EE 121: Introduction to Digital Communication Systems

Final Exam Spring 2003

- Please put your name on the front page.
- Answer all the questions on the blank pages provided.
- Explain clearly the steps in your answers. Yes/no answers with no explanations get no marks.
- Waterfall error probability curves are provided at the back for your use throughout the exam. Please reference the appropriate plots when you use them. (You may want to take a look at them now.)

[10] 1.

[6] a). Explain briefly what these concepts mean:

i) entropy

ii) uniquely decodable codes

iii) Nyquist sampling theorem

iv) Nyquist criterion

v) bi-orthogonal modulation

vi) ISI

[4] b) Is there any relationship between the delay spread and coherence bandwidth in a wireless channel? How about between delay spread and coherence time? For each case, if there is a relationship, explain. If not, give an example of a wireless channel in which the two parameters can be specified separately.

[10] 2. Suppose you have calculated the transmit power link budget for a wireless system, assuming

- uncoded binary pulse amplitude modulation (binary PAM, also known as BPSK)
- an error probability of 10^{-4}
- transmission over an AWGN channel.

[5] a) Suppose now there is flat Rayleigh fading and coherent detection is performed. How much extra transmit power is needed? (This is called the *fade margin.*) State clearly how you come up with your answer.

[5] b) Suppose now each symbol is repeated twice and interleaved over different coherence time periods. Compared to (a), how much is the fade margin reduced by?

[15] 3.

A passband communication system uses ideal sinc pulses and signal at Nyquist rate.

[3] a) What is the difference, in terms of symbols transmitted in the in-phase and quadrature-phase, between BPSK (binary phase-shift keying, also known as binary PAM) and QPSK in such a system? Draw a system diagram for each.

[4] b) Compare the spectral efficiency (in bits/s/Hz) and E_b/N_0 performance of BPSK and QPSK over an AWGN channel, for the same *symbol* error probability p_e. (You can assume that p_e is small in making any approximations. Also, you may find the approximation $Q(x) \sim \exp(-x^2/2)$ useful.)

[3] c) Suppose the in-phase and quadrature-phase components of each QPSK symbol carry two independent information bits . Repeat the comparison in part (a) between BPSK and QPSK but now for the same *bit* error probability.

[5] d) Consider the discrete-time baseband equivalent flat Rayleigh fading channel:

$$y_n = h_n x_n + w_n,$$

where $w_n \sim \mathcal{CN}(0, N_0)$ and $h_n \sim \mathcal{CN}(0, 1)$. In class, we have stated that for coherent BPSK, the symbol error probability is:

$$p_e = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}}\right).$$

Using part (c) or otherwise, compute the probability of bit error for coherent QPSK over the Rayleigh flat fading channel. assuming again that the in-phase and quadrature phases carry independent information bits. Again, what is the difference in E_b/N_0 performance for the same bit error probability?

[10] 4. Consider a wireless channel with only one line-of-sight path which arrives after a delay of τ. Communication is over a passband at carrier frequency $f_c = 1900$ MHz and bandwidth $W = 1$ MHz. Assume ideal sinc functions are used as transmit pulses.

[4] a) The optimal demodulator matches filter to the transmit waveform and sample at the correct times. Since τ is unknown in practice, it has to be estimated so that the sample times can be determined, a process known as *timing recovery* or *symbol synchronization.* In terms of the given system parameters, give an order-of-magnitude approximation of how accurate τ have to be estimated for proper timing recovery.

[5] b) Since we are communicating on a passband, the delay τ will also cause a phase rotation of the baseband transmitted signal. To do *coherent* detection, this phase has to be estimated as well, a process known as *carrier recovery.* In terms of the given system paramters, give an order-of-magnitude approximation of how accurate τ have to be estimated for proper carrier recovery.

[1] c) In practice τ varies as the mobile moves, so τ have to be continuous tracked for ongoing timing and carrier recovery. From your answers in part (a) and (b), which do you think it's a more difficult problem, timing or carrier recovery? Explain.

[27] 5.

I. A sampled complex baseband-equivalent channel:

$$y_n = hx_n + w_n,$$

where $h \sim \mathcal{CN}(0,1)$ and $\{w_n\}$ is a sequence of independent $\mathcal{CN}(0, N_0)$ noise.

[2] a) What are the assumptions on the passband physical channel for which this would a reasonable baseband model? Why is it reasonable to model the channel gain h as $\mathcal{CN}(0,1)$?

[1] b) Suppose h is unknown to the receiver. Can you transmit information using BPSK? How about QPSK? Why?

[4] c) Design a scheme such that the receiver can demodulate without knowing h. Give the receiver structure. Give an expression for the error probability.

II. Consider now another sampled baseband equivalent channel, given by:

$$y_n = e^{j\theta} x_n + w_n,$$

where $w_n \sim \mathcal{CN}(0, N_0)$, independent over time, and θ is a random variable uniformly distributed in $[0, 2\pi]$ independent of the additive noise.

[3] a) Give an example of a passband physical channel for which this is a reasonable baseband model. What is the interpretation of θ? Why is it reasonable to model θ as uniformly distributed in $[0, 2\pi]$?

[1]b) Suppose θ is unknown to the receiver. Can you transmit information using BPSK? How about QPSK? Why?

[4] c) Design a scheme such that the receiver can demodulate without knowing θ. Give the receiver structure. Give an expression for the error probability. It's ok if it's not in closed form.

[4] d) Qualitatively compare the performance in part I c) and part II c). Which channel is harder to communicate over? Explain.

III. Let us now consider yet another baseband channel:

$$\begin{aligned} y_n^A &= h_A x_n + w_n^A \\ y_n^B &= h_B x_n + w_n^B \end{aligned}$$

where $h_A, h_B \sim \mathcal{CN}(0,1)$ are independent, and w_n^A, w_n^B are independent additive $\mathcal{CN}(0, N_0)$ noise, independent of h_A and h_B.

[2] a) Give an example of a physical scenario for which this is a reasonable channel model. In your scenario, under what condition is it reasonable to model the channel gains h_A h_B as independent?

[5] b) Design a scheme that can exploit both the received signals $\{y_n^A\}$ and $\{y_n^B\}$ in detecting the transmitted symbols, but without needing to know the gains h_A and h_B.

[3] c) Qualitatively compare the performance of this scheme with that in part I c). Explain the nature of the performance improvement, if any.

[28] 6. Communication is done over the baseband $[-W, W]$ through an M-ary orthogonal code in conjunction with binary PAM modulation of each coded symbol. The transmit pulse is a raised-cosine waveform with a roll-off factor of 10%. The orthogonal code is implemented through the Hadamard matrix H_M, defined recursively as

$$H_1 = [1] \qquad H_M = \begin{bmatrix} H_{M/2} & H_{M/2} \\ H_{M/2} & -H_{M/2} \end{bmatrix}, \quad M \geq 2.$$

The questions below refer to a system for *general* M, unless otherwise stated.

[2] a) Draw a block diagram of the transmitter and receiver for this communication system. Give the optimal receiver structure. Write down the code vectors for the example of $M = 4$.

[3] b) What is the symbol rate, data rate and spectral efficiency of this system?

[2] c) Suppose the energy in the transmit pulse for each coded symbol is E_s. Find the energy per bit E_b.

[4] d) Give an explicit upper bound on the detection error probability of the orthogonal code as a function of E_b/N_0. (You can use the fact that $Q(x) \leq \exp(-x^2/2)$.) Using your upper bound, give an upper bound on the E_b/N_0 required for a given error probability p_e. In the rest of the question, you can use this upper bound as a proxy for the actual error probability.

[4] d) Suppose now instead of the orthogonal code, we use a more naive *repetition* code where each information bit is repeated K times before performing the PAM modulation. Give the optimal receiver structure for detecting the information bits. Compute the error probability p_e as a function of E_b/N_0. Does your answer depend on K?

[2] e) Take $p_e = 10^{-3}$ and $M = 64$. Using parts (c) and (d) or otherwise: how much higher in E_b/N_0 is required for the repetition code to attain the same error probability as the orthogonal code? (This is called the *coding gain* of the orthogonal code.)

f) In the IS-95 standard , orthogonal coding is used in conjunction with repetition. Specifically information bits are coded by an M-ary orthogonal code, and each coded symbol is then repeated K times before the PAM modulation.

[2] i) Draw a block diagram for the transmitter and receiver. If we think of the composition of the orthogonal code together with repetition as a single code, write down the code vectors for the example of $M = 4$ and $K = 2$.

[2]ii) Give the optimal receiver structure.

[5] iii) Show that the performance of this system is equivalent to one using an orthogonal code but without repetition. Hence or otherwise, compute an explicit upper bound on the error probability of this system in terms of E_b/N_0, and relate the E_b/N_0 requirement to a given error probability p_e.

[2] iv) An alternative, more complex design is to use a MK-ary orthogonal code without repetition. For error probability $p_e = 10^{-3}$, $M = 64$, $K = 4$, how much performance gain in terms of E_b/N_0 requirement does this buy?