{j75°} - 3e^{-j75°} + 4e^{-j90°} = 1.29 + j4.83 - 0.78 - j4.83 - j4 = 3.76\angle 82°$$

So our result is:

$$3.76\cdot\cos(\omega t + 82°)$$

<u>Problem 8</u>

Part a)

Convert the sine term into a cosine term:

$$v(t) = 100\cdot\sin\left(200t + 30°\right) = 100\cdot\cos(200t + 30° - 90°) = 100\cdot\cos(200t - 60°)$$

We express the voltage and current as phasors now:

$v = 100\angle -60°$ and $i = 1\angle 30°$

$$Z = \frac{v}{i} = \frac{100\angle -60°}{1\angle 30°} = 100\angle -90°$$

This is a capacitor. Since we know $\omega = 200$, we calculate the capacitor to be:

$$\frac{1}{\omega C} = 100 \Rightarrow C = 50 \ \ \mu F$$

Part b)

$v = 500\angle 50°$ and $i = 2\angle -40°$

$Z = 250\angle 90°$

This is an inductor, with value:

$\omega L = 250 \Rightarrow L = 1.25 \ \ H$

Part c)

$v = 100\angle 30°$ and $i = 1\angle -60°$

$Z = 100\angle 90°$

This is an inductor, with value:

$\omega L = 100 \Rightarrow L = 0.5 \ \ H$

<u>Problem 9</u>

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

If $\omega = 500$:

$$Z = 50 + j(500)(100\times 10^{-3}) + \frac{1}{j(500)(10\times 10^{-6})} = 50 - j150 \ \ \Omega$$

If $\omega = 1000$:

$$Z = 50 + j(1000)(100 \times 10^{-3}) + \frac{1}{j(1000)(10 \times 10^{-6})} = 50\Omega$$

If $\omega = 2000$:

$$Z = 50 + j(500)(100 \times 10^{-3}) + \frac{1}{j(500)(10 \times 10^{-6})} = 50 + j150 \ \ \Omega$$

<u>Problem 10</u>

The capacitor's impedance is:

$$Z_C = \frac{1}{j \cdot 500 \cdot 2 \times 10^{-6}} = -j1000 = 1000\angle -90°$$

The overall impedance is:

$$Z = 1000 - j1000 = 1414\angle -45°$$

The current then is:

$$i = \frac{v}{Z} = \frac{10\angle 0°}{1414\angle -45°} = 7.1 \times 10^{-3} \angle 45°$$

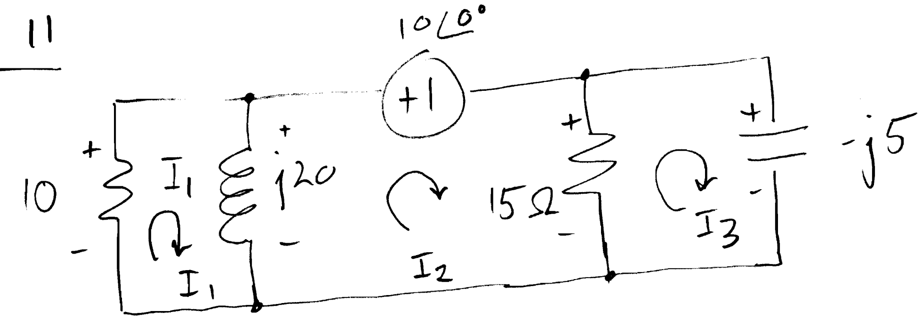
The voltage across the resistor is:

$$v_R = iR = (7.1 \times 10^{-3} \angle 45°) \cdot 1000 = 7.07\angle 45°$$

The voltage across the capacitor:

$$v_C = iZ_C = (7.1 \times 10^{-3} \angle 45°)(1000\angle -90°) = 7.07\angle -45°$$

prob 11



$$10 I_1 = j20(I_1 - I_2)$$

$$j20(I_2 - I_1) = 10 + 15(I_2 - I_3)$$

$$15(I_3 - I_2) = -j5 \cdot I_3$$

$$\begin{bmatrix} 10-j20 & j20 & 0 \\ -j20 & -15+j20 & 15 \\ 0 & -15 & 15+j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.94 \angle 150.4° \\ 1.05 \angle 177.0° \\ 0.997 \angle 158.6° \end{bmatrix}$$

$$v_1 = -I_1 \cdot R = -(0.94 \angle 150.4°)10 = 9.4 \angle -29.6°$$

$$v_2 = I_3 \cdot Z_c = (0.997 \angle 158.6°)(-5 \angle -90°)$$

$$= 4.99 \angle -111.4°$$

Check:

$$v_1 - v_2 = (9.4 \angle -29.6°) - (4.99 \angle -111.4°) = 10 \checkmark$$

Problem 12

Part a)

$$Z = \left(\frac{1}{j\omega C}\right) \| \left(R + j\omega L\right) = \frac{\left(\frac{1}{j\omega C}\right)\left(R + j\omega L\right)}{\frac{1}{j\omega C} + R + j\omega L} = \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC}$$

Part b)

Remembering that:

$$\frac{1}{a + jb} = \frac{a}{a^2 + b^2} - j\frac{b}{a^2 + b^2}$$

We write:

$$Z = \frac{\left(R + j\omega L\right)\left(1 - \omega^2 LC - j\omega RC\right)}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2} = \frac{R\left(1 - \omega^2 RC\right) + \omega^2 LRC}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2} + j\frac{\left(1 - \omega^2 LC\right)\omega L - \omega R^2 C}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2}$$

$$|Z| = \sqrt{\frac{\left(R\left(1 - \omega^2 RC\right) + \omega^2 LRC\right)^2}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2} + \frac{\left(\left(1 - \omega^2 LC\right)\omega L - \omega R^2 C\right)^2}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2}}$$
$$= \frac{\sqrt{\left(R\left(1 - \omega^2 RC\right) + \omega^2 LRC\right)^2 + \left(\left(1 - \omega^2 LC\right)\omega L - \omega R^2 C\right)^2}}{\left(1 - \omega LC\right)^2 + \left(\omega RC\right)^2}$$

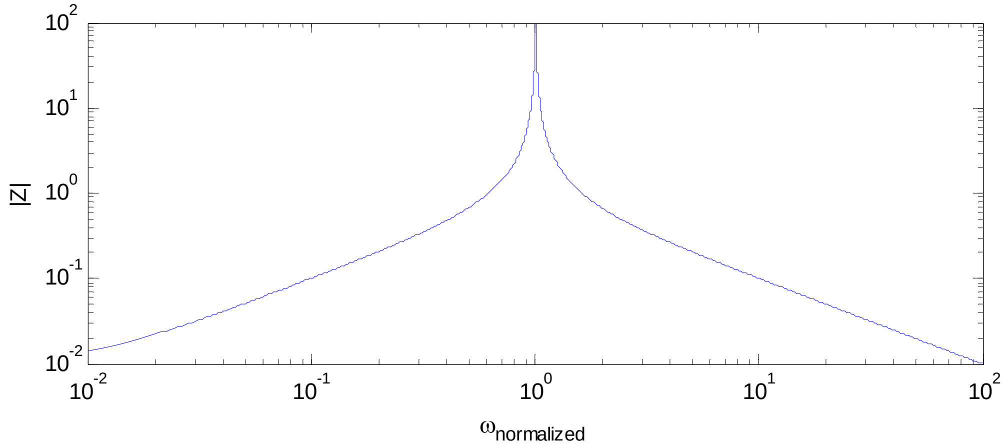
Part c)

If we assume that $\left(RC\right)^2 << LC$, then our denominator becomes zero when:

$$1 - \omega^2 LC = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

Part d)

To plot $|Z|$, select L=1, C=1, and R=0.01. This way, our assumption is justified.



Problem 13

For the circuit in Figure a), the limits are:

$$v_{\omega \to 0} = 0$$

$$v_{\omega \to \infty} = \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}$$

For the circuit in Figure b), the limits are:

$$v_{\omega \to 0} = 0$$

$$v_{\omega \to \infty} = 0$$

The parasitics have a greater effect at higher frequencies.