Quick introduction to phasor analysis for the RC filter lab

Very brief introduction to phasors

Using phasors to analyze a low-pass
RC filter circuit

“PHASORS”

You can solve AC circuit analysis problems that involve

Circuits with linear elements (R, C, L) plus independent and dependent voltage and/or current sources operating at a single angular frequency $\omega = 2\pi f$ (radians/s) such as $v(t) = V_0\cos(\omega t)$ or $i(t) = I_0\cos(\omega t)$

By using any of Ohm's Law, KVL and KCL equations, doing superposition, nodal or mesh analysis, AND

Using instead of the functions of time below on the left, the phasor expressions below on the right:

RESISTOR I-V relationship in terms of

v(t) and i(t) OR phasor voltage $\mathbf{V_R}$ and phasor current $\mathbf{I_R}$ (not functions of time)

$v_R = i_R R$ .... $\mathbf{V_R} = \mathbf{I_R} R$ where R is the resistance in ohms,

$\mathbf{V_R}$ = phasor voltage, $\mathbf{I_R}$ = phasor current

(boldface indicates complex quantity)

CAPACITOR I-V relationship in terms of

v(t) and i(t) OR phasor voltage $\mathbf{V_C}$ and phasor current $\mathbf{I_C}$ (not functions of time)

$i_C = C dv_C/dt$ .... $\mathbf{I_C} = \mathbf{V_C} / \mathbf{Z_C}$, with capacitive impedance $\mathbf{Z_C} = 1/\mathbf{j}\omega C$, where

$\mathbf{j} = (-1)^{1/2}$ and boldface type indicates a complex quantity

Thus, $\mathbf{I_C} = \mathbf{j}\omega C \mathbf{V_C}$

[We'll see later that the ω comes from the time derivative of a sinusoidal voltage $v_c(t) = V_C\cos(\omega t)$, etc.]

INDUCTOR I-V relationship in terms of

v(t) and i(t) OR phasor voltage $\mathbf{V_L}$ and phasor current $\mathbf{I_L}$ (not functions of time)

$v_L = L di_L/dt$ .... $\mathbf{V_L} = \mathbf{I_L}\mathbf{Z_L}$, with inductive impedance $\mathbf{Z_L} = \mathbf{j}\omega L$, where

$\mathbf{j} = (-1)^{1/2}$ and boldface type indicates a complex quantity

Thus, $\mathbf{V_L} = \mathbf{j}\omega L \mathbf{I_L}$

[We'll see later that the ω comes from the time derivative of a sinusoidal current $i_L(t) = I_L\cos(\omega t)$, etc.]

$v_R = i_R R$
$i_C = C dv_C/dt$
$v_L = L di_L/dt$

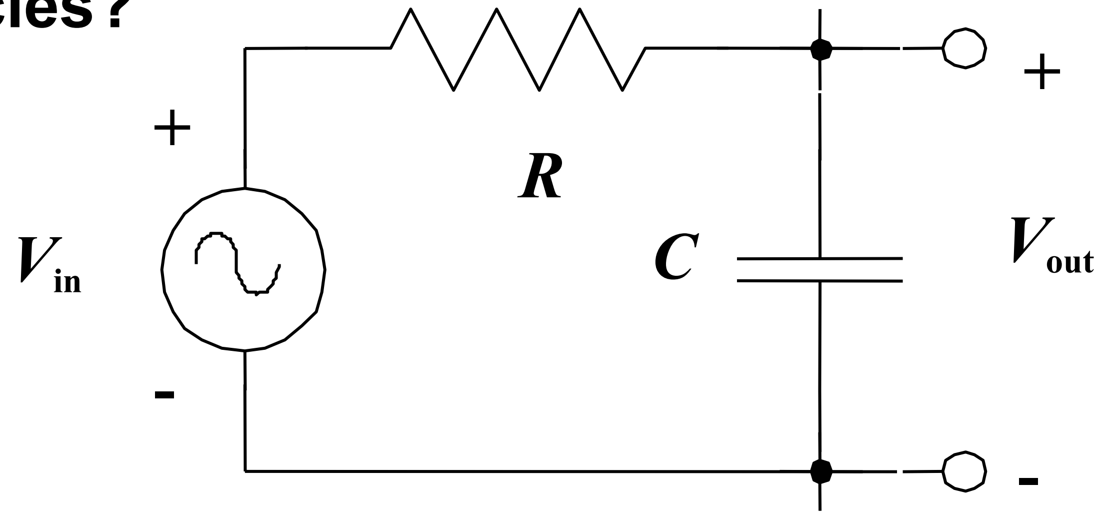
Use with steady or transient sources

$\mathbf{V_R} = \mathbf{I_R} R$
$\mathbf{I_C} = \mathbf{V_C}/\mathbf{Z_C} = (\mathbf{j}\omega C)\mathbf{V_C}$
$\mathbf{V_L} = \mathbf{I_L}\mathbf{Z_L} = (\mathbf{j}\omega L)\mathbf{I_L}$

Use with single-frequency sinusoidal sources

Using phasors to analyze a low-pass RC filter

Consider the circuit shown below. We want to use phasors and complex impedances to find how the ratio $|V_{out}/V_{in}|$ varies as the frequency of the input sinusoidal source changes. This circuit is a filter; how does it treat the low frequencies and the high frequencies?



Assume the input voltage is $v_{in}(t) = V_{in}\cos(\omega t)$ and represent it by the phasor $\mathbf{V_{in}}$. A phasor current **I** flows clockwise in the circuit.

Write KVL: $-\mathbf{V}_{in} + \mathbf{I}R + \mathbf{I}\mathbf{Z}_C = 0 = -\mathbf{V}_{in} + \mathbf{I}(R + \mathbf{Z}_C)$

The phasor current is thus $\mathbf{I} = \mathbf{V}_{in}/(R + \mathbf{Z}_C)$

The phasor output voltage is $\mathbf{V}_{out} = \mathbf{I}\,\mathbf{Z}_C$.

Thus $\mathbf{V}_{out} = \mathbf{V}_{in}[\mathbf{Z}_C /(R + \mathbf{Z}_C)]$

If we are only interested in the dependence upon frequency of the <u>magnitude</u> of $(\mathbf{V}_{out} / \mathbf{V}_{in})$, we can write

$$|\mathbf{V}_{out} / \mathbf{V}_{in}| = |\mathbf{Z}_C/(R + \mathbf{Z}_C)| = 1/|1 + R/\mathbf{Z}_C|$$

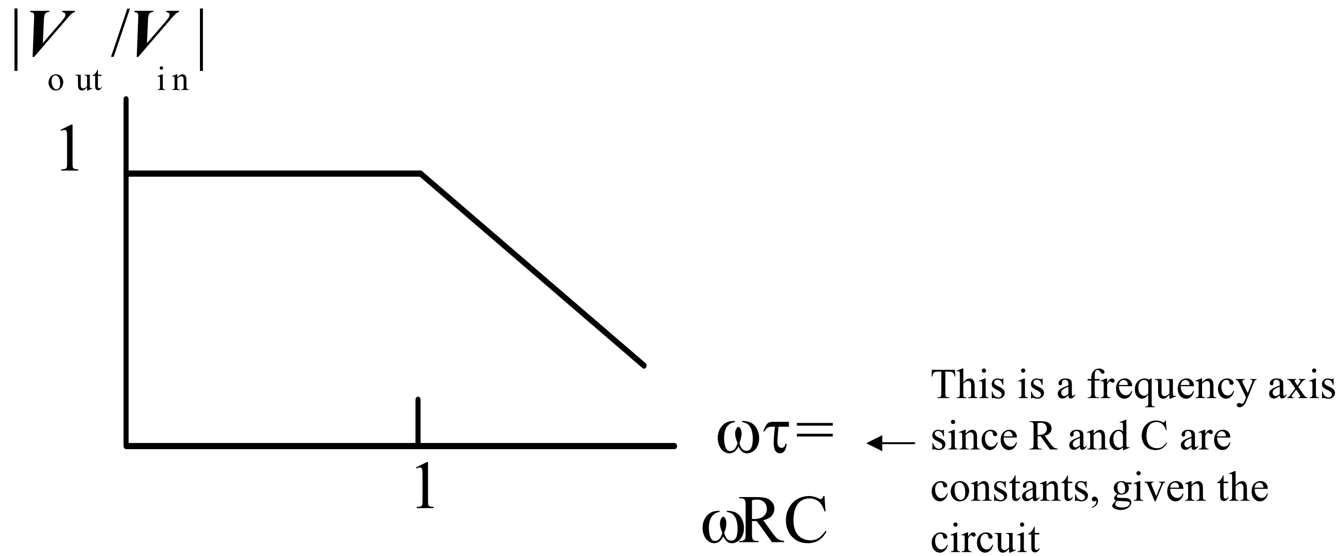
Substituting for $\mathbf{Z}_C$, we have $1 + R/\mathbf{Z}_C = 1 + j\omega RC$, whose magnitude is the square root of $(\omega RC)^2 + 1$. Thus,

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

Explore the Result

If $\omega RC \ll 1$ (low frequency) then $| \mathbf{V}_{out} / \mathbf{V}_{in} | = 1$
If $\omega RC \gg 1$ (high frequency) then $| \mathbf{V}_{out} / \mathbf{V}_{in} | \sim 1/\omega RC$

If we plot $| \mathbf{V}_{out} / \mathbf{V}_{in} |$ vs. ωRC we obtain roughly the plot below, which was plotted on a log-log plot:



The plot shows that this is a low-pass filter. Its cutoff frequency is at the frequency ω for which $\omega RC = 1$.

Decibel: Logarithmic measure for power, voltage and current ratios

Power: To express a power, P, relative to a reference power, $P_{reference}$, we define the power in decibels as: $\text{Power P in decibels (dB)} = 10 \log_{10}(P/P_{reference})$. [E.g., if P = 2 mW and $P_{reference}$ = 1 mW, $P = 10 \log_{10}(2/1) = 10 \log_{10}(2) = 10 \times 0.301 = 3$ dB re 1 mW.]

Voltage and current: Suppose that the voltage V (or current I) appears across (or flows in) a resistance whose value is R. The corresponding power dissipated, P, is then V^2/R or I^2R. We can similarly relate the reference voltage or reference current to the reference power as

$$P_{reference} = (V_{reference})^2/R \text{ or } P_{reference} = (I_{reference})^2R..$$

Hence

$$\text{Voltage V in dB} = 20 \log_{10}(V/V_{reference})$$
$$\text{Current I in dB} = 20 \log_{10}(I/I_{reference})$$

Similarly, the power, voltage or current gain of an amplifier or a filter can be represented in terms of input and output quantities as

$$\text{Power gain} = 10 \log_{10}(P_{out}/P_{in})$$
$$\text{Voltage gain} = 20 \log_{10}(V_{out}/V_{in})$$
$$\text{Current gain} = 20 \log_{10}(I_{out}/Iin).$$

[E.g., a voltage amplifier whose output voltage is 1000 times its input voltage has a voltage gain of $20 \log_{10}(1000)$ = 60 dB, and a filter whose output voltage is 1/10 of its input voltage has a voltage gain of -20 dB (or a voltage loss of 20 dB (or a).]