EE 100/42 Spring 2006

Homework #7 Solutions

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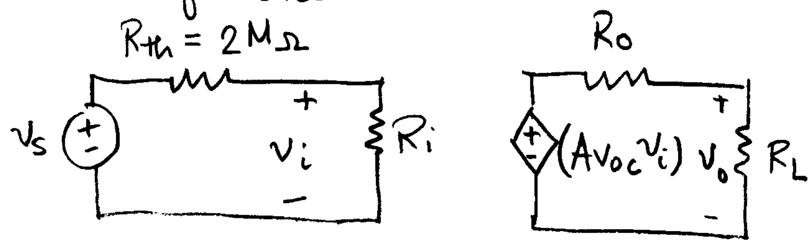
P 11. 2

An amplifier can be characterized by:

(1) input impedance
(2) output impedance
(3) gain (i.e. open-circuit voltage gain)

P 11.6

The equivalent circuit is:



Given:

$$\begin{cases} R_i = 1\,M\Omega \\ R_o = 1\,K\Omega \\ R_L = 1\,K\Omega \end{cases}$$

$V_S = 3\cos(200\pi t)\,mV$

$A_{voc} = -10^4$

$$V_i(t) = \frac{R_i}{R_i + R_{th}} \cdot V_S = \frac{10^6}{10^6 + 2\times 10^6}\left(3\times 10^{-3}\cos(200\pi t)\right)$$

$$= 10^{-3}\cos(200\pi t)\ V = \cos(200\pi t)\ mV$$

$$V_o(t) = A_{voc} V_i \cdot \frac{R_L}{R_o + R_L} = -10^4\left[10^{-3}\cos(200\pi t)\right]\left(\frac{1000}{1000+1000}\right)$$

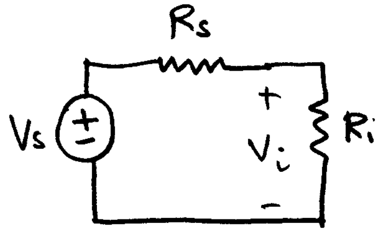
$$\boxed{V_o(t) = -5\cos(200\pi t)\ V}$$

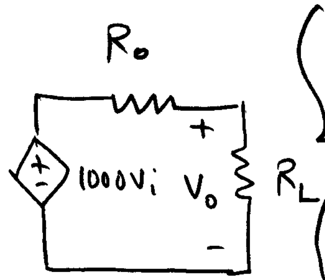
$$P_i = \frac{V_i^2(rms)}{R_i} = \frac{\left(10^{-3}/\sqrt{2}\right)^2}{10^6} = \frac{1}{2}\ pW$$

$$P_o = \frac{V_o^2(rms)}{R_L} = \frac{\left(5/\sqrt{2}\right)^2}{10^3} = 12.5\ mW$$

$$G = \frac{P_o}{P_i} = \boxed{25\times 10^9}$$

P 11.12





$$\left\{\begin{array}{l} A_{voc}=1000 \\ R_i = 20\,k\Omega \\ R_o = 2\,\Omega \\ R_s = 10\,k\Omega \\ R_L = 8\,\Omega \end{array}\right.$$

$$V_o = 1000V_i\left(\frac{R_L}{R_o+R_L}\right)$$

$$A_v = \frac{V_o}{V_i} = 1000\left(\frac{R_L}{R_o+R_L}\right) = 1000\left(\frac{8}{2+8}\right) = \boxed{800}$$

$$V_i = V_s\left(\frac{R_i}{R_s+R_i}\right)$$

$$\frac{V_i}{V_s} = \frac{R_i}{R_s+R_i} = \frac{20\times10^3}{20\times10^3+10\times10^3} = \frac{2}{3}$$

$$A_{vs} = \frac{V_o}{V_s} = \frac{\cancel{V_i}}{V_s}\cdot\frac{V_o}{\cancel{V_i}} = \frac{V_i}{V_s}\cdot A_v = \left(\frac{2}{3}\right)(800) = \boxed{533.3}$$

$$A_i = \frac{V_o/R_L}{V_i/R_i} = \frac{V_o}{V_i}\cdot\frac{R_i}{R_L} = A_v\cdot\frac{R_i}{R_L} = 800\left(\frac{20^k}{8}\right)$$

$$= \boxed{2\times10^6}$$

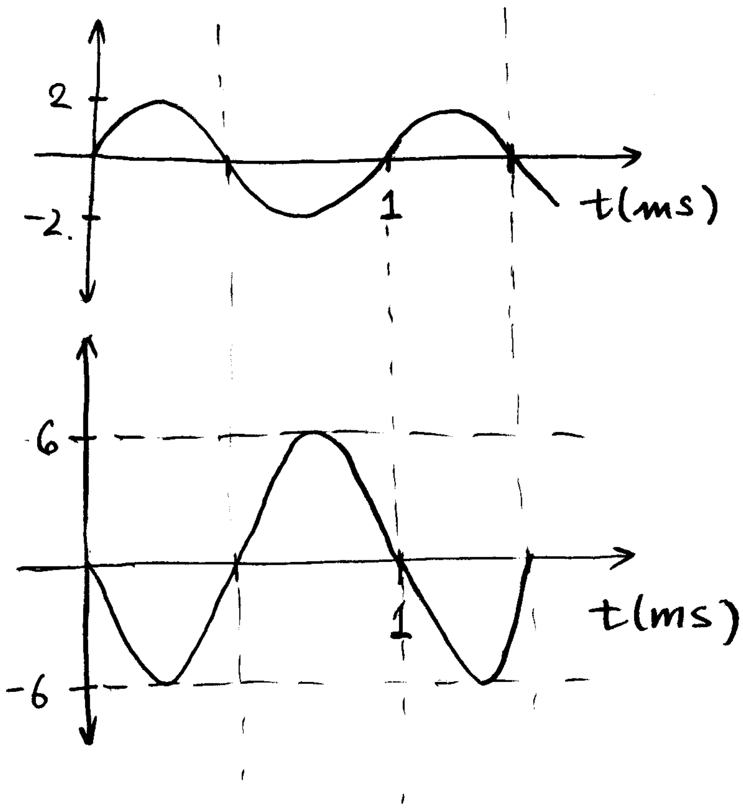
$$G = A_v\cdot A_i = 800(2\times10^6) = \boxed{1.6\times10^9}$$

P 14.8 Inverting Amplifier

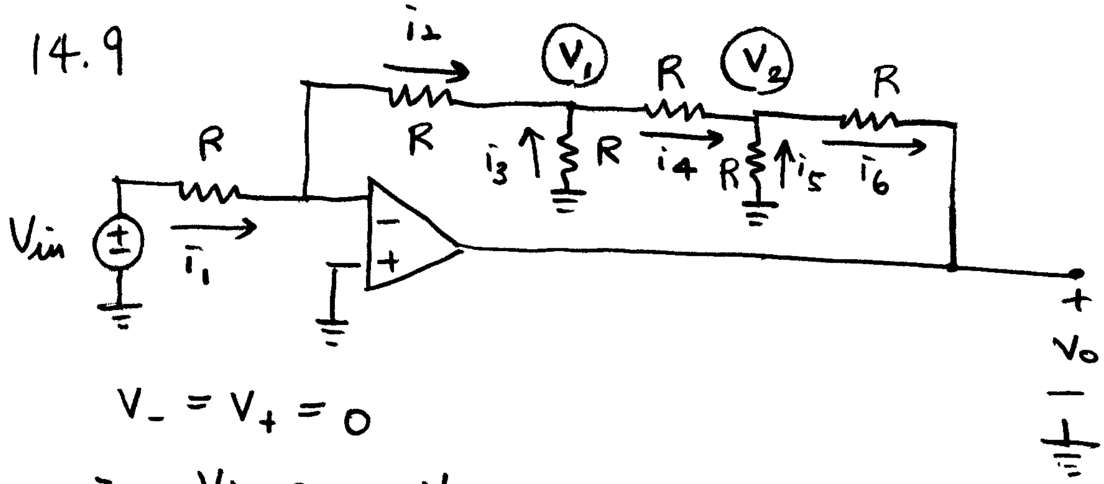
$$A_v = -\frac{R_2}{R_1} = -3$$

$$V_o(t) = -3(2\cos(2000\pi t)) = -6\cos(2000\pi t)\ V$$

$v_{in}(t)$



P 14.9



$$V_- = V_+ = 0$$

$$i_1 = \frac{V_{in}-0}{R} = \frac{V_{in}}{R}$$

$$i_1 = i_2$$

$$i_2: \quad \frac{0-V_1}{R} = \frac{V_{in}}{R}$$

$$V_1 = -V_{in}$$

$$i_3 = \frac{0-V_1}{R} = \frac{-V_1}{R} = \frac{V_{in}}{R}$$

$$i_4 = i_2 + i_3 = \frac{V_{in}}{R} + \frac{V_{in}}{R} = \frac{2V_{in}}{R}$$

$$i_4: \quad \frac{V_1 - V_2}{R} = \frac{2V_{in}}{R} \Rightarrow V_1 - V_2 = 2V_{in}$$

$$V_2 = V_1 - 2V_{in} = -V_{in} - 2V_{in} = -3V_{in}$$

$$i_5: \quad \frac{0-V_2}{R} = \frac{3V_{in}}{R}$$

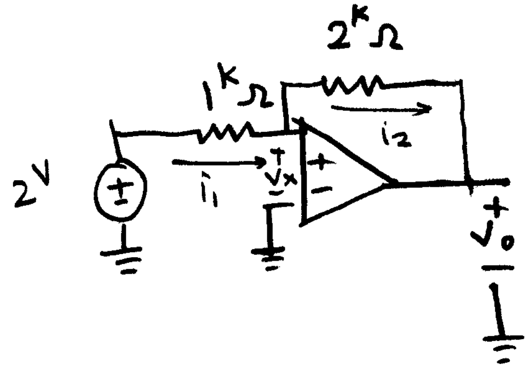
$$i_6 = i_4 + i_5 = \frac{2V_{in}}{R} + \frac{3V_{in}}{R} = \frac{5V_{in}}{R}$$

$$i_6: \quad \frac{V_2 - V_o}{R} = \frac{5V_{in}}{R}$$

$$V_2 - V_o = 5V_{in} \Rightarrow V_o = V_2 - 5V_{in} = -3V_{in} - 5V_{in} = -8V_{in}$$

$$\boxed{A_v = \frac{V_o}{V_{in}} = -8}$$

P 14.10



This circuit has <u>positive</u> feedback, and the output swings from -10^V to $+10^V$.

Writing a current equation in terms of V_x and V_o:

$$i_1 = i_2$$

$$\frac{2 - V_x}{1^K} = \frac{V_x - V_o}{2^K}$$

$$\frac{2}{1^K} - \frac{V_x}{1^K} = \frac{V_x}{2^K} - \frac{V_o}{2^K}$$

$$\frac{2}{1^K} + \frac{V_o}{2^K} = \frac{V_x}{1^K} + \frac{V_x}{2^K}$$

$$V_x\left(\frac{1}{1^K} + \frac{1}{2^K}\right) = \frac{2}{1^K} + \frac{V_o}{2^K}$$

$$V_x = \frac{\frac{2}{1^K} + \frac{V_o}{2^K}}{\frac{1}{1^K} + \frac{1}{2^K}} = 1.3333 + 0.3333\, V_o$$

For $V_o = 10^V$

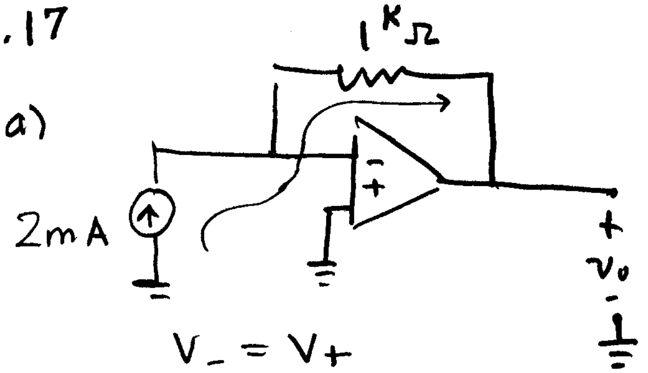
$$V_x = 1.3333 + 0.3333(10^V) \approx \boxed{4.6^V} \quad (4.3^V \text{ o.K.})$$

For $V_o = -10^V$

$$V_x = 1.3333 + 0.3333(-10^V) \approx \boxed{-2^V}$$

P 14.17

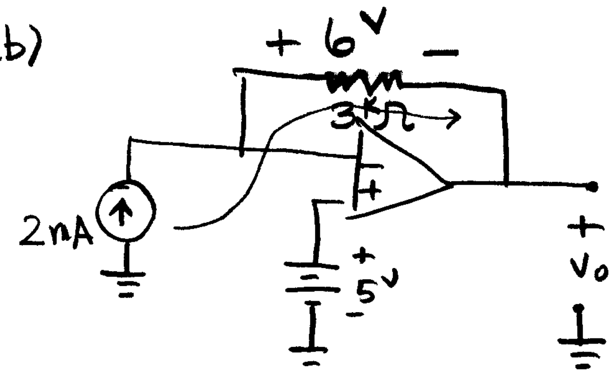
(a)



$V_- = V_+$

$V_o = -2\text{ mA}(1^{k}) = \boxed{-2^{V}}$

(b)

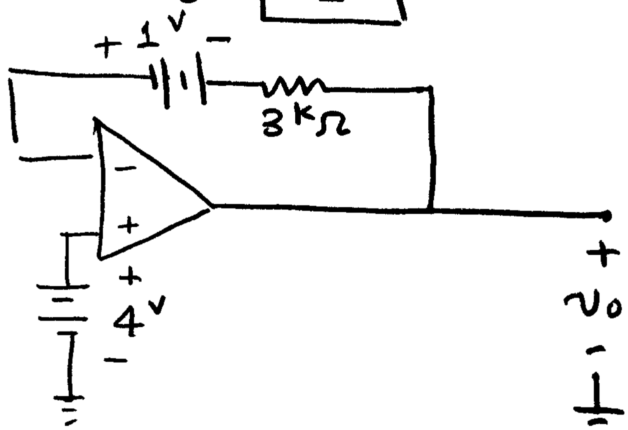


Voltage across $1^{k}\Omega$: $2\text{ mA}(3^{k}) = 6^{V}$

$V_- = V_+$

$V_o = -6 + 5 = \boxed{-1^{V}}$

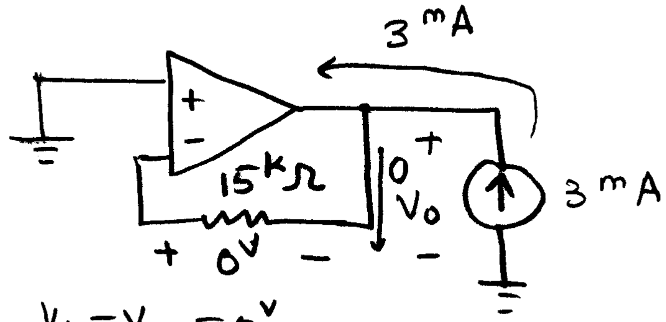
(c)



No current flows through the $3^{k}\Omega$ resistor.

$V_o = 4 - 1 = \boxed{3^{V}}$

(d)



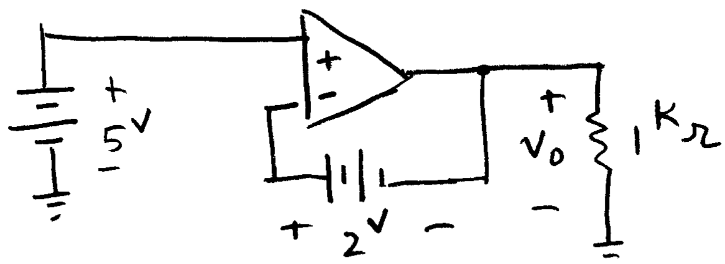
$V_+ = V_- = 0^V$

$\therefore$ No current may flow into V_+ and V_- terminals.

Thus, there is no current flowing across the $15^{k}\Omega$ resistor.

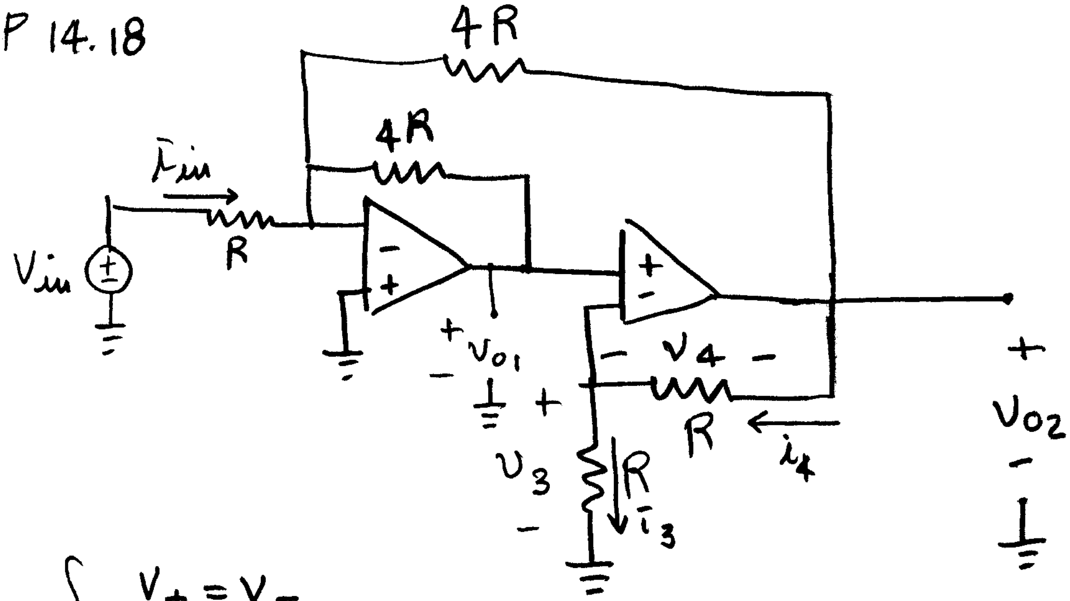
$$\boxed{V_o = 0^V}$$

(e)



$$V_o = 5 - 2 = \boxed{3^V}$$

P 14.18



$$\begin{cases} V_+ = V_- \\ v_{o1} = v_3 \end{cases}$$

$$\begin{cases} i_4 = i_3 = \dfrac{v_{o1}}{R} \\ V_4 = V_3 = v_{o1} \end{cases}$$

$$v_{o2} = V_3 + V_4 = 2v_{o1} \quad (1)$$

$$i_{in} + \frac{V_{o1}}{4R} + \frac{V_{o2}}{4R} = 0$$

$$i_{in} = \frac{V_{in}}{R}$$

$$\frac{V_{in}}{R} + \frac{V_{o1}}{4R} + \frac{V_{o2}}{4R} = 0 \quad (2)$$

For A_1:

Substitute into (2): $v_{o2} = 2v_{o1}$ (1)

$$\frac{V_{in}}{R} + \frac{V_{o1}}{4R} + \frac{2V_{o1}}{4R} = 0$$

$$\frac{V_{in}}{\cancel{R}} = \frac{-3V_{o1}}{4\cancel{R}}$$

$$\boxed{A_1 = \frac{v_{o1}}{v_{in}} = -4/3}$$

For A_2:

$$A_2 = \frac{v_{o2}}{v_{in}} = \frac{2V_{o1}}{V_{in}} = 2\left(-\frac{4}{3}\right)$$

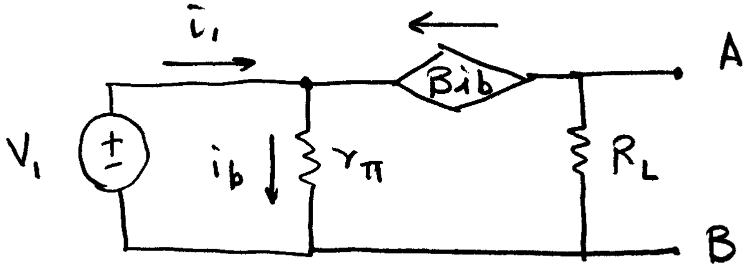
$$\boxed{A_2 = \frac{-8}{3}}$$

(4.9) $V_1 = 2^V$

$r_\pi = 2.5^k\Omega$

$R_L = 5^k\Omega$

$\beta = 100$



Find i_1.

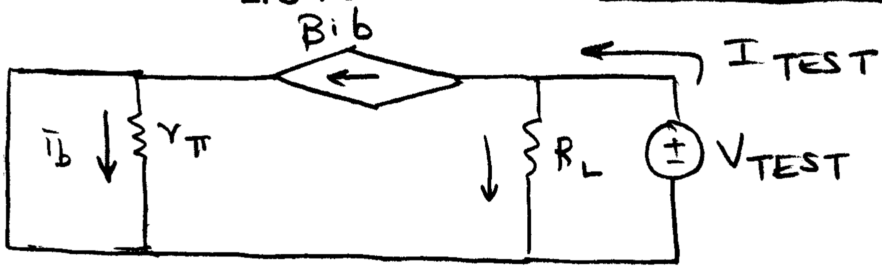
$$i_1 + \beta i_b = i_b$$

$$i_1 = i_b - \beta i_b = i_b(1-\beta)$$

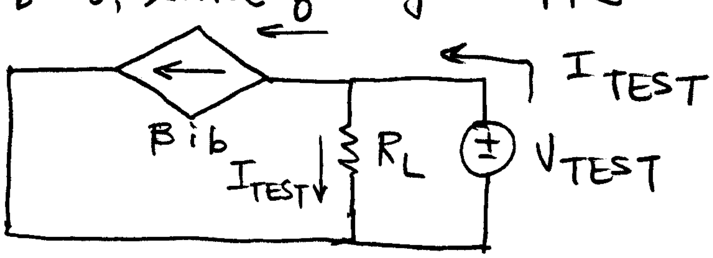
$$i_b = \frac{V_1}{r_\pi} = \frac{2^V}{2.5^k\Omega}$$

$$i_1 = \frac{2^V}{2.5^k\Omega}(1-100) = \boxed{-79.2\,mA}$$

(4.11)



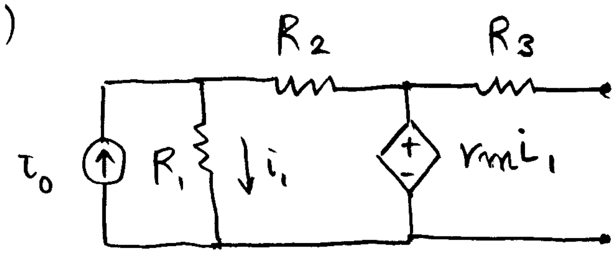
ib = 0, since voltage suppy is shorted.



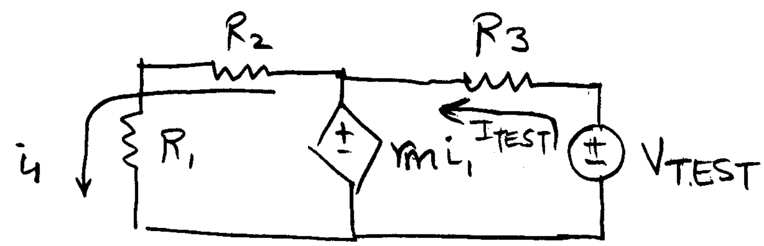
$$V_{TEST} = I_{TEST} \cdot R_L$$

$$\boxed{R_{th} = \frac{V_{TEST}}{I_{TEST}} = R_L}$$

(4.12)



open current sources.



KVL: $V_{TEST} - I_{TEST} R_3 - r_m i_1 = 0$

i_1 must flow through R_2:

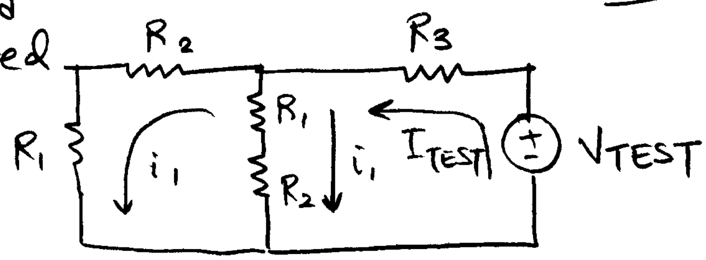
$$r_m i_1 - i_1 R_2 - i_1 R_1 = 0$$

① $r_m = R_1 + R_2$ OR. ② If assume $r_m \neq R_1 + R_2$

$i = 0$

$$\boxed{R_{th} = R_3}$$

Circuit could be represented as:



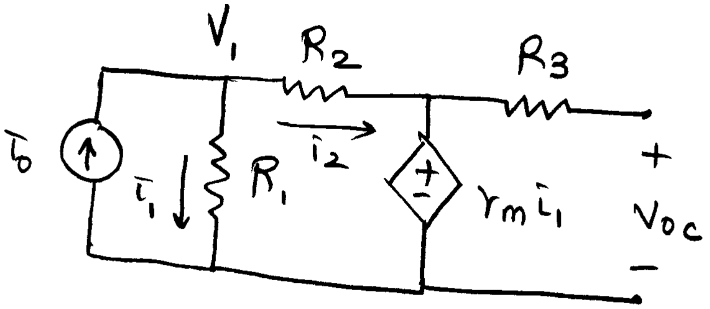
$$I_{TEST} = i_1 + i_1 = 2i_1$$

$$i_1 = \frac{I_{TEST}}{2}$$

$$V_{TEST} - I_{TEST} R_3 - r_m \left(\frac{I_{TEST}}{2}\right) = 0$$

$$V_{TEST} = I_{TEST}\left(R_3 + \frac{r_m}{2}\right)$$

$$R_{th} = \frac{V_{TEST}}{I_{TEST}} = \boxed{R_3 + \frac{r_m}{2}}$$



No current flowing across R_3, open circuit.

$\therefore V_{oc} = r_m i_1$

$i_0 = i_1 + i_2$

$$i_0 = \frac{V_1}{R_1} + \frac{V_1 - r_m i_1}{R_2}$$

$$V_1 = i_1 R_1$$

$$i_0 = \frac{i_1 \cancel{R_1}}{\cancel{R_1}} + \frac{i_1 R_1 - r_m i_1}{R_2}$$

$$i_0 = i_1 \left(1 + \frac{R_1 - r_m}{R_2}\right)$$

$$i_0 = i_1 \left(\frac{R_2 + R_1 - r_m}{R_2}\right)$$

$$i_1 = i_0 \left(\frac{R_2}{R_2 + R_1 - r_m}\right)$$

$$\therefore V_{oc} = r_m i_1 = r_m \left(\frac{R_2}{R_2 + R_1 - r_m}\right) i_0$$

$$\boxed{V_{Th} = V_{oc} = \frac{i_0 r_m R_2}{R_1 + R_2 - r_m}}$$