

1. (Reading Assignment)

Sections 6.2, 6.3, 6.6 and 6.8 of Chapter 6 as well as sections 11.1 and 11.2 of Chapter 11: Hambley 3rd edition.

2. Problem 5.53

Since the reactance is negative, the load is capacitive

$$\text{Power delivered is } P_d = I_{rms}^2 \times R = (15)^2 \times 100 = 22.5kW$$

$$\text{Reactive power is } P_r = I_{rms}^2 \times X = (15)^2 \times (-50) = -11.25kVAR$$

Power factor is

$$P_f = \cos(\mathbf{q}) = \cos\left[\tan^{-1}\left(\frac{P_r}{P_d}\right)\right] = \cos[\tan^{-1}(-0.5)] = \cos[26.57^\circ] = 89.44\%$$

3. Problem 5.56

First let us compute the rms current (I_{rms})

$$I = \frac{240\sqrt{2}\angle 50^\circ - 220\sqrt{2}\angle 30^\circ}{1 + j2} = 52.\angle 52.71^\circ \Rightarrow I_{rms} = 36.8A$$

Knowing the current (I_{rms}), we can now compute the power for each source (absorbed or delivered)

- Power delivered by source A

$$P_A = 240I_{rms} \times \cos(50 - 52.71) = 8.8kW$$

$$Q_A = 240I_{rms} \times \sin(50 - 52.71) = -0.42KVAR$$

- Power dissipated/absorbed by source B

$$P_B = 220I_{rms} \times \cos(30 - 52.71) = 7.5kW$$

$$Q_B = 220I_{rms} \times \sin(30 - 52.71) = -3.1KVAR$$

- Power dissipated/absorbed by the resistor

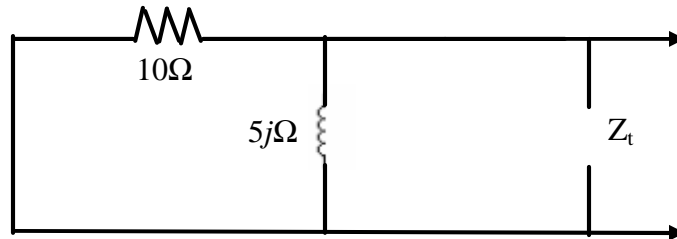
$$P_R = I_{rms}^2 \times R = (36.8)^2 \times 1\Omega = 1.35kW$$

- Power dissipated/absorbed by the inductor

$$P_r = I_{rms}^2 \times X = (36.8)^2 \times (2\Omega) = 2.7kVAR$$

4. Problem 5.68

We are asked to find the Thevenin and Norton of the circuit given:
After Zeroing the sources, we have:



Now we can compute the Thevenin impedance as follow:

$$Z_t = \frac{1}{1/10 + 1/j5} = 4.472 \angle 63.43^\circ = 2 + j4$$

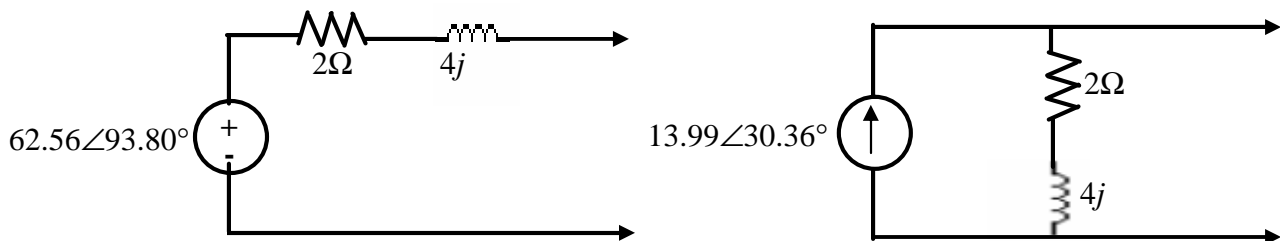
Using KCL to compute for the current flowing at the upper end of the current source under open circuit conditions, we get:

$$\frac{V_{oc} - 100 \angle 45^\circ}{10} + \frac{V_{oc}}{j5} = 5, \text{ Solving for } V_{oc}, \text{ we get}$$

$$V_t = V_{oc} = 62.56 \angle 93.80^\circ$$

$$\text{And } I_t = \frac{V_t}{Z_t} = 13.99 \angle 30.36^\circ$$

Therefore, the Thevenin and Norton equivalences are given by:



- The maximum power that this circuit can deliver to a load with complex impedance is computed as follow:

$$P_{load} = I_{rms-load}^2 \times Z_{load}$$

$$\text{Given } Z_{load} = 2 - j4 \text{ and } I_{load} = \frac{V_t}{Z_t + Z_{load}} = \frac{62.56 \angle 93.80^\circ}{2 + j4 + 2 - j4} = 15.64 \angle 93.80^\circ$$

$$P_{load} = I_{rms-load}^2 \times Z_{load} = 244.6W$$

- The maximum power that this circuit can deliver to a load with complex impedance is computed as follow:

$$P_{load} = I_{rms-load}^2 \times Z_{load}$$

$$\text{Given } Z_{load} = R_{load} = 4.472 \text{ \& } I_{load} = \frac{V_t}{Z_t + Z_{load}} = \frac{62.56 \angle 93.80^\circ}{2 + j4 + 4.472} = 8.223 \angle -62.08^\circ$$

$$P_{load} = I_{rms-load}^2 \times R_{load} = 151.2W$$

5. Problem 6.9

The phasors for the steady state input and output of this filter are given as follow:

$$V_{in} = 2 \angle -25^\circ \text{ and } V_{out} = 1 \angle 20^\circ$$

Therefore, the complex value of the filter transfer function at $f=5,000\text{Hz}$ will be given as:

$$H(5,000) = \frac{V_{out}}{V_{in}} = 0.5 \angle 45^\circ$$

6. Problem 6.12

Given the input signal provided as

$$V_{in}(t) = 1 + 2 \cos(2000pt) + 3 \sin(3000pt) + 4 \cos(4000pt)$$

One can extract the following frequency components: 0, 1000, 1500 and 2000 Hz

The transfer function for each of these frequencies is compute by dividing the corresponding output to the input. This results in the following:

$$H(0) = \frac{3}{1} = 3, \quad H(1,000) = \frac{4 \angle 30^\circ}{2 \angle 0^\circ} = 2 \angle 30^\circ$$

$$H(1,500) = \frac{3 \angle 0^\circ}{3 \angle -90^\circ} = 1 \angle 90^\circ, \quad H(2,000) = \frac{0}{4 \angle 0^\circ} = 0$$

7. Problem 6.19

Given the input signal to be $V_{in}(t) = 1 + 2\cos(2000\pi t) + 3\sin(3000\pi t) + 4\cos(4000\pi t)$ with frequencies components of 250, 500 and 1000 Hz

The half power of the filter frequency is $f_b = \frac{1}{2\pi RC} = 500\text{Hz}$

The transfer function is given by equation 6.9 in the text book $H(f) = \frac{1}{1 + (f/f_b)}$

After computing the transfer function for the different component frequencies of V_{in} , we get the following:

$$H(250) = \frac{1}{1 + j(250/500)} = 0.89 \angle -26.57^\circ$$

$$H(500) = \frac{1}{1 + j(500/500)} = 0.707 \angle -45^\circ$$

$$H(1000) = \frac{1}{1 + j(1000/500)} = 0.44 \angle -63.43^\circ$$

Applying the appropriate value of the transfer function to each of these component of the input signal allows us to an expression for the output signal which results to:

$$V_{out}(t) = 4.472 \cos(500\pi t - 26.57^\circ) + 3.535 \cos(1000\pi t - 45^\circ) + 2.236 \cos(2000\pi t - 63.43^\circ)$$

8. Problem 6.33

To convert to decibels, we need to take 20 times the common logarithm of a transfer function. Therefore, we have:

$$20\log(0.5) = -6.021\text{dB} \quad \text{and} \quad 20\log(2) = +6.021\text{dB}$$