Name: ______________________________

UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering
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Professor David Tse Spring 2000

EECS 121 — FINAL EXAM

17 May 2000, 8:00-11:00 a.m.

Please write answers on blank pages only. Answer all 8 questions. Clear justifications of answers are needed. You can express the answers in terms of the Q-function.

Problem 1 (8 points)

Consider a binary source with letter probabilities p and $1 - p$. The symbols are independent

[2 pts.] **a)** Plot the entropy as a function of p.

[2 pts.] **b)** For what values of p is the entropy maximized? Minimized?

[2 pts.] **c)** Explain precisely the significance of entropy in the context of the data compression problem.

[2 pts.] **d)** Do your answers to b) make intuitive sense in the context of the data compression problem?

Problem 2 (8 points)

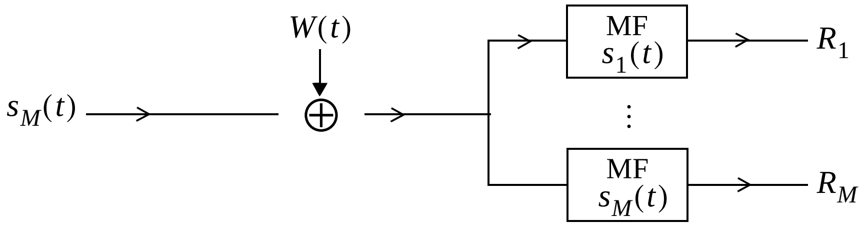
Consider a multi-hypothesis detection problem under additive white Gaussian noise.

[5 pts.] **a)** Give the structure of the Optimal Maximum Likelihood detector. Sketch the decision regions.

[3 pts.] **b)** Using your figure, explain the difference between the actual probability of error and the union bound.

Problem 3 (8 points)

A modulation scheme uses M waveforms $s_1(t), \ldots, s_M(t)$, which are transmitted via an AWGN channel. Suppose at the receiver we use a bank of matched filters, matched to the waveforms:



[2 pts.] **a)** Does $R_1, \ldots, R_M$ provide a set of sufficient statistics for the problem?

[3 pts.] **b)** Is the noise in $R_1, \ldots, R_M$ independent?

[3 pts.] **c)** Suppose $s_M(t)$ can be expressed as a linear combination of $s_1(t), \ldots, S_{M-1}(t)$. Does R_M provide further information beyond what is in $R_1, \ldots, R_{M-1}$ for the detection problem?

Problem 4 (8 points)

A source with dependent symbols has to be quantized.

[4 pts.] **a)** Is vector quantization better than scalar quantization?

[4 pts.] **b)** How about if the symbols are independent?

Explain your answers to both parts a) and b). Use figures when appropriate.

Problem 5 (8 points)

In a carrier modulated system, the additional SNR per bit to get one extra bit in PAM and PSK is about 6 dB, while for rectangular QAM is about 3 dB.

[3 pts.] **a)** Explain precisely what these statements mean.

[5 pts.] **b)** Explain why they are true in terms of minimum distance properties of the constellations.

Problem 6 (20 points)

IS-95 is the CDMA standard (Code-Division Multiple Access) for digital wireless cellular communications. Some parameters of the system:

carrier frequency	=	900 MHz
bandwidth	=	1.25 MHz
vehicle speed	=	up to 40 m/s
modulation scheme	=	4 PSK (QPSK)

Consider first the situation when the channel consists of a single direct signal path from the transmitter to the receiver.

[2 pts.] a) What is the maximum data rate that can be supported if ISI is to be avoided?

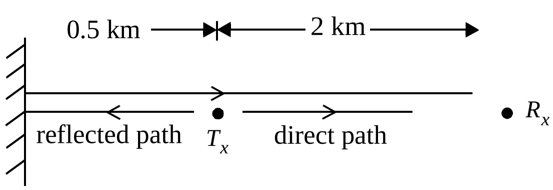
[2 pts.] b) What is the maximum data rate that can be supported if ISI is to be avoided and a raised cosine transmit filter with a roll-off factor of 10% is to be used?

[4 pts.] c) What is the SNR required to achieve a symbol error probability of 10^{-5}?

[4 pts.] d) The carrier phase rotates due to the motion of the transmitter. What is the fastest rate it can change?

[4 pts.] e) Based on your answers to parts a) and d), do you think it's easy to estimate and track the carrier phase? Explain. If so, give one technique to do it.

[2 pts.] f) Now suppose that in addition to the direct path, there is also a second path due to reflection, as shown:

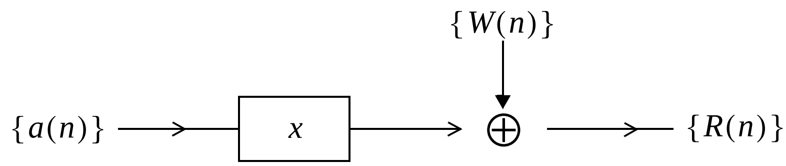


The *delay spread* of the channel is the time difference between the arrivals of the first and second paths at the receiver. Assuming the data rate in (a), what is the delay spread in terms of number of symbol times? Is the ISI introduced by the reflected path significant?

[2 pts.] g) Suppose we are operating in an indoor environment, and the distances in f) are 0.5 m and 2 m instead of 0.5 km and 2 km. Answer part f) again.

Problem 7 (20 points)

A discrete-time approximation for an ISI channel is as follows:



Here, $\{a(n)\}$ is the sequence of transmitted symbols, $a(n) = \pm 1$ equally likely; x is a 3-tap discrete-time causal impulse response of the combination of the transmit filter, channel response, and receiver filter and a symbol-rate sampler; $R(n)$ is the output of the sampler at time n; $\{W(n)\}$ is white Gaussian noise of psd $N_0 / 2$.

[4 pts.] **a)** This is only an approximate model of the underlying continuous time system, as $\{W(n)\}$ may not be white. Why?

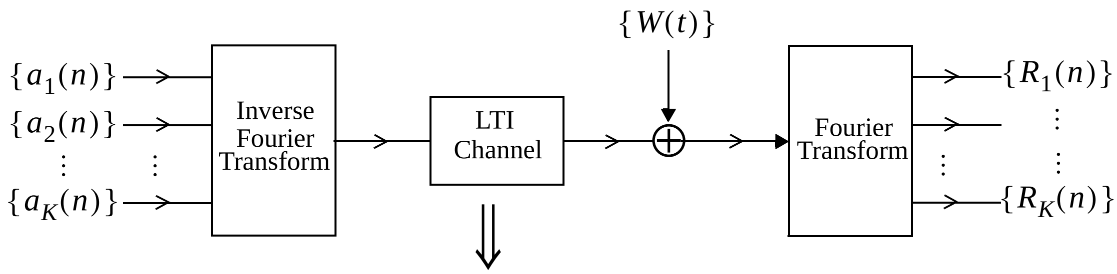
[6 pts.] **b)** Suppose we want to use a 3-tap equalizer to equalize $\{R(n)\}$. Can we achieve the perfect zero-forcing condition? If so, compute the zero-forcing equalizer. If not, find a good approximation. Derive an approximate expression for the probability of error if a symbol-by-symbol detector is used at the output of the equalizer.

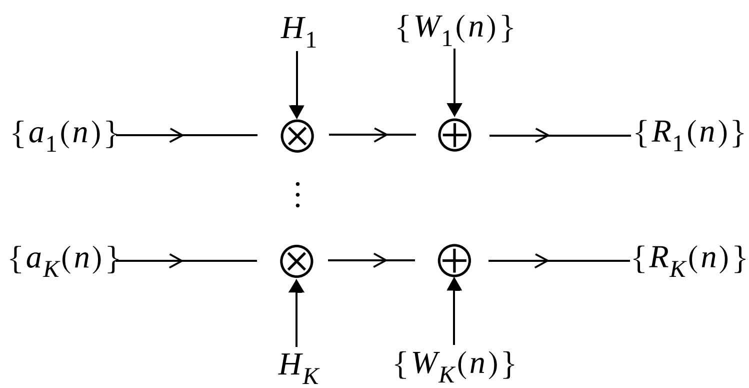
[6 pts.] **c)** Develop the optimal sequence detector based on $\{R(n)\}$. Draw the relevant state diagram and trellis. Describe the algorithm. Give an estimate of the complexity of the algorithm.

[4 pts.] **d)** Suppose $x(0) = 3, x(1) = -2, x(2) = 1$. Estimate the probability of sequence detection error in part (c).

Problem 8 (20 points)

One approach towards communication over ISI channels is a *multi-carrier system* (also called OFDM), as shown:





By appropriate signal processing at the transmitter and at the receiver, the ISI channel is converted into K AWGN channels, such that the k^{th} channel (called a subcarrier) is given by

$$R_k(n) = H_k a_k(n) + W_k(n)$$

where:

$a_k(n)$ = transmitted symbol on k^{th} subcarrier at time n ($a_k(n) = \pm 1$)

$R_k(n)$ = received symbol on k^{th} subcarrier at time n

$W_k(n)$ = white Gaussian noise $\sim N\left(0, \frac{N_0}{2}\right)$, independent over all n and k

H_k = time-invariant channel gain for the k^{th} subcarrier (assumed real)

A subcarrier is associated with a frequency f; H_k is the gain at frequency f.

A key component of an OFDM is the *channel estimator* to estimate the channel gains $H_1, \ldots, H_k$. We model each of the H_k's as mean zero, variance σ^2 rv's. One way of estimating H_k's are through the use of *training sequences*.

[6 pts.] **a)** First, we look at the problem of estimating H_k on each subcarrier separately. Focus on subcarrier 1 for the moment.

i) Suppose we transmit a training symbol $a_1(1) = +1$ at time t. Find the LLSE estimate of H_1.

ii) Suppose we transmit T training symbols $a_1(1) = a_1(2) = \dots a_1(T) = +1$. Find the LLSE estimate of H_1.

iii) What happens as $T \to \infty$?

[6 pts.] b) Suppose now that the H_k's are correlated, such that

$$E[H_i H_j] = \sigma^2 \rho^{|i-j|}$$

At time 1, we send a training symbol $a_k(1) = +1$ on all subcarriers. Is there any advantage of estimating $H_1, \dots, H_k$ *jointly* from $R_1(1), R_2(1), \dots, R_k(1)$? Explain. If so, derive a good estimator.

[5 pts.] c) In practice, only a certain subset of the subcarriers are reserved for training. Suggest a way to estimate *all* the channel gains. For the same performance, explain intuitively how the number of subcarriers reserved for training should depend on the parameter ρ?

[3 pts.] d) Is it a good idea to reserve a subset of carriers to transmit *only* training symbols at *all* times? Can you think of a better way?