[22] 1. Short questions

[5] (a) What is the key difference between a digital and an analog communication system? What are the advantages of a digital system over an analog system?

[3](b) What is the role of the sampling theorem in a digital communication system?

[7](c)Let $A_1, \ldots A_n$ be a set of events that partition the sample space and E be another event. Is it always true that:

$$P(E) = \sum_{i=1}^{n} P(E|A_i)?$$

If true, give a proof. If false, give a counter-example.

[7] (d) State the union bound. Is it always true that the union bound holds with equality whenever the events are mutually independent? If true, give a proof. If not, give a counter-example.

[26]2. Let $X_1, X_2, \ldots$ an i.i.d. source; each source symbol takes on K values, with $P(X = x_k) = p_k$ for $k = 1, \ldots, K$.

[3](a) What is the entropy rate of this source?

[8](b) Suppose the Huffman algorithm is applied on coding X. Derive a lower bound and an upper bound on the expected codeword length, both in terms of the entropy rate of the source.

[4](c) Suppose the source is binary with probability of $X_1 = 1$ to be 0.01. What is the ratio of the expected Huffman codeword length to the entropy rate of the source?

[11](d) Compute the improvement by coding over blocks of length 2 and blocks of length 3.

[28]3. Consider an AWGN channel:

$$y[n] = x[n] + w[n]$$

where $\{w[n]\}$ is an i.i.d. sequence of $N(0, \sigma^2)$ random noise. In class we consider detection based directly on the analog outputs $y[n]$'s. In many communication systems, however, only a quantized version of the analog output is available, generated by the front-end A/D. In that case, detection is based on $Q(y[n])$ instead of $y[n]$, where $Q(\cdot)$ is a quantizer. For concreteness, we consider the 1-bit quantizer $Q(y) = \mathrm{sgn}(y)a$, where $\mathrm{sgn}(y)$ is the sign of y and a is a constant.

[3] (a) Suppose each $x[n]$ is independent BPSK, equally likely to be $+\sqrt{E}$ or $-\sqrt{E}$. Is there any loss in performance in using $Q(y[n])$ instead of $y[n]$ in the detection of $x[n]$? Explain.

[4](b) Redo (a) if $x[n]$ is more likely to be $+\sqrt{E}$ than $-\sqrt{E}$.

[4](c)Redo (a) if $x[n]$ is 4-PAM.

[17](d) Now suppose we use BPSK, equally likely to be $+\sqrt{E}$ or $-\sqrt{E}$, but in a repetition code, where each BPSK symbol is repeated 3 times. Is there any loss in performance in using the quantized outputs instead of the analog outputs themselves to perform the detection? If not, explain. If so, compute approximately the additional energy per bit (in dB) required to achieve the same target error probability because we are using quantized outputs instead of the outputs themselves? (You can assume the target error probability is small and use appropriate approximations.)

[24] 4. You are given a bandwidth of 1 MHz and a Gaussian channel with noise variance per symbol of σ^2. You want to design a communication link that delivers *at least* R bits/s with an error probability of *no more* than 10^{-4}. You can use M-ary PAM symbols, $M = 2, 4, 8$ or 16, in conjunction with a repetition code of any length k. You can also use a M-ary orthogonal code, $M = 2, 4, 8, 16, 32, 64$ or a M-ary biorthogonal code, $M = 2, 4, 8, 16, 32$. You can also use either of these codes in conjunction with a repetition code, just like in the IS-95 system (and also considered in the discussion.)

Pick a good scheme that uses small energy per bit for

[12](a) $R = 4$ Mbits/s.

[12](b) $R = 150$ kilobits/s

You can use the plots on the following page to help you. Be sure to explain clearly how you arrive at your choice and how other choices are ruled out. Also, state the E_b/σ^2 for the scheme you have chosen in each of the two cases.