## Op-Amps Experiment Theory

## 1. Objective

The purpose of these experiments is to introduce the most important of all analog building blocks, the operational amplifier ("op-amp" for short). This handout gives an introduction to these amplifiers and a smattering of the various configurations that they can be used in. Apart from their most common use as amplifiers (both inverting and non-inverting), they also find applications as buffers (load isolators), adders, subtractors, integrators, logarithmic amplifiers, impedance converters, filters (low-pass, high-pass, band-pass, band-reject or notch), and differential amplifiers. So let's get set for a fun-filled adventure with op-amps!

## 2. Introduction: Amplifier Circuit

Before jumping into op-amps, let's first go over some amplifier fundamentals. An amplifier has an input port and an output port. (A port consists of two terminals, one of which is usually connected to the ground node.) In a linear amplifier, the output signal $=A \times$ input signal, where $A$ is the amplification factor or "gain." Depending on the nature of the input and output signals, we can have four types of amplifier gain: voltage gain (voltage out / voltage in), current gain (current out / current in), transresistance (voltage out / current in) and transconductance (current out / voltage in). Since most op-amps are voltage/voltage amplifiers, we will limit the discussion here to this type of amplifier. The circuit model of an amplifier is shown in Figure 1 (center dashed box, with an input port and an output port). The input port plays a passive role, producing no voltage of its own, and is modelled by a resistive element $R_{i}$ called the input resistance. The output port is modeled by a dependent voltage source $A V_{i}$ in series with the output resistance $R_{o}$, where $V_{i}$ is the potential difference between the input port terminals. Figure 1 shows a complete amplifier circuit, which consists of an input voltage source $V_{s}$ in series with the source resistance $R_{s}$, and an output "load" resistance $R_{L}$. From this figure, it can be seen that we have voltage-divider circuits at both the input port and the output port of the amplifier. This requires us to re-calculate $V_{i}$ and $V_{o}$ whenever a different source and/or load is used:

$$
\begin{equation*}
V_{i}=\left(\frac{R_{i}}{R_{s}+R_{i}}\right) V_{s} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
V_{o}=\left(\frac{R_{L}}{R_{o}+R_{L}}\right) A V_{i} \tag{2}
\end{equation*}
$$



Figure 1: Circuit model of an amplifier circuit.

## 3. The Operational Amplifier: Ideal Op-Amp Model

The amplifier model shown in Figure 1 is redrawn in Figure 2 showing the standard op-amp notation. An op-amp is a "differential to single-ended" amplifier, i.e. it amplifies the voltage difference $V_{p}-V_{n}=V_{i}$ at the input port and produces a voltage $V_{o}$ at the output port that is referenced to the ground node of the circuit in which the op-amp is used.


Figure 2: Standard op-amp


Figure 3: Ideal op-amp

The ideal op-amp model was derived to simplify circuit analysis and is commonly used by engineers for first-order approximation calculations. The ideal model makes three simplifying assumptions:

> Gain is infinite: $A=\infty$
> Input resistance is infinite: $R_{i}=\infty$

## Output resistance is zero: $R_{o}=0$

Applying these assumptions to the standard op-amp model results in the ideal op-amp model shown in Figure 3. Because $R_{i}=\infty$ and the voltage difference $V_{p}-V_{n}=V_{i}$ at the input port is finite, the input currents are zero for an ideal op-amp:

$$
\begin{equation*}
i_{n}=i_{p}=0 \tag{6}
\end{equation*}
$$

Hence there is no loading effect at the input port of an ideal op-amp:

$$
\begin{equation*}
V_{i}=V_{s} \tag{7}
\end{equation*}
$$

In addition, because $R_{o}=0$, there is no loading effect at the output port of an ideal op-amp:

$$
\begin{equation*}
V_{o}=A \times V_{i} \tag{8}
\end{equation*}
$$

Finally, because $\mathrm{A}=\infty$ and $V_{o}$ must be finite, $V_{i}=V_{p}-V_{n}=0$, or

$$
\begin{equation*}
V_{p}=V_{n} \tag{9}
\end{equation*}
$$

Note: Although Equations 3-5 constitute the ideal op-amp assumptions, Equations 6 and 9 are used most often in solving op-amp circuits.


Figure 4a: Non-inverting amplifier


Figure 4b: Voltage transfer curve of non-inverting amplifier


Figure 5b: Voltage transfer curve of voltage follower


Figure 6b: Voltage transfer curve of inverting amplifier


Figure 4c: Realistic transfer curve of non-inverting amplifier


Figure 5c: Realistic transfer curve of voltage follower


Figure 6c: Realistic transfer curve
of inverting amplifier

## 4. Non-Inverting Amplifier

An ideal op-amp by itself is not a very useful device, since any finite non-zero input signal would result in infinite output. (For a real op-amp, the range of the output signal is limited by the positive and negative power-supply voltages.) However, by connecting external components to the ideal opamp, we can construct useful amplifier circuits.

Figure 4a shows a basic op-amp circuit, the non-inverting amplifier. The triangular block symbol is used to represent an ideal op-amp. The input terminal marked with a " + " (corresponding to $V_{p}$ ) is called the non-inverting input; the input terminal marked with a "-" (corresponding to $V_{n}$ ) is called the inverting input.

To understand how the non-inverting amplifier circuit works, we need to derive a relationship between the input voltage $V_{\text {in }}$ and the output voltage $V_{\text {out }}$. For an ideal op-amp, there is no loading effect at the input, so

$$
\begin{equation*}
V_{p}=V_{i} \tag{10}
\end{equation*}
$$

Since the current flowing into the inverting input of an ideal op-amp is zero, the current flowing through $R_{1}$ is equal to the current flowing through $R_{2}$ (by Kirchhoff's Current Law -- which states that the algebraic sum of currents flowing into a node is zero -- to the inverting input node). We can therefore apply the voltage-divider formula find $V_{n}$ :

$$
\begin{equation*}
V_{n}=\left(\frac{R_{1}}{R_{1}+R_{2}}\right) V_{\text {out }} \tag{11}
\end{equation*}
$$

From Equation 9, we know that $V_{i n}=V_{p}=V_{n}$, so

$$
\begin{equation*}
V_{\text {out }}=\left(1+\frac{R_{2}}{R_{1}}\right) V_{\text {in }} \tag{12}
\end{equation*}
$$

The voltage transfer curve ( $V_{\text {out }}$ vs. $V_{\text {in }}$ ) for a non-inverting amplifier is shown in Figure 4b. Notice that the gain $\left(V_{\text {out }} / V_{\text {in }}\right)$ is always greater than or equal to one.

The special op-amp circuit configuration shown in Figure 5a has a gain of unity, and is called a "voltage follower." This can be derived from the non-inverting amplifier by letting $R_{1}=\infty$ and $R_{2}=$ 0 in Equation 12. The voltage transfer curve is shown in Figure 5b. A frequently asked question is why the voltage follower is useful, since it just copies input signal to the output. The reason is that it isolates the signal source and the load. We know that a signal source usually has an internal series resistance ( $R_{s}$ in Figure 1, for example). When it is directly connected to a load, especially a heavy (high conductance) load, the output voltage across the load will degrade (according to the voltagedivider formula). With a voltage-follower circuit placed between the source and the load, the signal source sees a light (low conductance) load -- the input resistance of the op-amp. At the same time, the load is driven by a powerful driving source -- the output of the op-amp.

## 5. Inverting Amplifier

Figure 6a shows another useful basic op-amp circuit, the inverting amplifier. It is similar to the noninverting circuit shown in Figure 4 a except that the input signal is applied to the inverting terminal via $R_{1}$ and the non-inverting terminal is grounded. Let's derive a relationship between the input voltage $V_{\text {in }}$ and the output voltage $V_{\text {out }}$. First, since $V_{n}=V_{p}$ and $V_{p}$ is grounded, $V_{n}=0$. Since the current flowing into the inverting input of an ideal op-amp is zero, the current flowing through $R_{1}$ must be equal in magnitude and opposite in direction to the current flowing through $R_{2}$ (by Kirchhoff's Current Law):

$$
\begin{equation*}
\frac{V_{\text {in }}-V_{n}}{R_{1}}=\frac{V_{\text {out }}-V_{n}}{R_{2}} \tag{13}
\end{equation*}
$$

Since $V_{n}=0$, we have:

$$
\begin{equation*}
V_{\text {out }}=\left(-\frac{R_{2}}{R_{1}}\right) V_{\text {in }} \tag{14}
\end{equation*}
$$

The gain of inverting amplifier is always negative, as shown in Figure 6 b.

## 6. Operation Circuit

Figure 7 shows an operation circuit, which combines the non-inverting and inverting amplifier. Let's derive the relationship between the input voltages and the output voltage $V_{\text {out }}$. We can start with the non-inverting input node. Applying Kirchhoff's Current Law, we obtain:

$$
\begin{equation*}
\frac{V_{B 1}-V_{p}}{R_{B 1}}+\frac{V_{B 2}-V_{p}}{R_{B 2}}+\frac{V_{B 3}-V_{p}}{R_{B 3}}=\frac{V_{p}}{R_{B}} \tag{15}
\end{equation*}
$$

Applying Kirchhoff's Current Law to the inverting input node, we obtain:

$$
\begin{equation*}
\frac{V_{A 1}-V_{n}}{R_{A 1}}+\frac{V_{A 2}-V_{n}}{R_{A 2}}+\frac{V_{A 3}-V_{n}}{R_{A 3}}=\frac{V_{n}-V_{\text {out }}}{R_{A}} \tag{16}
\end{equation*}
$$

Since $V_{n}=V_{p}$ (from Equation 9), we can combine Equations 15 and 16 to get

$$
\begin{equation*}
V_{\text {out }}\left(\frac{R_{A^{\prime}}}{R_{A}}\right)=\left(\frac{V_{B 1}}{R_{B 1}}+\frac{V_{B 2}}{R_{B 2}}+\frac{V_{B 3}}{R_{B 3}}\right) R_{B^{\prime}}-\left(\frac{V_{A 1}}{R_{A 1}}+\frac{V_{A 2}}{R_{A 2}}+\frac{V_{A 3}}{R_{A 3}}\right) R_{A^{\prime}} \tag{17}
\end{equation*}
$$

where

$$
R_{A^{\prime}}=\frac{1}{\frac{1}{R_{A}}+\frac{1}{R_{A 1}}+\frac{1}{R_{A 2}}+\frac{1}{R_{A 3}}}
$$

and

$$
R_{B^{\prime}}=\frac{1}{\frac{1}{R_{B}}+\frac{1}{R_{B 1}}+\frac{1}{R_{B 2}}+\frac{1}{R_{B 3}}}
$$

Thus this circuit adds $V_{B 1}, V_{B 2}$ and $V_{B 3}$ and subtracts $V_{A 1}, V_{A 2}$ and $V_{A 3}$. Different coefficients can be applied to the input signals by adjusting the resistors. If all the resistors have the same value, then

$$
\begin{equation*}
V_{\text {out }}=\left(V_{B 1}+V_{B 2}+V_{B 3}\right)-\left(V_{A 1}+V_{A 2}+V_{A 3}\right) \tag{18}
\end{equation*}
$$



Figure 7: Operation circuit

## 7. Integrator

By adding a capacitor in parallel with the feedback resistor $R_{2}$ in an inverting amplifier as shown in Figure 8, the op-amp can be used to perform integration. An ideal or lossless integrator ( $R_{2}=\infty$ ) performs the computation $V_{\text {out }}=\frac{-1}{R_{1} C} \int V_{\text {in }} d t$. Thus a square wave input would cause a triangle wave output. However, in a real circuit $\left(R_{2}<\infty\right)$ there is some decay in the system state at a rate proportional to the state itself. This leads to exponential decay with a time constant of $\tau=R_{2} C$.


Figure 8: Integrator

## 8. Differentiator

By adding a capacitor in series with the input resistor $R_{1}$ in an inverting amplifier, the op-amp can be used to perform differentiation. An ideal differentiator $\left(R_{1}=0\right)$ has no memory and performs the computation $V_{\text {out }}=-R_{2} C \frac{d V_{\text {in }}}{d t}$. Thus a triangle wave input would cause a square wave output. However, a real circuit $\left(R_{1}>0\right)$ will have some memory of the system state (like an lossy integrator) with exponential decay of time constant $\tau=R_{1} C$.

## 9. Differential Amplifier

Figure 9 shows the differential amplifier circuit. As the name suggests, this op-amp configuration amplifies the difference of two input signals.

$$
\begin{equation*}
V_{\text {out }}=\left(V_{+}-V_{-}\right) \frac{R_{2}}{R_{1}} \tag{20}
\end{equation*}
$$

If the two input signals are the same, the output should be zero, ideally. To quantify the quality of the amplifier, the term Common Mode Rejection Ratio (CMRR) is defined. It is the ratio of the
output voltage corresponding to the difference of the two input signals to the output voltage corresponding to "common part" of the two signals. A good op-amp has a high CMRR.


Figure 9: Differential amplifier

## 10. Frequency Response of Op-Amp

The "frequency response" of any circuit is the magnitude of the gain in decibels ( dB ) as a function of the frequency of the input signal. The decibel is a common unit of measurement for the relative loudness of a sound or, in electronics, for the relative magnitude of two power levels. (A decibel is one-tenth of a "Bel", a seldom-used unit named for Alexander Graham Bell, inventor of the telephone.) The gain expressed in dB is $20 \log _{10}|\mathrm{G}|$. The frequency response of an op-amp is a low pass characteristic (passing low-frequency signals, attenuating high-frequency signals), Figure 10.


Figure 10: Frequency response of op-amp.

The bandwidth is the frequency at which the power of the output signal is reduced to half that of the maximum output power. This occurs when the power gain $G$ drops by 3 dB . In Figure 10, the
bandwidth is B Hz . For all op-amps, the Gain*Bandwidth product is a constant. Hence, if the gain of an op-amp is decreased, its operational bandwidth increases proportionally. This is an important trade-off consideration in op-amp circuit design. In Sections 3 through 8 above, we assumed that the op-amp has infinite bandwidth.

## 10. More on Op-Amps

All of the above op-amp configurations have one thing in common. There exists a path from the output of the op-amp back to its inverting input. When the output is not "railed" to a supply voltage, negative feedback ensures that the op-amp operates in the linear region (as opposed to the saturation region, where the output voltage is "saturated" at one of the supply voltages). Amplification, addition/subtraction, and integration/differentiation are all linear operations. Note that both AC signals and DC offsets are included in these operations, unless we add a capacitor in series with the input signal(s) to block the DC component.

## 11. The LMC 6482 Operational Amplifier

Guidelines for usage:

- Two power-supply voltages (one positive, one negative) must be connected to the LM741. The input voltage should fall within the range between the two power-supply voltages.
- The power-supply voltages limit the output voltage swing. The voltage-transfer characteristics of the non-inverting amplifier, the voltage follower and the inverting amplifier are illustrated in Figures 4c, 5 b and 5 c , respectively. The output swing also depends on the load resistor $R_{L}$. In order for the LMC 6482 to work like an ideal op-amp, don't connect too heavy a load (resistor of low resistance) to it. Avoid continuous operation under maximum current output; otherwise it can burn your finger!
- The input resistance of the LMC 6482 is not tremendously large, so resistors like $R_{1}$ and $R_{2}$ shown in Figure 4 and 6 should not be too large. Of course they should not be too small, since neither the external signal source nor the LMC 6482 itself may have sufficient current driving capacity.


## LMC 6482 pins:

The op-amp is packaged in an epoxy mini-DIP ("dual inline package") which has 4 pins emerging from each of the two sides of the package. The most important part of the identification is to note that Pin 1 is marked by a small circular indentation in the lower left-hand corner of the package when viewed from the top side with the manufacturer's marking reading upright. The pin numbers on ICs always start with Pin 1 being this labeled pin and increase counter-clockwise as read from the top side of the package. Thus on this particular package, Pin 8 is directly across from Pin 1, and Pin 5 is diagonally across from Pin 1.

You will need to use only 5 of the 8 pins. These are: Inverting Input - Pin2, Non-Inverting Input Pin3, Negative Supply - Pin4, Output - Pin6 and Positive Supply - Pin7. Other pins are available for an optional offset null adjustment (Pins $1 \& 5$ ), which is unnecessary for the circuits used in this experiment. Do not ground these pins, as this will result in failure of the op-amp. Pin 8 has no internal connection into the silicon chip inside.

## Op-Amps: Experiment Guide

In this lab, we are going to study operational amplifiers and circuits with op-amps. The op-amp chip that we are going to use is LMC 6482 from National Instrument. The configuration of the chip is shown below. It has two amplifiers in one chip with 8 pins. The pin configuration is also shown in the same figure (There is a node on the chip indicating pin 1). The power supply to the chip is -4 V for $\mathrm{V}^{-}$and +6 V for $\mathrm{V}^{+}$in this lab (Maximum $\mathrm{V}^{+}-\mathrm{V}^{-}$is 30 V ). For more information, please refer to the device specification.


## Part 1: Noninverting Amplifier

## (a) DC measurements:

(1) Build up the noninverting amplifier as shown in Fig 1. Use +25 V channel and -25 V channel of the DC power supply for the $\mathrm{V}_{\mathrm{DD}}$ and $\mathrm{V}_{\mathrm{SS}}$, the +25 V should be set up to +6 V and -25 V channel should be set up to -4 V . Use 6 V channel of the DC power supply for $\mathrm{V}_{\mathrm{in}}$, and measure both input and output using oscilloscope. $R_{1}$ is $5 k$ and $R_{2}$ is $5 k$. Change $V_{\text {in }}$ from 2 V to 3 V to verify the proper amplification range of DC inputs.
(2) Fix the DC input 0.5 V , measure the amplifier gain $\left(\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}\right)$ for $\mathrm{R}_{2}=2 \mathrm{k}, 5 \mathrm{k}, 10 \mathrm{k} \Omega\left(\right.$ turn $\left.\mathrm{R}_{2}\right)$ and compare with the calculated gain. (You need to take out the pot from the circuit to measure its value.)

## (b) AC measurement:

(1) Now, set the input signal to a $1 \mathrm{kHz}, 0.5 \mathrm{~V}_{\mathrm{PP}}, 0 \mathrm{VDC}$ offset (on the function generator display) Sine wave from the function generator. Use a $10 \mathrm{k} \Omega$ potentiometer as $\mathrm{R}_{2}$. Adjust $\mathrm{R}_{2}$ to see the gain change. Can you get a gain less than unity by turning $\mathrm{R}_{2}$ ? Why?
(2) Turn the potentiometer $\mathrm{R}_{2}$ until the gain is 2 and then adjust the Vpp and DC offset to the input signal. Observe the input and output waveforms as you vary the DC offset for large Vpp (say 2.5 V ).

## (c) Clipping:

(1) Turn the potentiometer until the gain is $\sim 2$ and then add a DC offset to the input signal. Observe the input and output waveforms as you vary the DC offset from 0VDC to +1.0 VDC (the FG should display 0 VDC to +0.5 VDC ).
(2) Draw the input and output for a case that gives clipping, and include measured voltages of the peak, trough, and saturation levels referenced to ground. Explain the conditions under which distortion occurred.

## Do not disassemble this circuit. Remove Vi and change $\mathbf{R}_{\mathbf{2}}$ to a fixed 4.7k to be used in Part 2.



Fig 0: Grounding Unused Op-amps


Fig 1:Non-inverting Amplifier

## Part 2: Inverting Amplifier

Using the unused op-amp of the IC, build the inverting amplifier as shown below (please use the unused op-amp now). While you are building a circuit, it is safer for the circuits if you turn the DC power supply OUTPUT OFF. Let the input signal be a $1 \mathrm{kHz}, 1.0 \mathrm{~V}_{\mathrm{PP}}$ sine wave, 0 VDC offset, turn R2 to max. What's happening to the output signal? Adjust the input offset to make the output more complete. Now adjust the potentiometer and observe the resulting change in the amplitude and offset of the output. Adjust these two parameters until the gain is at its maximum and there's no clipping. What range of gain do you have in this circuit? Verify the correct amplification of both AC and DC signals. What is the phase difference between Vout and Vin and where is it from? Is it a delay?


## Part 3: Cascaded connection

Now we will study a cascade connection of two amplifiers. Connect the output of the inverting amp to act as the input voltage for the non-inverting amp. Use $\mathrm{R}_{2}=10 \mathrm{k}$ in the inverting circuit and $\mathrm{R}_{2}{ }^{\prime}=5 \mathrm{k}$ in the noninverting circuit. The input signal should be a $1 \mathrm{kHz}, 50 \mathrm{mV} \mathrm{PP}^{\text {(on the function }}$ generator display) sine wave and you have to pick the correct offset for the circuit to amplify linearly. Adjust the input signal to make sure there is no clipping in the circuit. Measure the gain of each stage separately and then the overall gain of this cascaded circuit.

## Part 4: Integrator

Put a 0.1 uF capacitor instead of $\mathrm{R}_{2}$ in a new inverting amplifier (Fig 3) and measure the time constant. Use a $60 \mathrm{~Hz}, 500 \mathrm{mV}$ PP square wave as input. After getting the waveforms and triggering correct, measure time constant RC (how will you measure it? Hint: your prelab question 4).

Compare measured time constant with theory. Now change the function generator back to a sine wave input, sweep frequency from 1 Hz to 100 kHz and observe the change of the gain with frequency.

## Note on op-amp integrator

The circuit in figure 4 violates one of the cardinal rules of op-amp circuit design - "there must always be a DC feedback path to the inverting input or the op-amp output will go to the rail." The general problem with this integrator circuit is that a small error current, input offset current, will be integrated by the capacitor to be large output voltages, and eventually drive the op-amp output into saturation. The LMC $6482 \mathrm{op-amp}$ you are using has remarkably low input offset currents, so that you may not see this effect in a short time. If you want to see this effect, ask your TA for another pin-compatible op-amp such as the LM6142, substitute in the integrator circuit, and see if you observe any difference in the average DC level of the output. (Typically, a real integrator is made with a zero-reset, or a large resistor in parallel with the integrator capacitor).

## Part 5: Differentiator

Build the inverting amplifier but put 0.1 uF capacitor in stead of $\mathrm{R}_{1}$ as shown in Fig 4. Use $\mathrm{R}_{2}=5 \mathrm{k}$ Input a 500 Hz 500 mVpp triangle wave. Zoom into the waveform to measure time constant RC (Hint: prelab question 5). Compare measured time constant with theory. Add DC offset to the input signal, is there any change on the output signal? Why? What happens when the input is a triangle wave?

