EE 121 - Introduction to Digital Communications
Midterm Practice Problems on Linear Codes

March 15, 2008

1. Let $\mathbf{x}_1$ be an arbitrary code word in a forward error correction (FEC) code of block length N and let x_0 be the all-zero code word corresponding to the all-zero information sequence. Show that for each $n \leq N$, the number of code words at distance n from x_1 is the same as the number of code words at distance n from x_0.

2. Show that if a FEC code has an odd minimum weight, adding a parity check digit that is a check on every digit in the code, increases the minimum weight by 1.

3. Consider two FEC codes. Code 1 is generated by the rule

$$\begin{aligned} x_1 &= u_1 \\ x_2 &= u_2 \\ x_3 &= u_3 \\ x_4 &= u_1 \oplus u_2 \\ x_5 &= u_1 \oplus u_3 \\ x_6 &= u_2 \oplus u_3 \\ x_7 &= u_1 \oplus u_2 \oplus u_3. \end{aligned}$$

Code 2 is the same except that $x_6 = u_2$.

(a) Write down the generator matrix and parity check matrix for code 1.

(b) Write out a decoding table for code 1, assuming a BSC with crossover probability $\epsilon < 1/2$.

(c) Give an exact expression for the probability of decoding error for code 1 and for code 2. Which is larger?

(d) Find $d_{\min}$ for code 1 and for code 2.

(e) Give a counterexample to the conjecture that if one (N, L) parity check code has a larger minimum distance than another (N, L) parity check code, it has a smaller error probability on a BSC.

4. Prove or disprove: There is a binary linear code with $N = 12$, $K = 7$ and $d_{\min} = 5$.